Window And Backwards Decoding Achieve the Same Sum Rate for the Fading Cooperative Gaussian Multiple Access Channel

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***Abstract*— For a two user fading Gaussian multiple access channel with user cooperation, we show that window decoding achieves the same sum rate as backwards decoding, when the encoding is done using block Markov superposition coding. We prove this result by showing that when optimum power control is employed, the additional constraints on the sum rate imposed by the use of window decoding instead of backwards decoding are never active. While doing so, we also provide some properties of power allocation that is jointly optimal with block Markov coding and window decoding.**

I. Introduction

Due to the additive nature of the wireless channel, the signals from a set of communicating users in a wireless network are superposed onto each other. In conventional wireless network design, this fact has invariably been viewed as a problem to be avoided by means of medium access strategies. However, what is traditionally viewed as multiuser interference, is in fact free side information, and can be taken advantage of by employing clever cooperative encoding strategies.

A multiple access channel (MAC) with generalized feedback constitutes a very good model for wireless systems since it models the over-heard information by the transmitters. In particular, a two user MAC with generalized feedback [1] is described by $(\mathcal{X}_1 \times \mathcal{X}_2, P(y, y_1, y_2|x_1, x_2), \mathcal{Y} \times \mathcal{Y}_1 \times \mathcal{Y}_2)$, where user 1 has access to channel output Y_1 and user two has access to channel output Y_2. For this channel, an achievable rate region was obtained in [1], by using a superposition block Markov encoding scheme, together with backwards decoding [2], where the receiver waits to receive all B blocks of codewords before decoding.

Recently, Sendonaris, Erkip and Aazhang have employed these encoding and decoding strategies in a Gaussian MAC in the presence of fading, leading to user cooperation diversity and higher rates [3]. In this setting, the transmitters form their codewords not only based on their own information, but also on the information they have received from each other. It is assumed in [3] that channel state information (CSI) for each link is known to the corresponding receiver on that link, and also phase of the channel state is known at the transmitters so that coherent combining gain is attained. The achievable rate region is shown to improve significantly over the capacity region of MAC with non-cooperating transmitters, especially when the channels between the two users are relatively good on average.

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For a fading Gaussian MAC with perfect CSI at the transmitters and the receiver, the power control policies that are jointly optimal with block Markov coding and backwards decoding were characterized in [4]. There, it was shown that, by employing power control, block Markov coding strategy is significantly simplified, in that at any given channel state, one of the codeword components of each user has to be assigned zero power, i.e., based on the relative channel qualities, only two out of possible three components need to be transmitted. Meanwhile, the rates achievable by this simple joint power control and user cooperation strategy are shown to improve notably over channel non-adaptive schemes.

In decoding messages encoded by block Markov superposition coding, backwards decoding is not the only option. Window decoding [5] is a powerful alternative, since it needs to wait for only one block to start decoding, as opposed to the backwards decoding which requires all the blocks to be received, and starts decoding from the last block. Window decoding has been recently studied in detail in the case of relay channels [6], [7], where it was shown to achieve the same rate as backwards decoding.

Window decoding for MAC with generalized feedback was first studied in [5], and more recently in [8]. It was shown in these works that the rate region achievable by window decoding is potentially inferior to that achievable by backwards decoding, due to the additional number of constraints that are needed to correctly decode the cooperative information while fresh information is still being injected.

In this paper, we consider a two user fading cooperative Gaussian MAC with complete channel state information at the transmitters and the receiver, and average power constraints on the transmit powers. In our model, the transmitters can adapt their coding strategies as a function of the channel states, by adjusting their transmit powers. Under these assumptions, we show that the sum rate achievable by window decoding is in fact identical to that achievable by backwards decoding, when optimal power allocation is used in conjunction with block Markov superposition encoding. We achieve this result by showing that the power control policy which is optimal for backwards decoding, also turns out to be optimal for window decoding, as the additional constraints that are introduced due to window decoding never become active with this power allocation. We further show that window decoding with and without stripping also achieve the same sum rate, when optimal power allocation is employed.

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II. Channel Model and Encoding Strategy

We consider a two user fading Gaussian MAC, where both the receiver and the transmitters receive noisy versions of the transmitted messages. The system is modelled by,

$$y_0 = \sqrt{h_{10}}x_1 + \sqrt{h_{20}}x_2 + z_0 \quad (1)$$

$$y_1 = \sqrt{h_{21}}x_2 + z_1 \quad (2)$$

$$y_2 = \sqrt{h_{12}}x_1 + z_2 \quad (3)$$

where x_i is the symbol transmitted by node i, y_i is the symbol received at node i, the receiver being denoted by $i = 0$; z_i is the zero-mean additive white Gaussian noise at node i, having variance σ_i^2, and $\sqrt{h_{ij}}$ are the random fading coefficients, the instantaneous realizations of which are assumed to be known by both the transmitters and the receiver. We assume that the channel variation is slow enough so that the fading parameters can be tracked accurately at the transmitters, yet fast enough to ensure that the long term ergodic properties of the channel are observed within the blocks of transmission.

This channel model is a special case of the MAC with generalized feedback [1], and its capacity region is not known to this date. However, it has been shown in several works that there exist cooperative schemes, which provide achievable rates well beyond the capacity region of the traditional MAC with no cooperation. The achievable rate regions are typically obtained by using what is so called the superposition block Markov encoding. In this paper, we employ a channel adaptive version of the block Markov encoding scheme, where the powers with which the cooperation information and the fresh information are transmitted are varied as functions of the channel state. The encoding strategy, which is a power controlled version of the scheme in [1], [3] is summarized below.

Codebook generation:

- Independently generate $2^{n(R_{12}+R_{21})}$ length n sequences $\mathbf{u}^n$, with entries from an iid unit Gaussian distribution, and assign each of these sequences to a distinct set of messages $\{w'_{12}, w'_{21}\} \in \{1, \cdots, 2^{nR_{12}}\} \times \{1, \cdots, 2^{nR_{21}}\}$, i.e., form $\mathbf{u}(w'_{12}, w'_{21})$
- For every $\mathbf{u}(w'_{12}, w'_{21})$, independently generate $2^{nR_{12}}$ sequences $\mathbf{x}_{12}^n$ from an iid unit Gaussian distribution and label them as $\mathbf{x}_{12}^n(w_{12}, \mathbf{u}(w'_{12}, w'_{21}))$, for $w_{12} \in \{1, \cdots, 2^{nR_{12}}\}$.
- For every $\mathbf{u}(w'_{12}, w'_{21})$, independently generate $2^{nR_{21}}$ sequences $\mathbf{x}_{21}^n$ from an iid unit Gaussian distribution and label them as $\mathbf{x}_{21}^n(w_{21}, \mathbf{u}(w'_{12}, w'_{21}))$, for $w_{21} \in \{1, \cdots, 2^{nR_{21}}\}$.
- For every pair $\mathbf{u}(w'_{12}, w'_{21})$ and $\mathbf{x}_{12}^n(w_{12}, \mathbf{u}(w'_{12}, w'_{21}))$, independently generate $2^{nR_{10}}$ sequences $\mathbf{x}_{10}^n$ from an i.i.d unit Gaussian distribution, and label them $\mathbf{x}_{10}^n(w_{10}, w_{12}, \mathbf{u}(w'_{12}, w'_{21}))$, for $w_{10} \in \{1, \cdots, 2^{nR_{10}}\}$.
- For every pair $\mathbf{u}(w'_{12}, w'_{21})$ and $\mathbf{x}_{21}^n(\mathbf{x}_{21}, \mathbf{u}(w'_{12}, w'_{21}))$, independently generate $2^{nR_{20}}$ sequences $\mathbf{x}_{20}^n$ from an i.i.d unit Gaussian distribution, and label them $\mathbf{x}_{20}^n(w_{20}, w_{21}, \mathbf{u}(w'_{12}, w'_{21}))$, for $w_{20} \in \{1, \cdots, 2^{nR_{20}}\}$.

Note that the codebook generation is identical to those in [1], [3], and is presented here in detail to allow for characterization of the achievable rate regions under several decoding scenarios. Encoding however, is done differently in that the codewords are scaled by variable power levels, thus adjusting the variance of each transmitted component.

Encoding:

The transmission of $B-1$ consecutive messages for each user, i.e., $w_1[b] = (w_{10}[b], w_{12}[b])$ and $w_2[b] = (w_{20}[b], w_{21}[b])$, $b = 1, \cdots, B-1$, is completed in B blocks. Here, the transmitters allocate some of their powers to establish some common information in each block, and in the next block, they coherently combine part of their transmitted codewords. In the presence of channel state information, the encoding in each block b is performed by

$$\begin{aligned} x_i^{(k)} = &\\ &\sqrt{p_{i0}(\mathbf{h}^{(k)})}x_{i0}^{(k)}(w_{i0}[b], w_{ij}[b], \mathbf{u}(w_{12}[b-1], w_{21}[b-1])) \\ &+ \sqrt{p_{ij}(\mathbf{h}^{(k)})}x_{ij}^{(k)}(w_{ij}[b], \mathbf{u}(w_{12}[b-1], w_{21}[b-1])) \\ &+ \sqrt{p_{u_i}(\mathbf{h}^{(k)})}u^{(k)}(w_{12}[b-1], w_{21}[b-1]) \end{aligned} \quad (4)$$

for $i, j \in \{1, 2\}$, $i \neq j$, and $k = 1, \cdots, n$, where $x_i^{(k)}$ denotes the kth entry of vector $\mathbf{x}_i$, and $\mathbf{h}^{(k)}$ is the channel realization during the transmission of the kth symbol. Here x_{i0} carries the fresh information intended for the receiver, x_{ij} carries the information intended for transmitter j for cooperation in the next block, and u is the common information sent by both transmitters for resolution of the remaining uncertainty from the previous block. Assuming that the fading process is stationary and ergodic, we can replace each sample $\mathbf{h}^{(k)}$ of the fading process with the corresponding random variable $\mathbf{h}$ that obeys the stationary distribution, and can therefore drop the time dependence, and convert time averages to statistical averages over the distribution of $\mathbf{h}$. Therefore, the transmit power levels associated with each component can be denoted by $p_{i0}(\mathbf{h})$, $p_{ij}(\mathbf{h})$ and $p_{u_i}(\mathbf{h})$, $i, j \in 1, 2,\ i \neq j$, which are required to satisfy the average power constraints,

$$E\left[p_{i0}(\mathbf{h}) + p_{ij}(\mathbf{h}) + p_{u_i}(\mathbf{h})\right] = E[p_i(\mathbf{h})] \leq \bar{p}_i \quad (5)$$

III. Window versus Backwards Decoding

Following the block Markov encoding, decoding may be done in several ways. Backwards decoding, which waits for all blocks to be received, and then decodes starting from the cooperation information in the last block, is perhaps the most widely studied one of these schemes [1]–[3]. This approach takes advantage of the fact that, by transmitting no fresh information $w_1(B)$ and $w_2(B)$ in the last block B, the receiver can decode the cooperation signal $\mathbf{u}(w_{12}(B-1), w_{21}(B-1))$ without any interference, and then use the signal received in the previous block to completely decode $w_{12}(B-1)$, $w_{21}(B-1)$, $w_{10}(B-1)$ and $w_{20}(B-1)$. Meanwhile, the cooperation information $\mathbf{u}(w_{12}(B-2), w_{12}(B-2))$ is also decoded. The disadvantage of this scheme however, is that it needs to wait for all the blocks to be received before starting the decoding process. For the fading Gaussian MAC

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with cooperating encoders, the rate region achievable by our proposed power controlled encoding scheme (4) followed by backwards decoding, is given by convex hull of R_1 and R_2 that satisfy $R_1 = R_{10} + R_{12}$ and $R_2 = R_{20} + R_{21}$, where

$$R_{12} \leq I(x_{12}; y_2 | x_2, u, \mathbf{h}) \quad (6)$$
$$R_{21} \leq I(x_{21}; y_1 | x_1, u, \mathbf{h}) \quad (7)$$
$$R_{10} \leq I(x_1; y_0 | x_2, x_{12}, x_{21}, u, \mathbf{h}) \quad (8)$$
$$R_{20} \leq I(x_2; y_0 | x_1, x_{12}, x_{21}, u, \mathbf{h}) \quad (9)$$
$$R_{10} + R_{20} \leq I(x_1, x_2; y_0 | x_{12}, x_{21}, u, \mathbf{h}) \quad (10)$$
$$R_{10} + R_{20} + R_{12} + R_{21} \leq I(x_1, x_2; y_0, \mathbf{h}) \quad (11)$$

The decoding delay can be significantly reduced if window decoding, which was introduced by Carleial in [5], is used instead of the backwards decoding. Window decoding has recently been studied in more detail, mostly in the context of relay channels, which are a special case of the MAC with generalized feedback, with only one user having a message to transmit to the destination. For relay channels, it has been shown that window decoding achieves the same rate as backwards decoding. Lately, Laneman and Gastpar [8] have employed window decoding in conjunction with block Markov encoding for an arbitrary MAC with generalized feedback, and obtained the achievable rate regions under two schemes: window decoding with and without stripping.

In window decoding, the decoding process begins after a delay of one block, and is carried out using a sliding window of two blocks: in block b, first the cooperation signal $\mathbf{u}(w_{12}(b-1), w_{21}(b-1))$ is decoded, and then the signal received in the previous block $b-1$ is used to completely decode $w_{12}(b-1)$, $w_{21}(b-1)$, $w_{10}(b-1)$ and $w_{20}(b-1)$. Depending on the order of decoding of these messages, the process is called window decoding with stripping (where the cooperation information, i.e., $w_{12}(b-1)$, $w_{21}(b-1)$ is decoded first, subtracted from the received signal, and fresh information is decoded afterwards); or window decoding without stripping (where no specific decoding order is imposed).

Despite of its much improved delay performance, the drawback of window decoding is that, unlike backwards decoding, the cooperative information $\mathbf{u}$ needs to be decoded first in the presence of interference from other codeword components, i.e., the rates achievable for the cooperative part of the messages w_{12} and w_{21} are in general potentially lower. This is due to the extra constraints imposed by window decoding for reliable decoding of the cooperative part of the message. For window decoding with stripping, these constraints, which are required in addition to the constraints for backwards decoding (6)-(11), are given by

$$R_{12} \leq I(u; y_0) + I(x_{12}; y_0 | x_{21}, u, \mathbf{h}) \quad (12)$$
$$R_{21} \leq I(u; y_0) + I(x_{21}; y_0 | x_{12}, u, \mathbf{h}) \quad (13)$$
$$R_{12} + R_{21} \leq I(u, x_{12}, x_{21}; y_0, |\mathbf{h}) \quad (14)$$

If window decoding without striping is used instead, after decoding $\mathbf{u}$ while treating all other components as interference, the fresh information and remaining cooperation information are decoded jointly, leading to the rate constraints

$$R_{12} + R_{10} \leq I(u; y_0) + I(x_1; y_0 | x_2, x_{21}, u, \mathbf{h}) \quad (15)$$
$$R_{21} + R_{20} \leq I(u; y_0) + I(x_2; y_0 | x_1, x_{12}, u, \mathbf{h}) \quad (16)$$
$$R_{12} + R_{10} + R_{20} \leq I(u; y_0) + I(u, x_1, x_2; y_0, | x_{21}, \mathbf{h}) \quad (17)$$
$$R_{21} + R_{10} + R_{20} \leq I(u; y_0) + I(u, x_1, x_2; y_0, | x_{12}, \mathbf{h}) \quad (18)$$

In what follows we will show that when our proposed power controlled encoding scheme (4) is used, the maximum sum of rates achievable by backwards decoding and window decoding are the same. In order to achieve this, we first need to express the rate constraints (6)-(18) in more detail in terms of the power components and channel states. For convenience, we define the effective channel gains normalized by the noise powers as $s_{ij} = h_{ij}/\sigma_j^2$. Also, for notational simplicity, we further define

$$A = 1 + s_{10}p_1(\mathbf{h}) + s_{20}p_2(\mathbf{h}) + 2\sqrt{s_{10}s_{20}p_{u_1}(\mathbf{h})p_{u_2}(\mathbf{h})}$$
$$B = 1 + s_{10}(p_{10}(\mathbf{h}) + p_{12}(\mathbf{h})) + s_{20}(p_{21}(\mathbf{h}) + p_{20}(\mathbf{h}))$$
$$C = 1 + s_{10}p_{10}(\mathbf{h}) + s_{20}p_{20}(\mathbf{h})$$

Then, it can be shown by standard properties of mutual information and entropy, and also by using standard results regarding the capacity of fading channels [9], that the achievable rate region for window decoding with stripping is given by the convex hull of rates $R_1 = R_{10} + R_{12}$ and $R_2 = R_{20} + R_{21}$, which obey equations (19) through (27) ((19) through (24) and (28) through (31) respectively for window decoding without stripping):

$$R_{12} < E\left[\log\left(1 + \frac{s_{12}p_{12}(\mathbf{h})}{s_{12}p_{10}(\mathbf{h}) + 1}\right)\right] \quad (19)$$
$$R_{21} < E\left[\log\left(1 + \frac{s_{21}p_{21}(\mathbf{h})}{s_{21}p_{20}(\mathbf{h}) + 1}\right)\right] \quad (20)$$
$$R_{10} < E\left[\log\left(1 + s_{10}p_{10}(\mathbf{h})\right)\right] \quad (21)$$
$$R_{20} < E\left[\log\left(1 + s_{20}p_{20}(\mathbf{h})\right)\right] \quad (22)$$
$$R_{10} + R_{20} < E\left[\log\left(C\right)\right] \quad (23)$$
$$R_1 + R_2 < E\left[\log\left(A\right)\right] \quad (24)$$
$$R_{12} < E\left[\log\left(\frac{A}{B}\right) + \log\left(\frac{C + s_{10}p_{12}(\mathbf{h})}{C}\right)\right] \quad (25)$$
$$R_{21} < E\left[\log\left(\frac{A}{B}\right) + \log\left(\frac{C + s_{20}p_{21}(\mathbf{h})}{C}\right)\right] \quad (26)$$
$$R_{12} + R_{21} < E\left[\log\left(\frac{A}{C}\right)\right] \quad (27)$$
$$R_1 < E\left[\log\left(\frac{A}{B}\right) + \log\left(1 + s_{10}(p_{10}(\mathbf{h}) + p_{12}(\mathbf{h}))\right)\right] \quad (28)$$
$$R_2 < E\left[\log\left(\frac{A}{B}\right) + \log\left(1 + s_{20}(p_{20}(\mathbf{h}) + p_{21}(\mathbf{h}))\right)\right] \quad (29)$$
$$R_1 + R_{20} < E\left[\log\left(\frac{A}{B}\right) + \log\left(C + s_{10}p_{12}(\mathbf{h})\right)\right] \quad (30)$$
$$R_2 + R_{10} < E\left[\log\left(\frac{A}{B}\right) + \log\left(C + s_{20}p_{21}(\mathbf{h})\right)\right] \quad (31)$$

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IV. Sum Rate Achievable by Window Decoding

The sum rate achievable by power controlled block Markov encoding (4), followed by backwards decoding can be obtained from equations (19)-(24), yielding (32) at the bottom of this page.

We have shown in [4] that, the optimal power allocation policy that maximizes the sum rate (32) has the following property:

Proposition 1: [4] The power control policy $\mathbf{p}^*(\mathbf{h})$ that maximizes (32), should satisfy

1) $p_{10}^*(\mathbf{h}) = p_{20}^*(\mathbf{h}) = 0$, if $s_{12} > s_{10}$ and $s_{21} > s_{20}$
2) $p_{10}^*(\mathbf{h}) = p_{21}^*(\mathbf{h}) = 0$, if $s_{12} > s_{10}$ and $s_{21} \leq s_{20}$
3) $p_{12}^*(\mathbf{h}) = p_{20}^*(\mathbf{h}) = 0$, if $s_{12} \leq s_{10}$ and $s_{21} > s_{20}$
4) $p_{12}^*(\mathbf{h}) = p_{21}^*(\mathbf{h}) = 0$
   OR
   $p_{10}^*(\mathbf{h}) = p_{21}^*(\mathbf{h}) = 0$, if $s_{12} \leq s_{10}$ and $s_{21} \leq s_{20}$
   OR
   $p_{12}^*(\mathbf{h}) = p_{20}^*(\mathbf{h}) = 0$

We will use this result to show that the sum rate optimal power control policy for window decoding is identical to that for backwards decoding, and both schemes achieve the same sum rate. Our main result is stated in the following theorem.

Theorem 1: For a fading Gaussian multiple access channel with two encoders employing the channel adaptive block Markov superposition encoding given by (4), let the maximum sum rates achievable by window decoding and backwards decoding be denoted by R_{w}^*, and R_{b}^*, respectively. Then, $R_{\text{w}}^* = R_{\text{b}}^*$. Furthermore, the maximum sum rates achievable by window decoding with and without stripping are identical.

Let $\mathbf{p}^*(\mathbf{h})$ be the power control policy that maximizes the sum rate R_{b} achievable by backwards decoding, given in (32). We will prove Theorem 1 in two steps, using the following two propositions.

Proposition 2: $R_{\text{b}}^* \triangleq R_{\text{b}}(\mathbf{p}^*(\mathbf{h})) \geq R_{\text{w}}(\mathbf{p}^*(\mathbf{h}))$.

Proof: Since the rate constraints for backwards decoding are a subset of the constraints for window decoding, for any valid power allocation policy $\mathbf{p}(\mathbf{h})$, $R_{\text{b}}(\mathbf{p}(\mathbf{h})) \geq R_{\text{w}}(\mathbf{p}(\mathbf{h}))$, and the result follows directly by choosing $\mathbf{p}(\mathbf{h}) = \mathbf{p}(\mathbf{h})^*$. ■

Proposition 3: $R_{\text{b}}(\mathbf{p}^*(\mathbf{h})) \leq R_{\text{w}}(\mathbf{p}^*(\mathbf{h}))$.

Proof: In our proof, we will focus on window decoding with stripping, since it is a special case of window decoding without stripping (with a specific decoding order), thus giving a sum rate at best as high as the sum rate without stripping. Hence, if we can prove the proposition for window decoding with stripping, the result will automatically follow for window decoding without stripping, which in general has a larger achievable rate region. We first start by listing all possible combinations of the constraints (19)-(27) (leaving out constraints (21) and (22), which are always dominated by (23), as far as sum rate is concerned) that will yield a bound on the sum rate R_{w}:

i) (24)
ii) (19)+(20)+(23)
iii) (23)+(27)
iv) (23)+(25)+(26)
v) (19)+(23)+(26)
vi) (20)+(23)+(25)

We will show that the bounds obtained by adding each listed set of equations are no tighter than the backwards decoding bound (32), when optimal power allocation policy is used, thereby proving Proposition 3.

i) The bound (24) is common to both backwards and window decoding, and therefore automatically satisfies the proposition, with equality.

ii) The bound (19)+(20)+(23) is also common to both backwards and window decoding, and therefore automatically satisfies the proposition, with equality.

iii) It is straightforward to check that the sum of right hand sides of (23) and (27) yield the right hand side of (24), and reduce to case i).

iv) Adding the right hand sides of the equations (23), (25) and (26), we obtain

$$\begin{aligned} &E\left[2\log\left(\frac{A}{B}\right) + \log\left(\frac{C + s_{10}p_{12}(\mathbf{h})}{C}\right) \right. \\ &\quad \left. + \log\left(\frac{C + s_{20}p_{21}(\mathbf{h})}{C}\right) + \log(C)\right] \\ &\geq E\left[\log\left(\frac{AC}{B}\right) + \log\left(\frac{C + s_{10}p_{12}(\mathbf{h}) + s_{20}p_{21}(\mathbf{h})}{C}\right)\right] \quad (33) \\ &= E\left[\log(A)\right] \quad (34) \end{aligned}$$

where, (33) follows from the concavity of the logarithm, and the fact that we threw away a positive term, and (34) follows by noting that the numerator of the second term in (33) is B. Hence, the bound on the sum rate obtained in this case is looser than $\log(A)$, i.e., the bound in case i).

v) Adding the right hand sides of the equations (19), (23) and (26), we obtain

$$\begin{aligned} E\Bigg[\log\left(\frac{A}{B}\right) + \log\left(1 + \frac{s_{12}p_{12}(\mathbf{h})}{s_{12}p_{10}(\mathbf{h}) + 1}\right) \\ + \log\left(\frac{C + s_{20}p_{21}(\mathbf{h})}{C}\right) + \log(C)\Bigg] \end{aligned} \quad (35)$$

By comparing with cases i)-iv), one can see that this bound is active only if its value is lower than $\log(A)$. Therefore, simplifying and rearranging the terms, it is easy to see that

$$R_{\text{sum}} = \min\left\{E\left[\log(A)\right], E\left[\log(C) + \log\left(1 + \frac{s_{12}p_{12}(\mathbf{h})}{s_{12}p_{10}(\mathbf{h}) + 1}\right) + \log\left(1 + \frac{s_{21}p_{21}(\mathbf{h})}{s_{21}p_{20}(\mathbf{h}) + 1}\right)\right]\right\} \quad (32)$$

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the bound is active if

$$\log\left(1+\frac{s_{12}p_{12}(\mathbf{h})}{s_{12}p_{10}(\mathbf{h})+1}\right)<\log\left(\frac{B}{C+s_{20}p_{21}(\mathbf{h})}\right) \quad (36)$$

In obtaining (36) we have forced the arguments of the expectations, rather than the expectations themselves, to be equal, thereby imposing a stricter condition. Yet, in what follows, we will see that this stricter condition is always satisfied.

Note that until now we have made no use of the properties of the optimal power allocation policy given by Proposition 1. This means that, bounds in cases i)-iv) are all looser, or equivalent, to the backwards decoding sum rate bound (32), regardless of the power control policy being used. In case v), we will need to use the fact that some components of $\mathbf{p}^*(\mathbf{h})$ are always zero, depending on the relative channel qualities.

- Let $s_{10} < s_{12}$ and $s_{20} < s_{21}$, in which case $p^*_{10}(\mathbf{h}) = p^*_{20}(\mathbf{h}) = 0$. Then, evaluating (36) at $\mathbf{p}^*(\mathbf{h})$, we get

$$\log(1+s_{12}p_{12}(\mathbf{h})) < \log\left(1+\frac{s_{10}p_{12}(\mathbf{h})}{1+s_{20}p_{21}(\mathbf{h})}\right) \quad (37)$$
$$\leq \log(1+s_{10}p_{12}(\mathbf{h})) \quad (38)$$
$$\leq \log(1+s_{12}p_{12}(\mathbf{h})) \quad (39)$$

  which is a contradiction, therefore, the bound is never active if $s_{10} < s_{12}$ and $s_{20} < s_{21}$.
- Let $s_{10} \geq s_{12}$ and $s_{20} < s_{21}$, in which case $p^*_{12}(\mathbf{h}) = p^*_{20}(\mathbf{h}) = 0$. Then, it is easy to check that both sides of (36) are equal to 0, the strict inequality is not satisfied, and the sum rate is bounded by $\log(A)$ as in cases i)-iv).
- Let $s_{10} < s_{12}$ and $s_{20} \geq s_{21}$ in which case $p^*_{10}(\mathbf{h}) = p^*_{21}(\mathbf{h}) = 0$. Then, for the bound to be active, we need,

$$\log(1+s_{12}p_{12}(\mathbf{h})) < \log\left(1+\frac{s_{10}p_{12}(\mathbf{h})}{1+s_{20}p_{20}(\mathbf{h})}\right) \quad (40)$$

  which is never possible, since $s_{10} < s_{12}$, and the bound is loose.
- Let $s_{10} \geq s_{12}$ and $s_{20} \geq s_{21}$ in which case $p^*_{12}(\mathbf{h}) = p^*_{21}(\mathbf{h}) = 0$ or $p^*_{12}(\mathbf{h}) = p^*_{20}(\mathbf{h}) = 0$ or $p^*_{10}(\mathbf{h}) = p^*_{21}(\mathbf{h}) = 0$. It is easy to check that the first two possible choices of powers give both sides of (36) equal to 0, thereby yielding no tighter bounds. On the other hand, we need to investigate the last choice in detail. In this case, (36) reduces to

$$1+s_{20}p_{20}(\mathbf{h}) < \frac{s_{10}}{s_{12}} \quad (41)$$

  However, this condition cannot be satisfied for an optimal $p^*_{10}(\mathbf{h})$, since, according to our derivations in [4], it is equivalent to the partial derivative of the sum rate with respect to p_{10} being always positive, which contradicts the optimality of $p^*_{10}(\mathbf{h}) = 0$. Therefore, we have shown that this rate constraint is also loose, and is dominated by the sum rate constraint for backwards decoding.

vi) Finally, we note that case vi) is completely symmetric with case v), and therefore the window decoding bound on the sum rate is looser or at best equivalent to that for backwards decoding. ■

The proof of Theorem 1 immediately follows by combining Propositions 2 and 3.

V. Conclusions

In this paper, we have addressed the problem of comparing the sum rates achievable by backwards decoding and window decoding, for a fading Gaussian multiple access channel with perfect channel state information at the transmitters and the receiver. We have employed a power controlled block Markov encoding scheme, followed by either backwards or window decoding, and investigated the rates achievable by optimally allocating the transmit powers for each codeword component as a function of the channel states. Our results can be summarized as follows:

1) Window decoding achieves the same sum rate as backwards decoding, when the powers are allocated optimally. This result is extremely important, since using window decoding, the decoding delay can be reduced from the B block delay of the backwards decoding, down to only a single block of transmission. Therefore, window decoding is always preferable as far as the total information rate in the system is concerned.

2) Since window decoding with stripping achieves the same maximum sum rate as backwards decoding, its more general version, window decoding without stripping, also achieves the same sum rate, therefore, under optimal power control, the sum rates achievable by window decoding with and without stripping are identical.

3) The sum rate maximizing power allocation policies for backwards and window decoding are also identical, thus, at any given channel state, two of the transmitted signal components need to be assigned zero powers, thereby simplifying the block Markov encoding strategy significantly.

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