Encoding/Decoding Strategies and Rate Regions for Cooperative Multiple Access Channels

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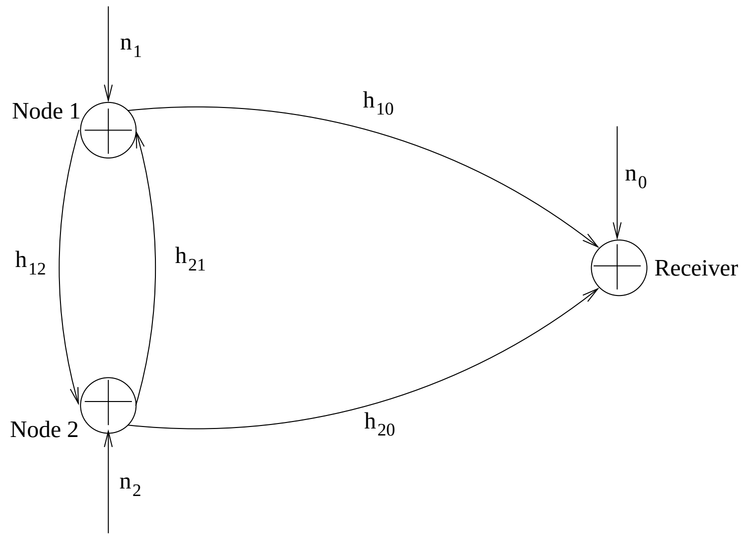
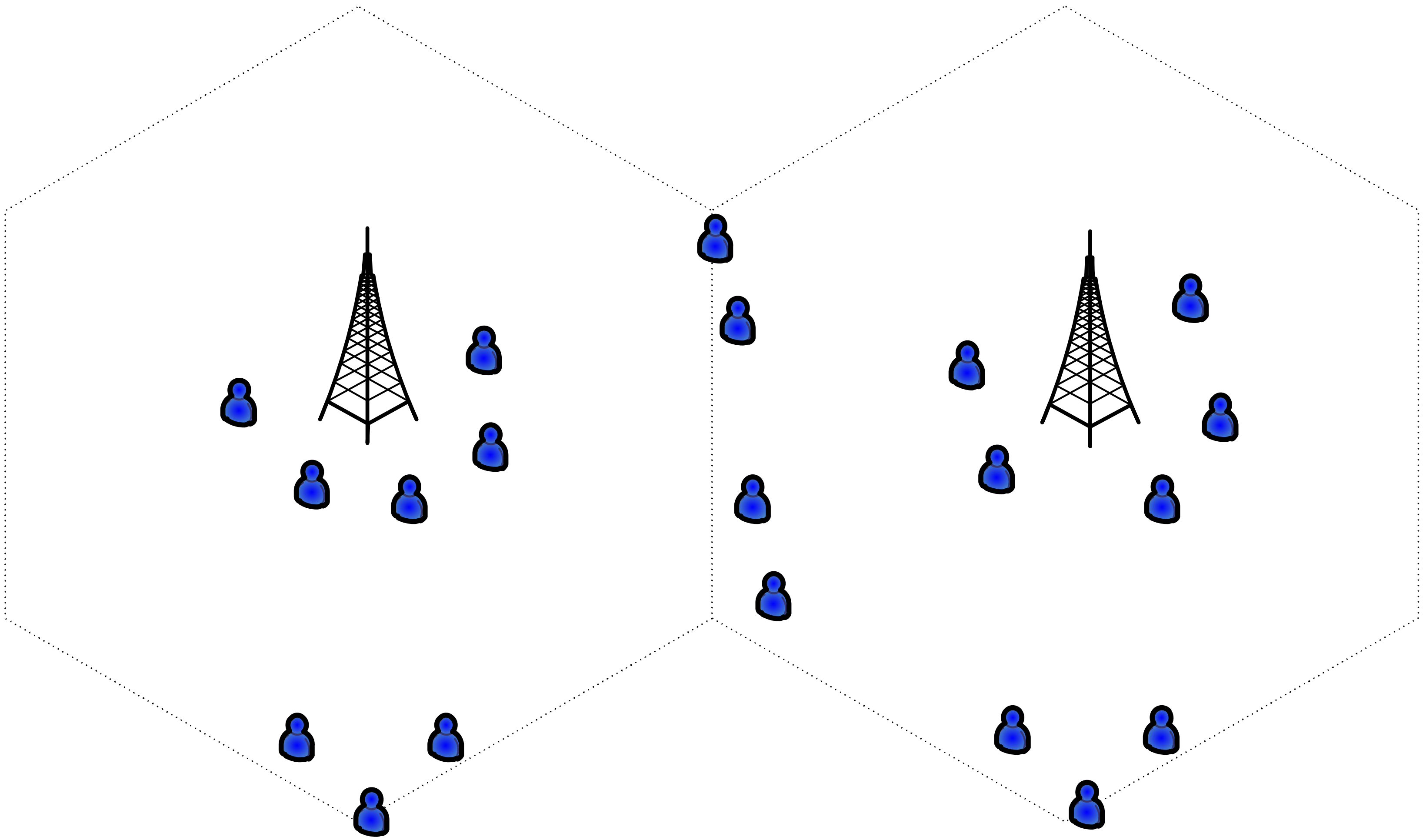
User Cooperation



$$Y_0 = h_{10}X_1 + h_{20}X_2 + n_0$$
$$Y_1 = h_{21}X_2 + n_1$$
$$Y_2 = h_{12}X_1 + n_2$$

- Interference is information.
- Some versions of all transmitted signals are received by all nodes.
- User cooperation: exploit overheard information to jointly design encoding, transmit, routing policies.
- Building block towards the analysis of larger networks.

Motivation



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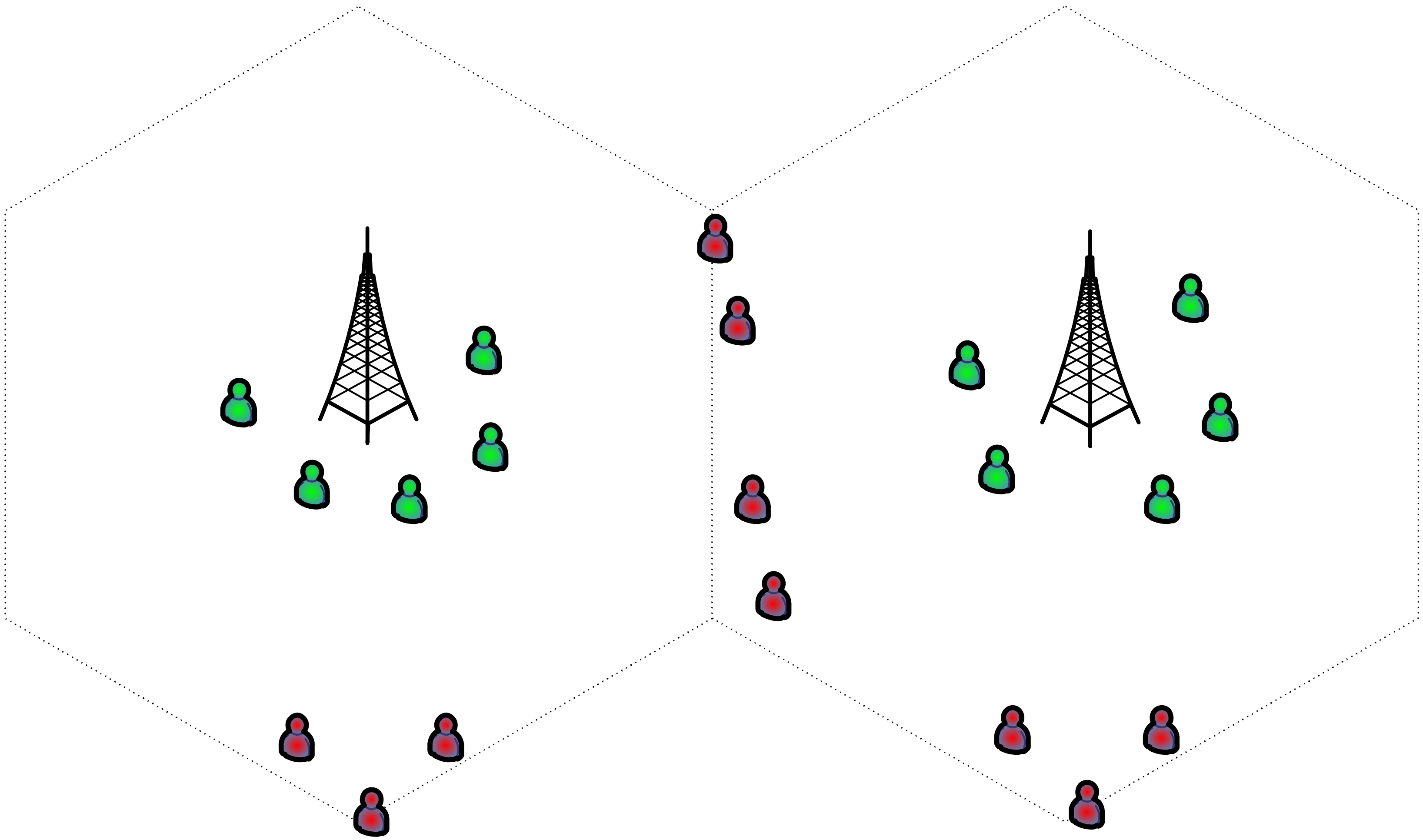

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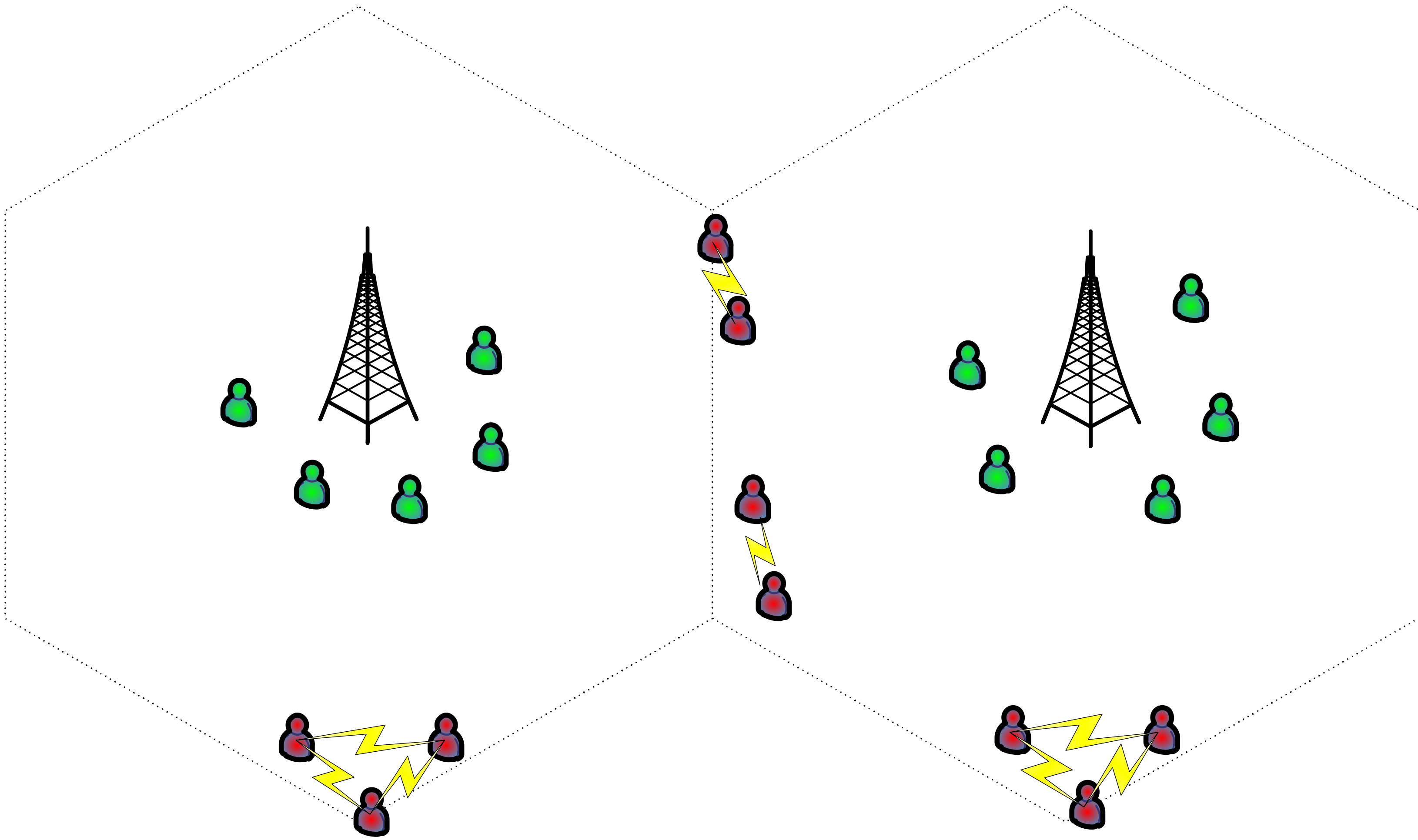

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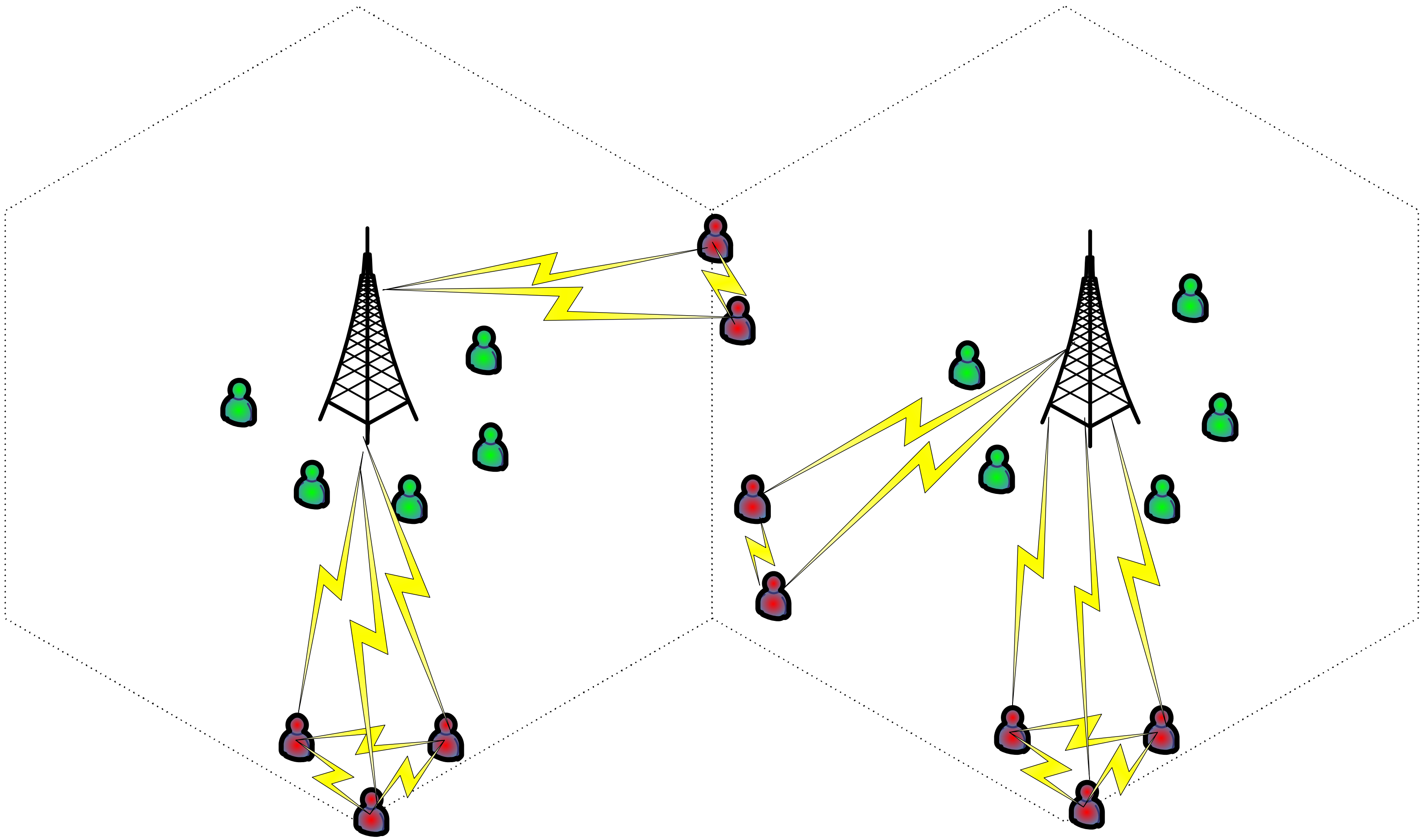

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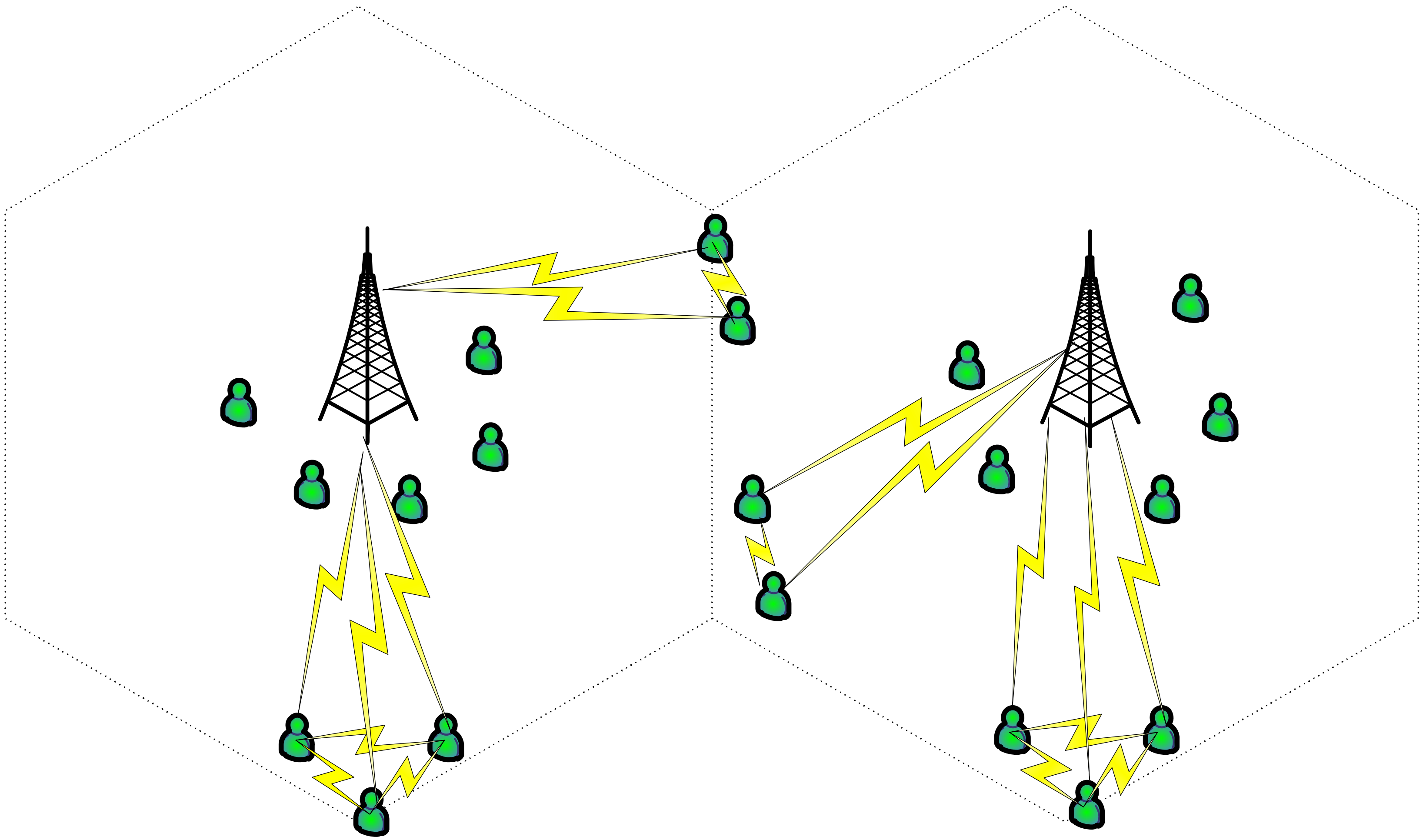
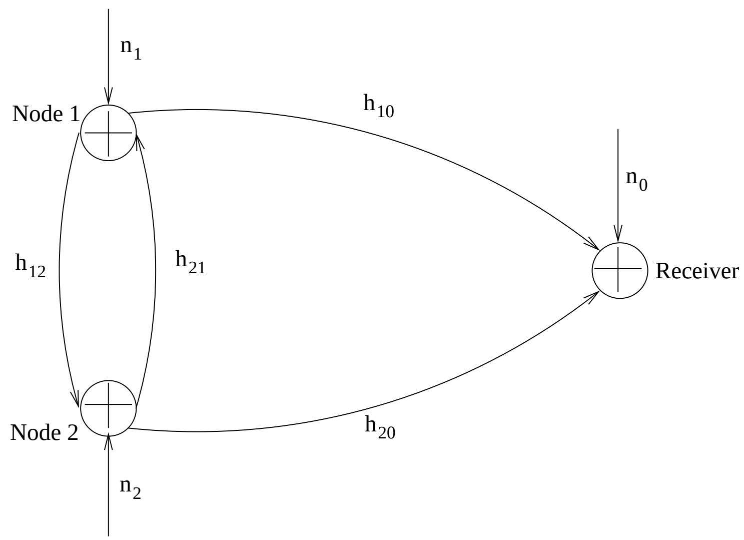


Optimum Power Allocation for the Two User Cooperative MAC



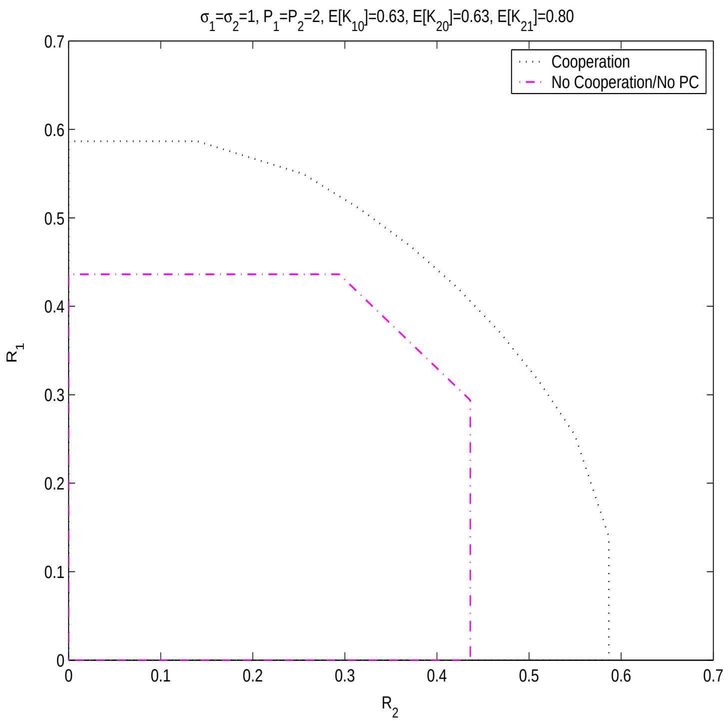
$$Y_0 = h_{10}X_1 + h_{20}X_2 + n_0$$
$$Y_1 = h_{21}X_2 + n_1$$
$$Y_2 = h_{12}X_1 + n_2$$

- Joint work with Sennur Ulukus

MAC with Generalized Feedback

- Gaussian MAC with cooperating encoders [Sendonaris, Erkip, Aazhang]
  - Special case of MAC with generalized feedback [Willems, van der Meulen, Schalkwijk]
- An achievable rate region is obtained by employing
  - Block Markov superposition encoding
    - ∗ Inject high rate fresh information to be resolved with the help of upcoming blocks.
    - ∗ Send resolution information for previous blocks.
  - Backward decoding
    - ∗ After receiving all blocks, decode the resolution information in the last block.
    - ∗ Using previously decoded resolution information, sequentially decode earlier blocks.

Gaussian MAC with User Cooperation – No Resource Allocation



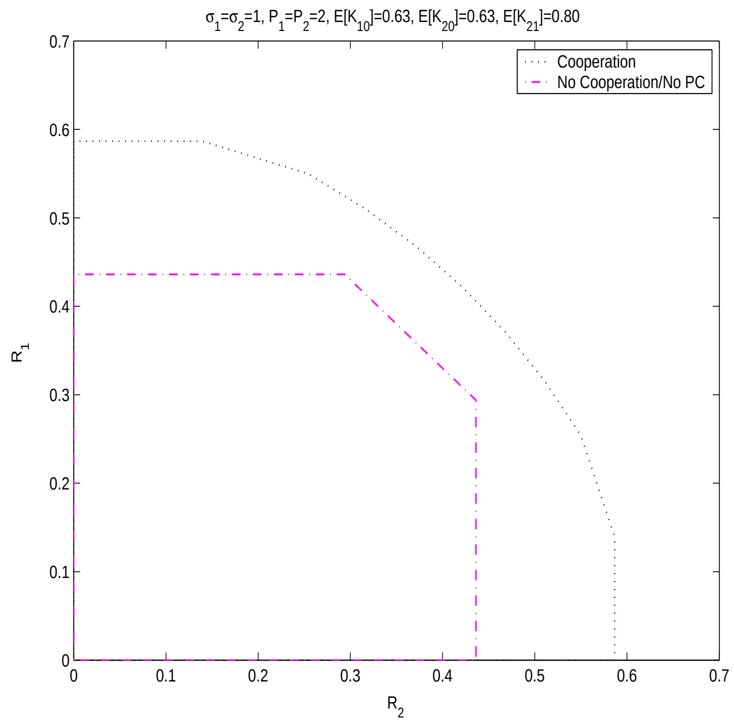
Block Markov superposition coding

- Build common information (X_{12}, X_{21})
- Cooperatively send (U)
- Inject new information (X_{10}, X_{20})

$$X_1 = \sqrt{p_{10}}X_{10} + \sqrt{p_{12}}X_{12} + \sqrt{p_{u1}}U$$
$$X_2 = \sqrt{p_{20}}X_{20} + \sqrt{p_{21}}X_{21} + \sqrt{p_{u2}}U$$

- Amplitude of the each channel's gain is assumed to be known at the corresponding receiver.
- Phases of all channel gains are assumed known at the receiver and the transmitters
  - Coherent combining.

Gaussian MAC with User Cooperation – Resource Allocation



Block Markov superposition coding

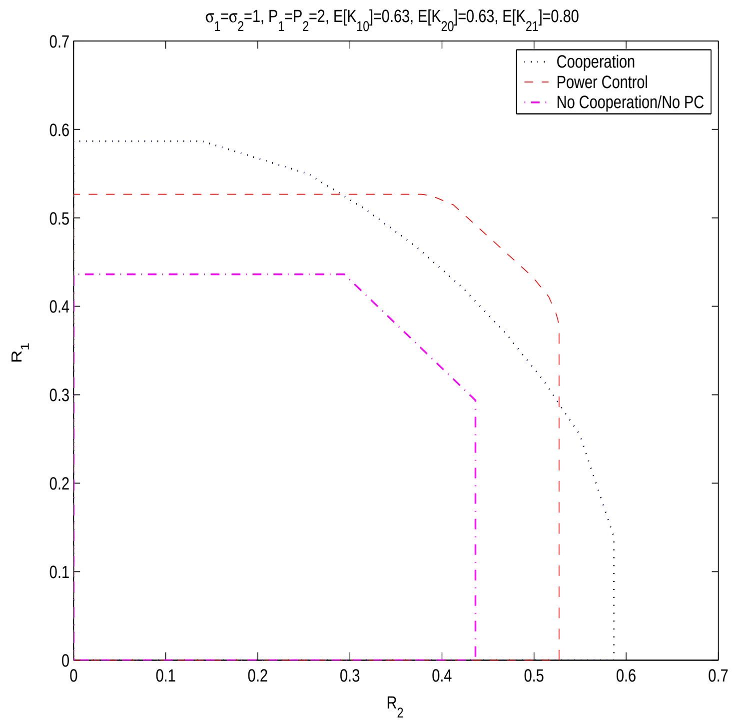
- Build common information (X_{12}, X_{21})
- Cooperatively send (U)
- Inject new information (X_{10}, X_{20})

$$X_1 = \sqrt{p_{10}(\mathbf{h})}X_{10} + \sqrt{p_{12}(\mathbf{h})}X_{12} + \sqrt{p_{u1}(\mathbf{h})}U$$

$$X_2 = \sqrt{p_{20}(\mathbf{h})}X_{20} + \sqrt{p_{21}(\mathbf{h})}X_{21} + \sqrt{p_{u2}(\mathbf{h})}U$$

- Complete channel state information at the transmitters and the receiver.
- Transmitted codewords can be modulated by channel adaptive power levels
  - Opportunistic cooperation and transmission – use available average power efficiently.

Gaussian MAC with User Cooperation – Resource Allocation



Block Markov superposition coding

- Build common information (X_{12}, X_{21})
- Cooperatively send (U)
- Inject new information (X_{10}, X_{20})

$$X_1 = \sqrt{p_{10}(\mathbf{h})} X_{10}$$

$$X_2 = \sqrt{p_{20}(\mathbf{h})} X_{20}$$

- Complete channel state information at the transmitters and the receiver.
- Transmitted codewords can be modulated by channel adaptive power levels
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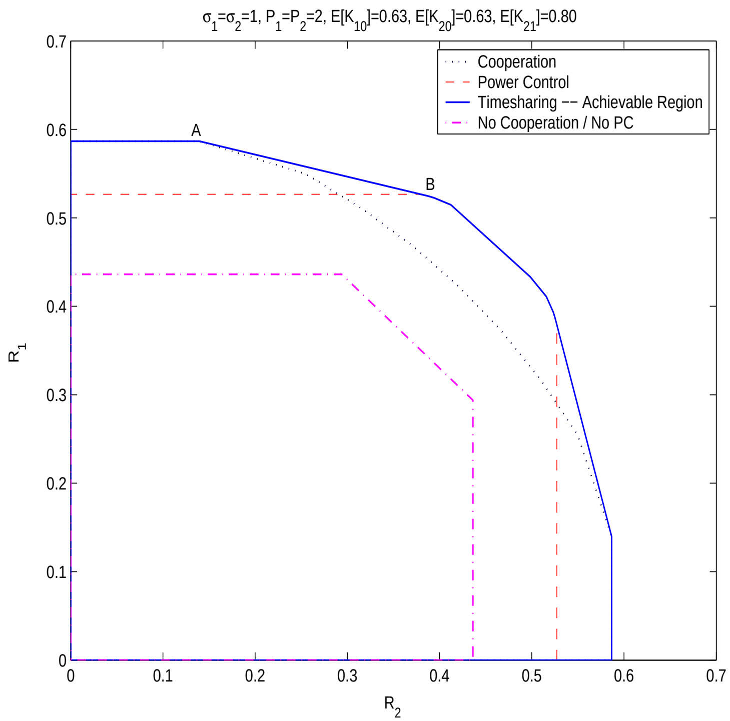
Gaussian MAC with User Cooperation – Resource Allocation



Block Markov superposition coding

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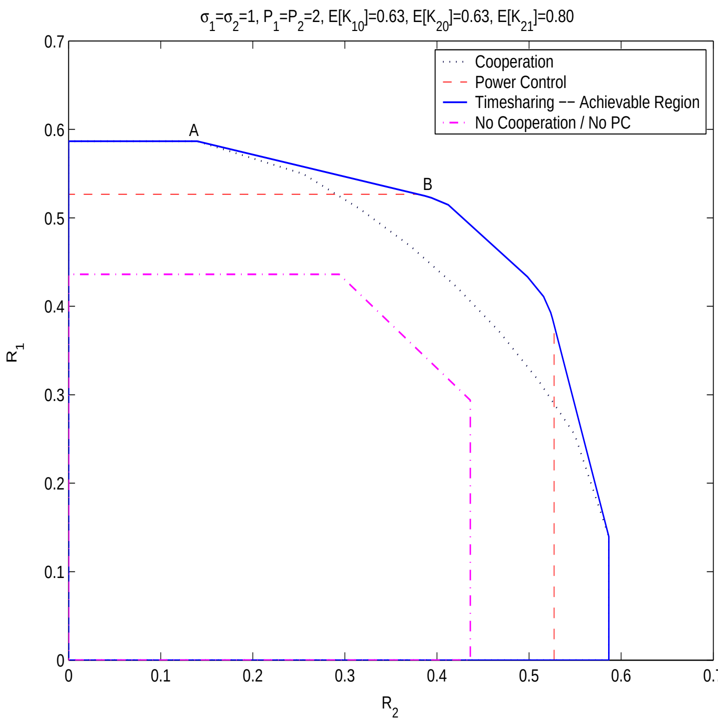
Gaussian MAC with User Cooperation – Resource Allocation



Block Markov superposition coding

- Build common information (X_{12}, X_{21})
- Cooperatively send (U)
- Inject new information (X_{10}, X_{20})

$$X_1 = \sqrt{p_{10}(\mathbf{h})}X_{10} + \sqrt{p_{12}(\mathbf{h})}X_{12} + \sqrt{p_{u1}(\mathbf{h})}U$$

$$X_2 = \sqrt{p_{20}(\mathbf{h})}X_{20} + \sqrt{p_{21}(\mathbf{h})}X_{21} + \sqrt{p_{u2}(\mathbf{h})}U$$

- Complete channel state information at the transmitters and the receiver.
- Transmitted codewords can be modulated by channel adaptive power levels
  - Opportunistic cooperation and transmission – use available average power efficiently.

Achievable Region of Rates with Power Control

- Union over all valid policies $E[p_{i0}(\mathbf{h}) + p_{ij}(\mathbf{h}) + p_{U_i}(\mathbf{h})] \leq \overline{p_i}$ of pairs $\{R_1, R_2\}$ that satisfy

$$R_1 < E\left[\log\left(1 + \frac{h_{12}p_{12}(\mathbf{h})}{h_{12}p_{10}(\mathbf{h}) + \sigma_2^2}\right) + \log\left(1 + \frac{h_{10}p_{10}(\mathbf{h})}{\sigma_0^2}\right)\right]$$

$$R_2 < E\left[\log\left(1 + \frac{h_{21}p_{21}(\mathbf{h})}{h_{21}p_{20}(\mathbf{h}) + \sigma_1^2}\right) + \log\left(1 + \frac{h_{20}p_{20}(\mathbf{h})}{\sigma_0^2}\right)\right]$$

$$R_1 + R_2 < \min\left\{E\left[\log\left(1 + \frac{h_{10}p_1(\mathbf{h}) + h_{20}p_2(\mathbf{h}) + 2\sqrt{h_{10}h_{20}p_{U_1}(\mathbf{h})p_{U_2}(\mathbf{h})}}{\sigma_0^2}\right)\right],\right.$$

$$\left. E\left[\log\left(1 + \frac{h_{12}p_{12}(\mathbf{h})}{h_{12}p_{10}(\mathbf{h}) + \sigma_2^2}\right) + \log\left(1 + \frac{h_{21}p_{21}(\mathbf{h})}{h_{21}p_{20}(\mathbf{h}) + \sigma_1^2}\right) + \log\left(1 + \frac{h_{10}p_{10}(\mathbf{h}) + h_{20}p_{20}(\mathbf{h})}{\sigma_0^2}\right)\right]\right\}$$

- Bounds not concave in power vector $\mathbf{p}(\mathbf{h}) = [p_{10}(\mathbf{h})\ p_{12}(\mathbf{h})\ p_{U_1}(\mathbf{h})\ p_{20}(\mathbf{h})\ p_{21}(\mathbf{h})\ p_{U_2}(\mathbf{h})]$

Properties of Sum-Rate-Optimal Power Allocation

Proposition 1 *Let the effective channel gains normalized by the noise powers be defined as* $s_{ij} = h_{ij}/\sigma_j^2$. *Then, for the power control policy* $\mathbf{p}^*(\mathbf{h})$ *that maximizes the sum rate, we need*

- $p_{10}^*(\mathbf{h}) = p_{20}^*(\mathbf{h}) = 0$, if $s_{12} > s_{10}$ and $s_{21} > s_{20}$
- $p_{10}^*(\mathbf{h}) = p_{21}^*(\mathbf{h}) = 0$, if $s_{12} > s_{10}$ and $s_{21} \leq s_{20}$
- $p_{12}^*(\mathbf{h}) = p_{20}^*(\mathbf{h}) = 0$, if $s_{12} \leq s_{10}$ and $s_{21} > s_{20}$
- $$\left.\begin{array}{c} p_{12}^*(\mathbf{h}) = p_{21}^*(\mathbf{h}) = 0 \\ \text{OR} \\ p_{10}^*(\mathbf{h}) = p_{21}^*(\mathbf{h}) = 0 \\ \text{OR} \\ p_{12}^*(\mathbf{h}) = p_{20}^*(\mathbf{h}) = 0 \end{array}\right\} \text{if } s_{12} \leq s_{10} \text{ and } s_{21} \leq s_{20}$$

Implications of the Optimal Power Allocation

- Block Markov superposition coding is simpler than originally thought.
  - Each transmitter either sends a cooperation signal or fresh information, but not both!
- The choice at each channel state “only” depends on the channel state.
  - Channel statistics, power constraints play no role in deciding which signals to transmit.
  - Except for the tiny little last case... which usually has very insignificant probability.
- The achivable rate expressions are greatly simplified, and are now concave.
- This simplified coding policy not only maximizes the sum rate, but also the individual rate constrains on R_1 and R_2, and is optimal in terms of the entire rate region.
- Concave optimization problem over a convex constraint set, but non-differentiable.

Simplified Rate Region – Example

- Assume $s_{12} > s_{10}$, $s_{21} > s_{20}$ to illustrate the simplified rate region.

$$R_1 < E\left[\log\left(1 + s_{12}p_{12}(\mathbf{h})\right)\right]$$
$$R_2 < E\left[\log\left(1 + s_{21}p_{21}(\mathbf{h})\right)\right]$$
$$R_1 + R_2 < \min\left\{E\left[\log\left(1 + s_{10}p_1(\mathbf{h}) + s_{20}p_2(\mathbf{h}) + 2\sqrt{s_{10}s_{20}p_{U_1}(\mathbf{h})p_{U_2}(\mathbf{h})}\right)\right],\right.$$
$$\left. E\left[\log\left(1 + s_{12}p_{12}(\mathbf{h})\right) + \log\left(1 + s_{21}p_{21}(\mathbf{h})\right)\right]\right\}$$

- Inequalities define either a pentagon like in the traditional MAC, or a triangle.
- All bounds concave in powers, and so is any weighted sum $\mu_1 R_1 + \mu_2 R_2$ at the corners.
- Sum rate not differentiable where the arguments of the min are equal.

Rate Maximization Using Subgradient Method

- Points on the rate region boundary can be obtained by maximizing $C_{\boldsymbol{\mu}} = \mu_1 R_1 + \mu_2 R_2$.
- The optimization problem for arbitrary priorities μ_1 and μ_2 is given by

$$\begin{aligned} \max_{\mathbf{p}(\mathbf{h})} \; & \mu_1 R_1 + \mu_2 R_2 \\ \text{s.t. } & E_{3,4}\left[p_{10}(\mathbf{h})\right] + E_{1,2}\left[p_{12}(\mathbf{h})\right] + E\left[p_{U_1}(\mathbf{h})\right] \leq \bar{p}_1 \\ & E_{2,4}\left[p_{20}(\mathbf{h})\right] + E_{1,3}\left[p_{21}(\mathbf{h})\right] + E\left[p_{U_2}(\mathbf{h})\right] \leq \bar{p}_2 \end{aligned}$$

- $\{R_1, R_2\}$ is the corner of the pentagon obtained for a given power allocation policy.
- Gradient of the objective function does not exist everywhere, find subgradient $\mathbf{g}$ instead

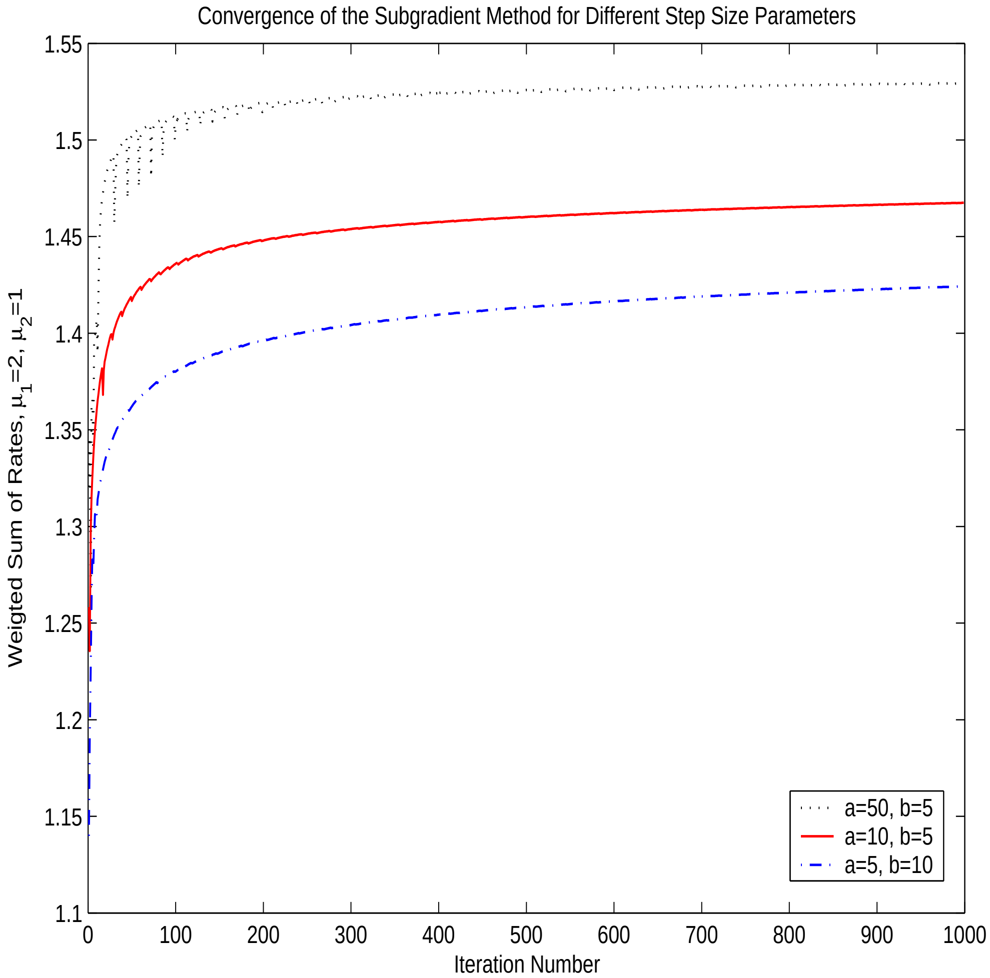
$$C_{\boldsymbol{\mu}}(\mathbf{p}') \leq C_{\boldsymbol{\mu}}(\mathbf{p}) + (\mathbf{p}' - \mathbf{p})\mathbf{g}$$

- Use projected subgradient method to maximize $C_{\boldsymbol{\mu}}$

$$\mathbf{p}(k+1) = \left[\mathbf{p}(k) + \alpha_k \mathbf{g}_k\right]^+$$

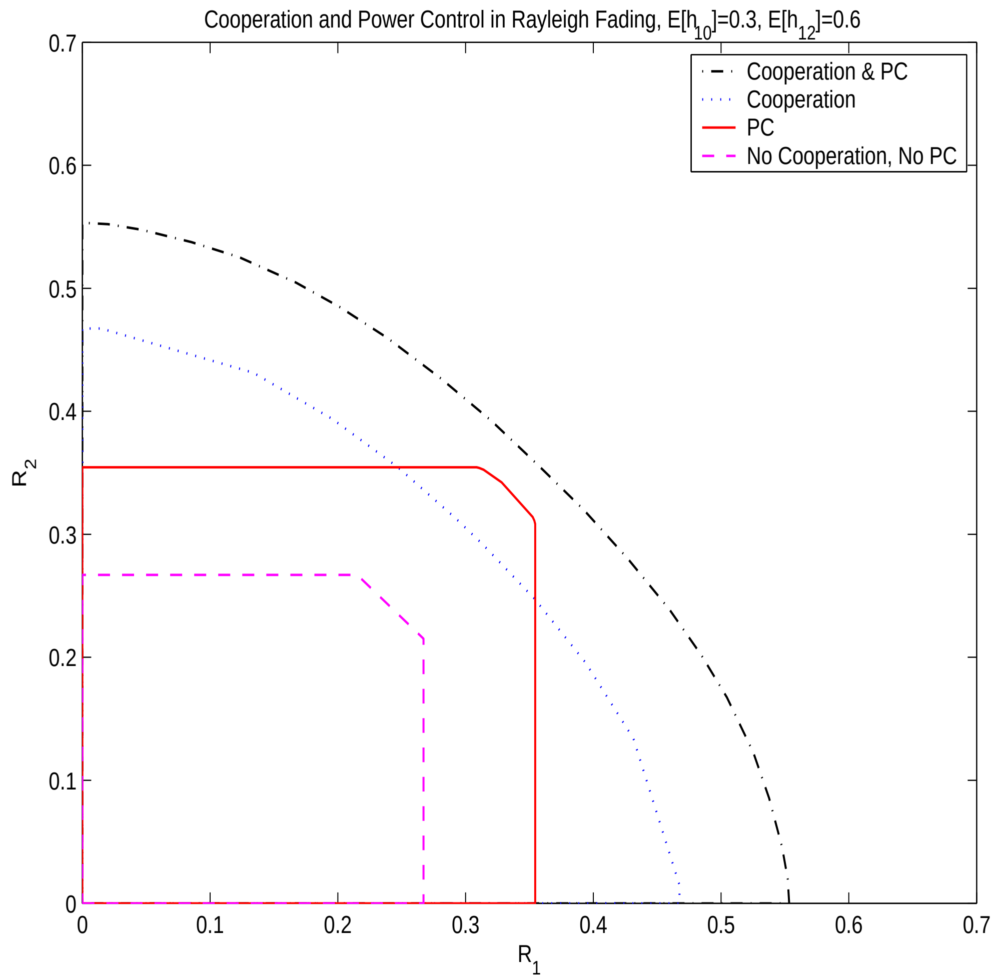
- Provably converges for a diminishing stepsize α_k [Shor].

Convergence of the Projected Subgradient Algorithm



- Rate of convergence depends on the stepsize parameter.
- Subgradient method need not give a monotonically increasing function value.

Achievable Rate Region for Joint Power Control and User Cooperation

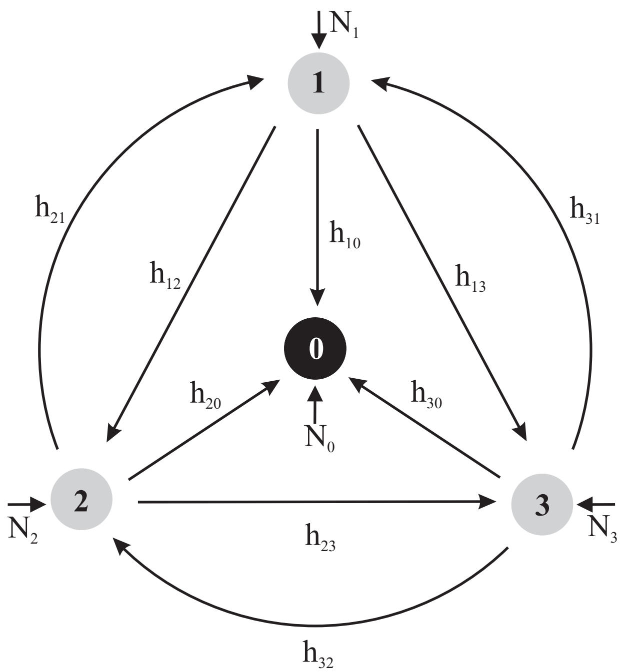


- Optimized power levels enlarge the achievable rate region significantly.

Summary and Conclusions

- Characterized the power control policies that are jointly optimal with Block Markov superposition coding.
- Using sub-gradient methods, obtained optimal power levels and corresponding rate region.
- Joint usage of cooperative diversity and time diversity: major improvements in capacity.
- Encoding and decoding is significantly simplified.
  - Transmitters send either cooperation or fresh information signals, but noth both.
- Optimal power policies also dictate MAC and routing policies
  - Cross layer design.

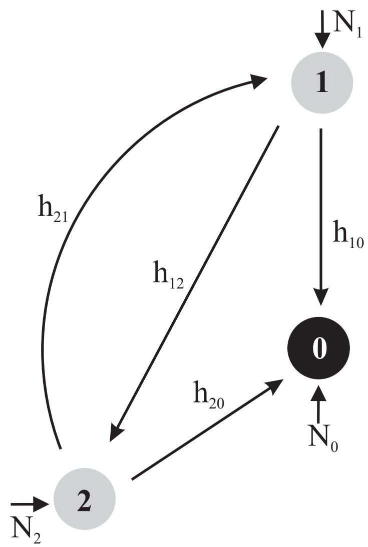
The Three User Cooperative Multiple Access Channel



$$Y_0 = h_{10}X_1 + h_{20}X_2 + h_{30}X_3 + N_0$$
$$Y_1 = h_{21}X_2 + h_{31}X_3 + N_1$$
$$Y_2 = h_{12}X_1 + h_{32}X_3 + N_2$$
$$Y_3 = h_{13}X_1 + h_{23}X_2 + N_3$$

- Joint work with Cagatay Edemen

Two User MAC-GF



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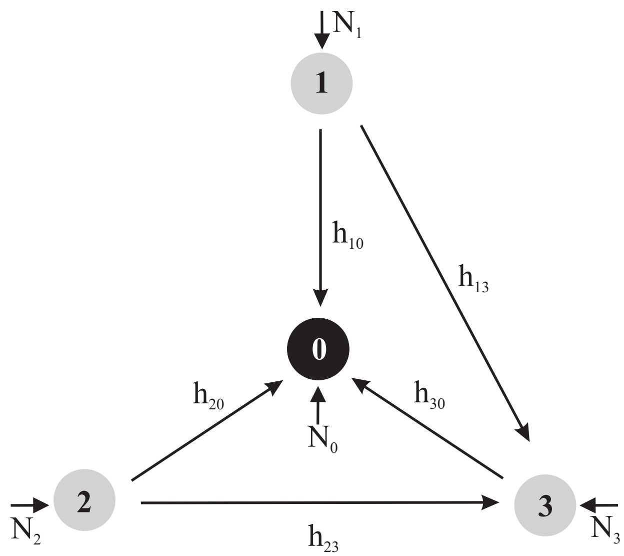
$$Y_1 = h_{21}X_2 + N_1$$
$$Y_2 = h_{12}X_1 + N_2$$

- Achievable rates obtained by block Markov Encoding [Sendonaris-Erkip-Aazhang 2003]

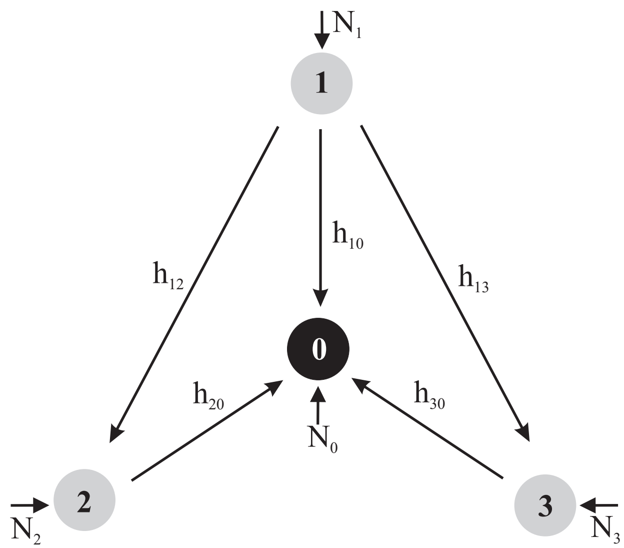
Multiple Access Relay Channel



$$Y_0 = h_{10}X_1 + h_{20}X_2 + h_{30}X_3 + N_0$$
$$Y_3 = h_{13}X_1 + h_{23}X_2 + N_3$$

- Multiple Access Relay Channel [Sankaranarayanan, Kramer, Mandayam 2004]

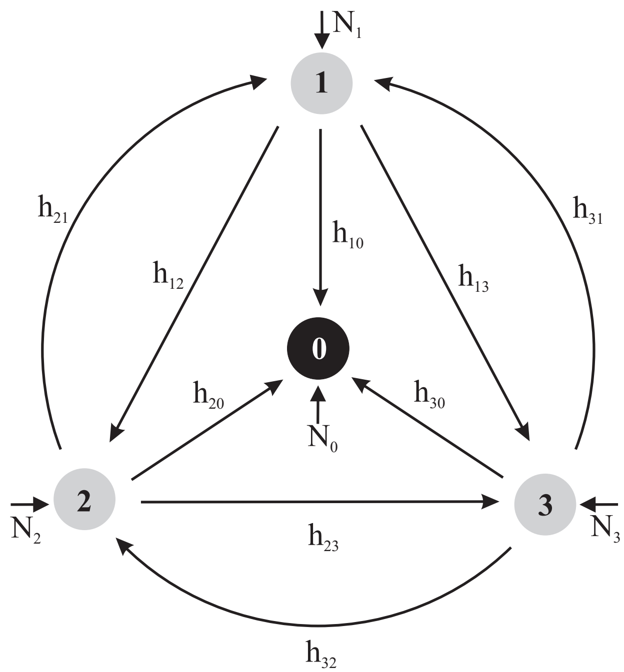
Multiple Relay Channel



$$Y_0 = h_{10}X_1 + h_{20}X_2 + h_{30}X_3 + N_0$$
$$Y_2 = h_{12}X_1 + N_2$$
$$Y_3 = h_{13}X_1 + N_3$$

- Multiple Relay Channel [Schein, Gallager 00],[Kramer, Gastpar, Gupta 03], [Xie, Kumar 05]

Three User Cooperative MAC



$$Y_0 = h_{10}X_1 + h_{20}X_2 + h_{30}X_3 + N_0$$
$$Y_1 = h_{21}X_2 + h_{31}X_3 + N_1$$
$$Y_2 = h_{12}X_1 + h_{32}X_3 + N_2$$
$$Y_3 = h_{13}X_1 + h_{23}X_2 + N_3$$

- Multiple users mutually cooperate: fairer scheme, potentially higher rates, more ad-hoc.

Block Markov Encoding - Two Users

- Two user cooperation: each user's message is divided into two sub-messages
  - $w_1 = (w_{10}, w_{12}), \quad w_2 = (w_{20}, w_{21})$
- Block Markov superposition coding
  - Build common information (X_{12}, X_{21})
  - Cooperatively send (U)
  - Inject new information (X_{10}, X_{20})

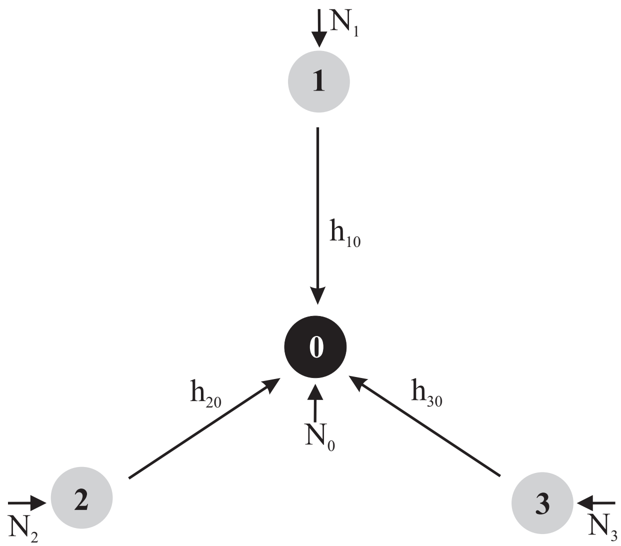
$$X_1 = \sqrt{p_{10}(\mathbf{h})}X_{10} + \sqrt{p_{12}(\mathbf{h})}X_{12} + \sqrt{p_{u1}(\mathbf{h})}U$$
$$X_2 = \sqrt{p_{20}(\mathbf{h})}X_{20} + \sqrt{p_{21}(\mathbf{h})}X_{21} + \sqrt{p_{u2}(\mathbf{h})}U$$

- When cooperative links stronger than direct links, should never send w_{i0} [Kaya-Ulukus 07]

Block Markov Encoding- Three Users

- Extension of Block Markov encoding to three user cooperation:
  - $w_1 = (w_{10}, w_{12}, w_{13}), \quad w_2 = (w_{20}, w_{21}, w_{23}), \quad w_3 = (w_{30}, w_{31}, w_{32})$
- Which messages should be decoded by which users?
  - Potentially too much interference.
- How should the cooperative signals be formed?
  - Should the users cooperate in pairs? Collectively?
- We propose a channel adaptive encoding/decoding approach.
- Drop w_{i0} for simplicity: assume stronger inter-user links as in two user case.

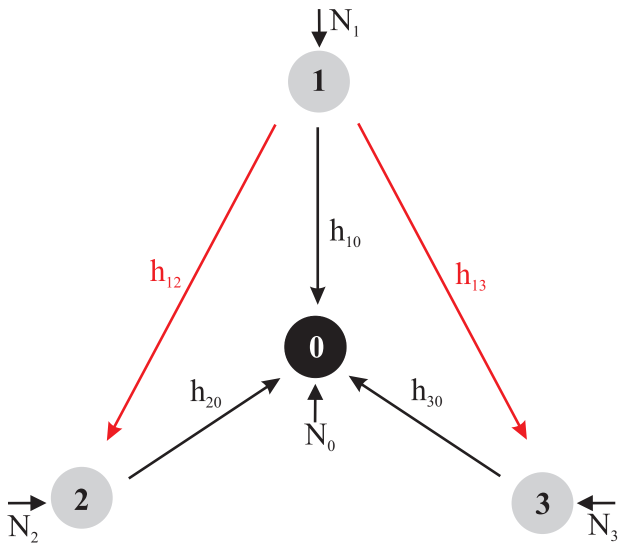
Users' Decoding Capability



- Normalization: $s_{ij} = h_{ij}/\sigma_j^2$
- Assumption:
  - $s_{ij} > s_{i0},\ \forall i, j \in \{1,2,3\},\ i \neq j$

User	Own Messages	Decoded Messages
1	w_{12}, w_{13}	
2	w_{21}, w_{23}	
3	w_{31}, w_{32}	

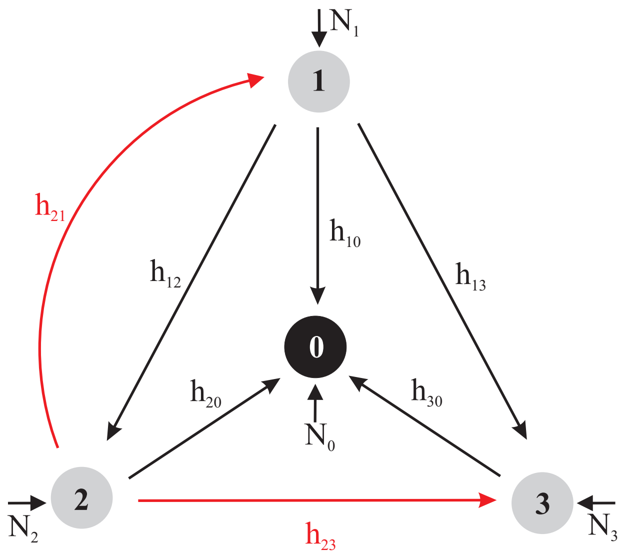
Users' Decoding Capability



- Normalization: $s_{ij} = h_{ij}/\sigma_j^2$
- Assumption:
  - $s_{ij} > s_{i0}, \ \forall i, j \in \{1, 2, 3\}, \ i \neq j$
  - $s_{12} > s_{13}$.

User	Own Messages	Decoded Messages
1	w_{12}, w_{13}	
2	w_{21}, w_{23}	w_{12}, w_{13}
3	w_{31}, w_{32}	w_{13}

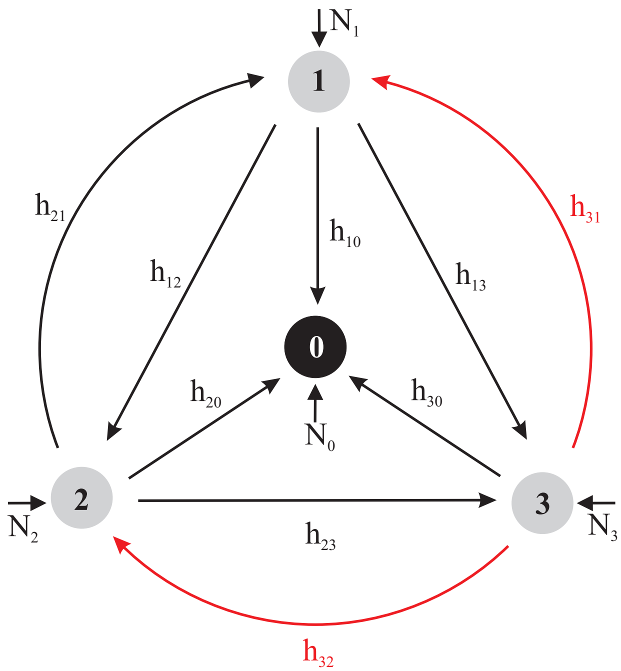
Users' Decoding Capability



- Normalization: $s_{ij} = h_{ij}/\sigma_j^2$
- Assumption:
  - $s_{ij} > s_{i0},\ \forall i, j \in \{1,2,3\},\ i \neq j$
  - $s_{12} > s_{13}$,
  - $s_{21} > s_{23}$

User	Own Messages	Decoded Messages
1	w_{12}, w_{13}	w_{21}, w_{23}
2	w_{21}, w_{23}	w_{12}, w_{13}
3	w_{31}, w_{32}	w_{13}, w_{23}

Users' Decoding Capability



- Normalization: $s_{ij} = h_{ij}/\sigma_j^2$
- Assumption:
  - $s_{ij} > s_{i0}, \ \forall i, j \in \{1, 2, 3\}, \ i \neq j$
  - $s_{12} > s_{13}$,
  - $s_{21} > s_{23}$
  - $s_{32} > s_{31}$

User	Own Messages	Decoded Messages
1	w_{12}, w_{13}	w_{21}, w_{23}, w_{31}
2	w_{21}, w_{23}	$w_{12}, w_{13}, w_{32}, w_{31}$
3	w_{31}, w_{32}	w_{13}, w_{23}

Block Markov Coding

User	Transmitted Codeword
1	$U(w'_{13}, w'_{23}, w'_{31})$, $U_1(w'_{12}, w'_{21}, U)$, $X_{12}(w_{12}, U_1, U)$, $X_{13}(w_{13}, U_1, U)$, $X_{10}(w_{10}, X_{12}, X_{13}, U_1, U)$
2	$U(w'_{13}, w'_{23}, w'_{31})$, $U_1(w'_{12}, w'_{21}, U)$, $U_3(w'_{32}, U)$, $X_{21}(w_{21}, U_1, U_3, U)$, $X_{23}(w_{23}, U_1, U_3, U)$, $X_{20}(w_{20}, X_{21}, X_{23}, U_1, U_3, U)$
3	$U(w'_{13}, w'_{23}, w'_{31})$, $U_3(w'_{32}, U)$, $X_{31}(w_{31}, U_3, U)$, $X_{32}(w_{32}, U_3, U)$, $X_{30}(w_{30}, X_{31}, X_{32}, U_3, U)$

$$X_1 = \sqrt{P_{10}}X_{10} + \sqrt{P_{12}}X_{12} + \sqrt{P_{13}}X_{13} + \sqrt{P_{1U_1}}U_1 + \sqrt{P_{1U}}U$$

$$X_2 = \sqrt{P_{20}}X_{20} + \sqrt{P_{21}}X_{21} + \sqrt{P_{23}}X_{23} + \sqrt{P_{2U_1}}U_1 + \sqrt{P_{2U_3}}U_3 + \sqrt{P_{1U}}U$$

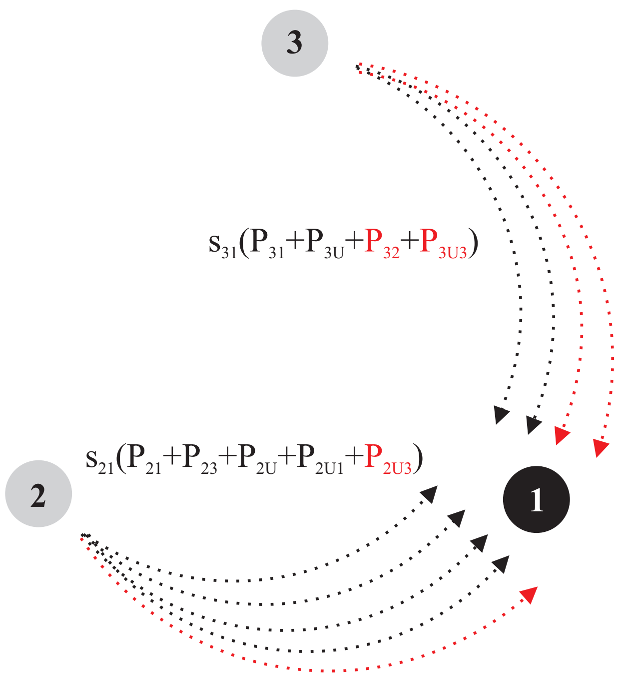
$$X_3 = \sqrt{P_{30}}X_{30} + \sqrt{P_{31}}X_{31} + \sqrt{P_{32}}X_{32} + \sqrt{P_{3U_3}}U_3 + \sqrt{P_{3U}}U$$

$$P_{10} + P_{12} + P_{13} + P_{1U_1} + P_{1U} \leq P_1$$

$$P_{20} + P_{21} + P_{23} + P_{2U_1} + P_{2U_3} + P_{2U} \leq P_2$$

$$P_{30} + P_{31} + P_{32} + P_{3U_3} + P_{3U} \leq P_3$$

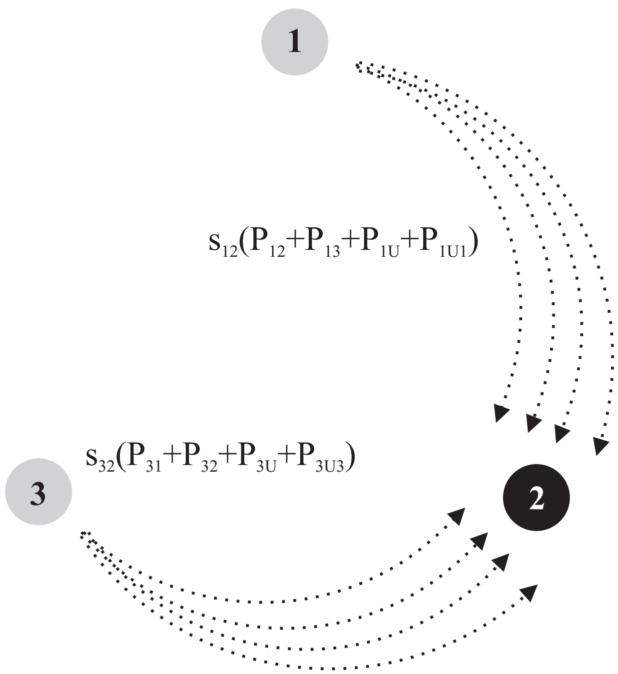
Rate Constraints for Error Free Decoding at User 1



$$
\begin{aligned}
R_{21} &< E\left[\log\left(1+s_{21}P_{21}/A\right)\right]\\
R_{23} &< E\left[\log\left(1+s_{21}P_{23}/A\right)\right]\\
R_{31} &< E\left[\log\left(1+s_{31}P_{31}/A\right)\right]\\
R_{2} &< E\left[\log\left(1+s_{21}(P_{21}+P_{23})/A\right)\right]\\
R_{21}+R_{31} &< E\left[\log\left(1+(s_{21}P_{21}+s_{31}P_{31})/A\right)\right]\\
R_{23}+R_{31} &< E\left[\log\left(1+(s_{21}P_{23}+s_{31}P_{31})/A\right)\right]\\
R_{2}+R_{31} &< E\left[\log\left(1+(s_{21}(P_{21}+P_{23})+s_{31}P_{31})/A\right)\right]
\end{aligned}
$$

- User 1 can decode w_{21}, w_{23} and w_{31} without error
- X_{32} and its cooperative version $U_3(w'_{32})$ are treated as noise at User 1
  - $A = s_{21}P_{2U_3} + s_{31}(P_{32}+P_{3U_3}) + 2\sqrt{s_{21}s_{31}P_{2U_3}P_{3U_3}} + 1$

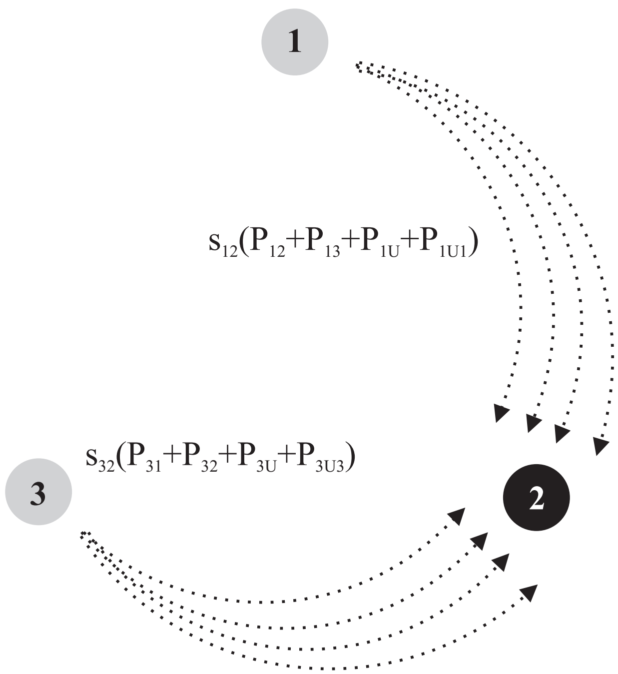
Rate Constraints for Error Free Decoding at User 2



$$R_{12} < E\left[\log\left(1+s_{12}P_{12}\right)\right]$$
$$R_{13} < E\left[\log\left(1+s_{12}P_{13}\right)\right]$$
$$R_{31} < E\left[\log\left(1+s_{32}P_{31}\right)\right]$$
$$R_{32} < E\left[\log\left(1+s_{32}P_{32}\right)\right]$$
$$R_{1} < E\left[\log\left(1+s_{12}(P_{12}+P_{13})\right)\right]$$
$$R_{12}+R_{31} < E\left[\log\left(1+s_{12}P_{12}+s_{32}P_{31}\right)\right]$$
$$R_{12}+R_{32} < E\left[\log\left(1+s_{12}P_{12}+s_{32}P_{32}\right)\right]$$

- User 2 can decode all transmitted signals.
- MAC capacity region with 4 independent messages,
  - no interference terms.

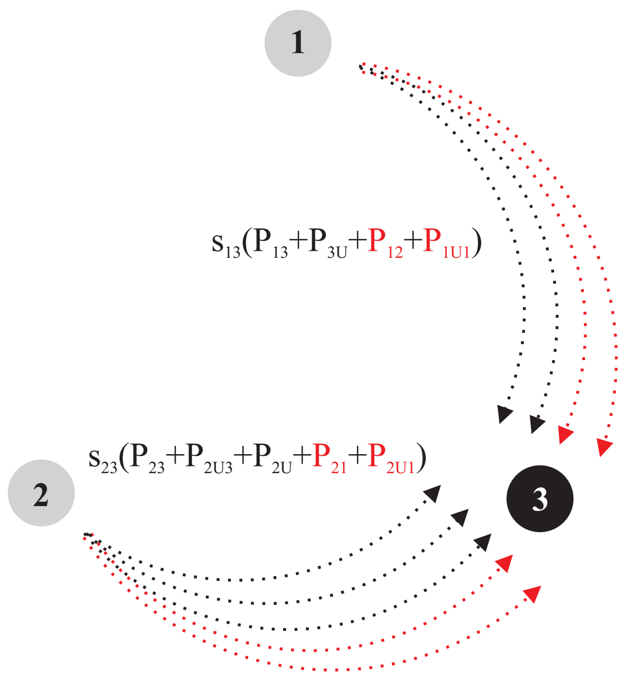
Rate Constraints for Error Free Decoding at User 2 (cnt'd)



$$R_{13} + R_{31} < E\left[\log\left(1 + s_{12}P_{13} + s_{32}P_{31}\right)\right]$$
$$R_{13} + R_{32} < E\left[\log\left(1 + s_{12}P_{13} + s_{32}P_{32}\right)\right]$$
$$R_{3} < E\left[\log\left(1 + s_{32}(P_{31} + P_{32})\right)\right]$$
$$R_{1} + R_{31} < E\left[\log\left(1 + s_{12}(P_{12} + P_{13}) + s_{32}P_{31}\right)\right]$$
$$R_{1} + R_{32} < E\left[\log\left(1 + s_{12}(P_{12} + P_{13}) + s_{32}P_{32}\right)\right]$$
$$R_{12} + R_{3} < E\left[\log\left(1 + s_{12}P_{12} + s_{32}(P_{31} + P_{32})\right)\right]$$
$$R_{13} + R_{3} < E\left[\log\left(1 + s_{12}P_{13} + s_{32}(P_{31} + P_{32})\right)\right]$$
$$R_{1} + R_{3} < E\left[\log\left(1 + s_{12}(P_{12} + P_{13}) + s_{32}(P_{31} + P_{32})\right)\right]$$

- User 2 can decode all transmitted signals.
- MAC capacity region with 4 independent messages,
  - no interference terms.

Rate Constraints for Error Free Decoding at User 3



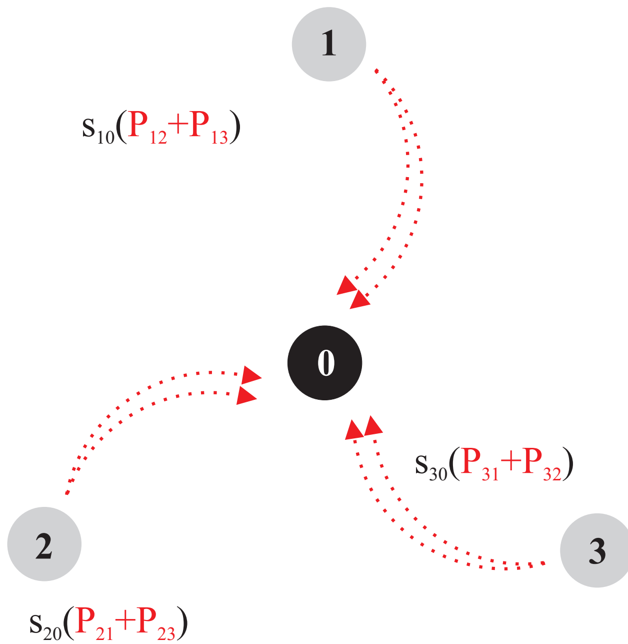
$$R_{13} < E\left[\log\left(1 + s_{13}P_{13}/B\right)\right]$$

$$R_{23} < E\left[\log\left(1 + s_{23}P_{23}/B\right)\right]$$

$$R_{13} + R_{23} < E\left[\log\left(1 + (s_{13}P_{13} + s_{23}P_{23})/B\right)\right]$$

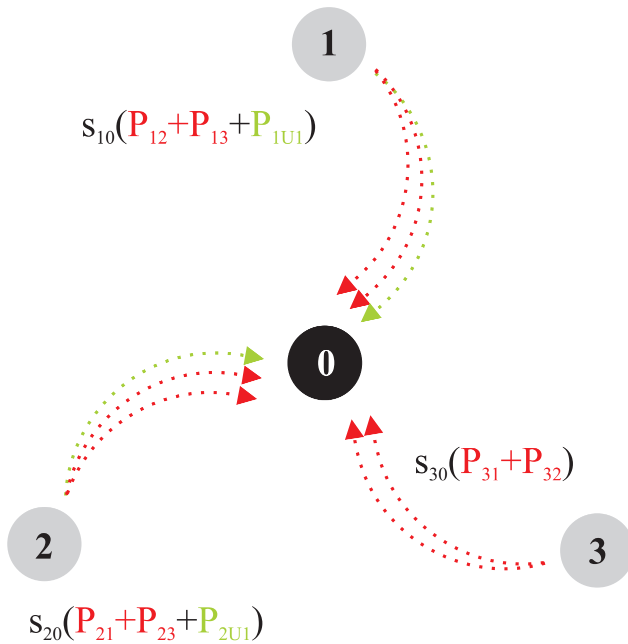
- User 3 can decode w_{13} and w_{23} without error
- X_{12}, X_{21} and their cooperative version $U_1(w'_{12}, w'_{21})$ are treated as noise component at User 3
  - $B = s_{13}(P_{12} + P_{1U_1}) + s_{23}(P_{21} + P_{2U_1}) + 2\sqrt{s_{13}s_{23}P_{1U_1}P_{2U_1}} + 1$

MAC Rate Constraints at Destination Side



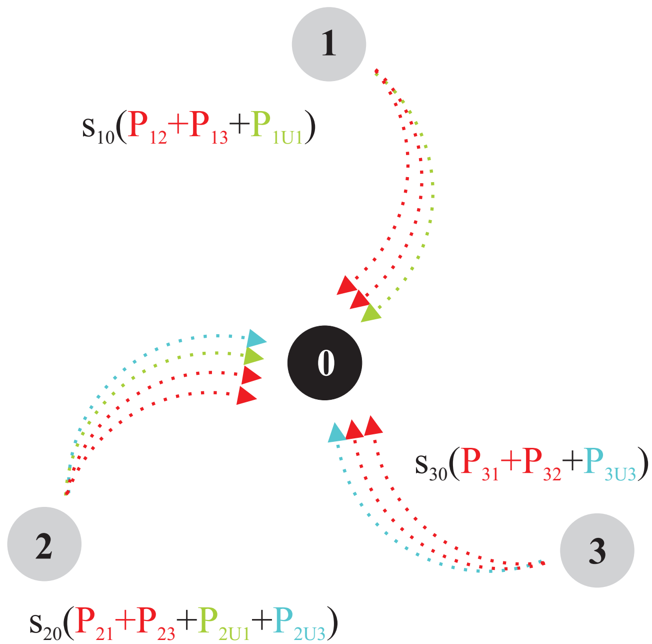
- The destination can decode all transmitted signals using backward decoding.

MAC Rate Constraints at Destination Side



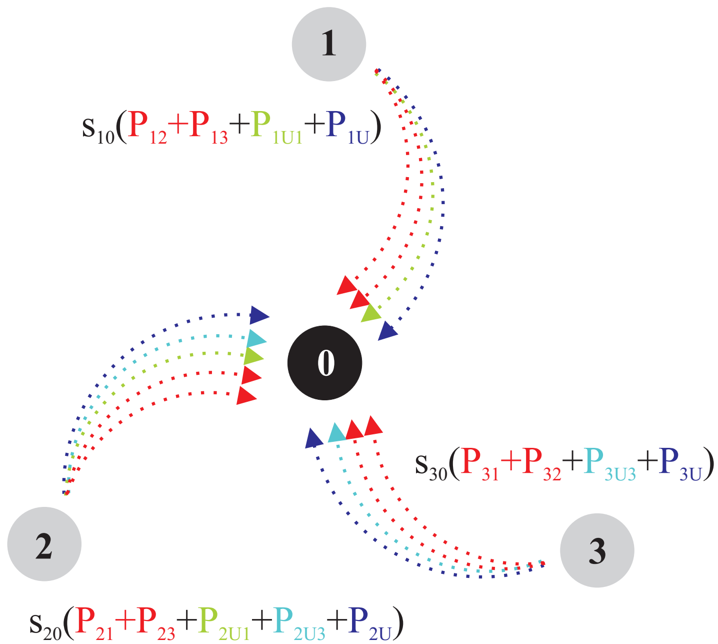
- The destination can decode all transmitted signals using backward decoding.

MAC Rate Constraints at Destination Side



- The destination can decode all transmitted signals using backward decoding.

MAC Rate Constraints at Destination Side

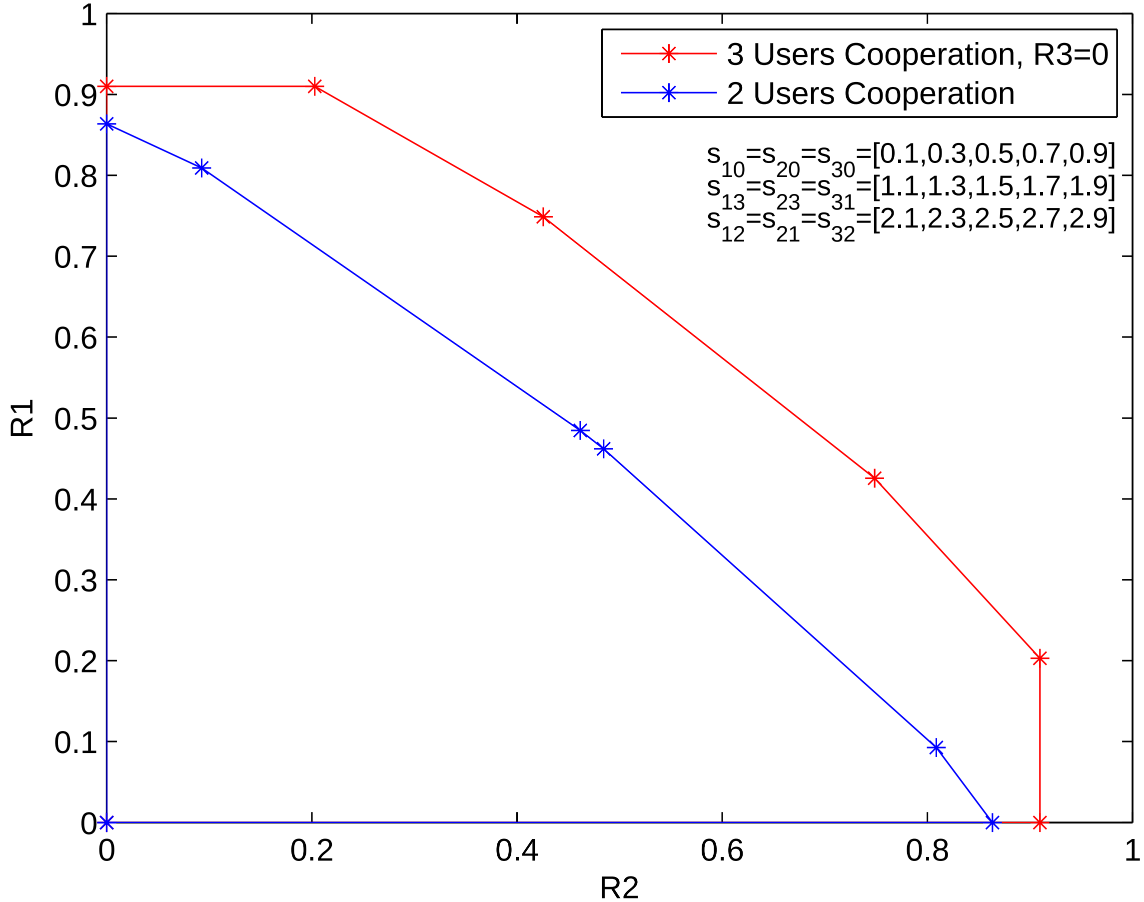


- The destination can decode all transmitted signals using backward decoding.

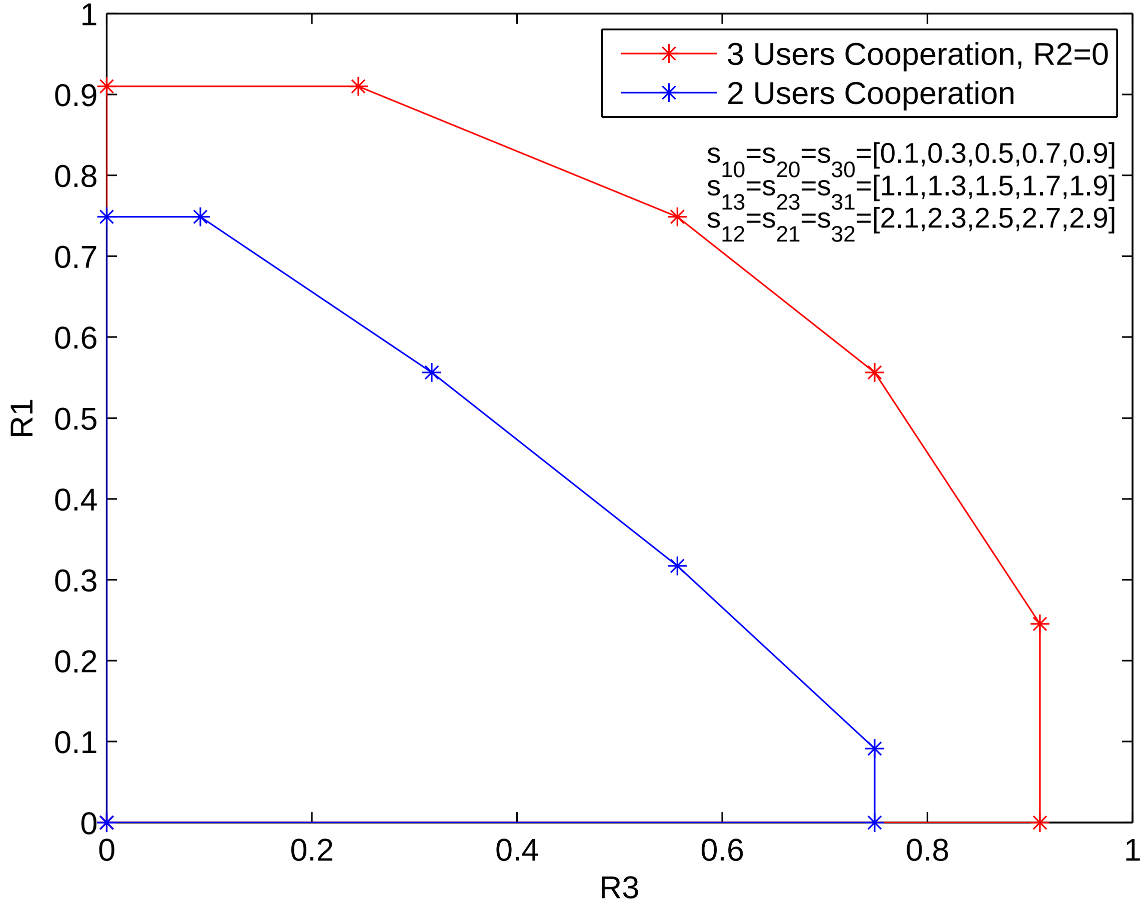
MAC Rate Constraints at Destination Side

$$
\begin{aligned}
R_{32} &< E\left[\log\left(1+s_{20}P_{2U_3}+s_{30}(P_{32}+P_{3U_3})+2\sqrt{s_{20}s_{30}P_{2U_3}P_{3U_3}}\right)\right]\\
R_{12}+R_{21} &< E\left[\log\left(1+s_{10}(P_{12}+P_{1U_1})+s_{20}(P_{21}+P_{2U_1})+2\sqrt{s_{10}s_{20}P_{1U_1}P_{2U_1}}\right)\right]\\
R_{13}+R_{23}+R_{31} &< E\left[\log\left(1+s_{10}(P_{13}+P_{1U})+s_{20}(P_{23}+P_{2U})+s_{30}(P_{31}+P_{3U})\right.\right.\\
&\qquad \left.\left.+2(\sqrt{s_{10}s_{20}P_{1U}P_{2U}}+\sqrt{s_{10}s_{30}P_{1U}P_{3U}}+\sqrt{s_{20}s_{30}P_{2U}P_{3U}})\right)\right]\\
R_{12}+R_{21}+R_{32} &< E\left[\log\left(1+s_{10}(P_{12}+P_{1U_1})+s_{20}(P_{21}+P_{2U_1}+P_{2U_3})+s_{30}(P_{32}+P_{3U_3})\right.\right.\\
&\qquad \left.\left.+2\sqrt{s_{10}s_{20}P_{1U_1}P_{2U_1}}+2\sqrt{s_{20}s_{30}P_{2U_3}P_{3U_3}}\right)\right]\\
R_{13}+R_{23}+R_{3} &< E\left[\log\left(1+s_{10}(P_{13}+P_{1U})+s_{20}(P_{23}+P_{2U}+P_{2U_3})+s_{30}P_{3}\right.\right.\\
&\qquad \left.\left.+2\sqrt{s_{20}s_{30}P_{2U_3}P_{3U_3}}+2(\sqrt{s_{10}s_{20}P_{1U}P_{2U}}+\sqrt{s_{10}s_{30}P_{1U}P_{3U}}+\sqrt{s_{20}s_{30}P_{2U}P_{3U}})\right)\right]\\
R_{1}+R_{2}+R_{31} &< E\left[\log\left(1+s_{10}P_{1}+s_{20}(P_{21}+P_{23}+P_{2U}+P_{2U_1})+s_{30}(P_{31}+P_{3U})\right.\right.\\
&\qquad \left.\left.+2\sqrt{s_{10}s_{20}P_{1U_1}P_{2U_1}}+2(\sqrt{s_{10}s_{20}P_{1U}P_{2U}}+\sqrt{s_{10}s_{30}P_{1U}P_{3U}}+\sqrt{s_{20}s_{30}P_{2U}P_{3U}})\right)\right]\\
R_{1}+R_{2}+R_{3} &< E\left[\log\left(1+s_{10}P_{1}+s_{20}P_{2}+s_{30}P_{3}\right.\right.\\
&\qquad \left.\left.+2\sqrt{s_{10}s_{20}P_{1U_1}P_{2U_1}}+2\sqrt{s_{20}s_{30}P_{2U_3}P_{3U_3}}+2(\sqrt{s_{10}s_{20}P_{1U}P_{2U}}+\sqrt{s_{10}s_{30}P_{1U}P_{3U}}+\sqrt{s_{20}s_{30}P_{2U}P_{3U}})\right)\right]
\end{aligned}
$$

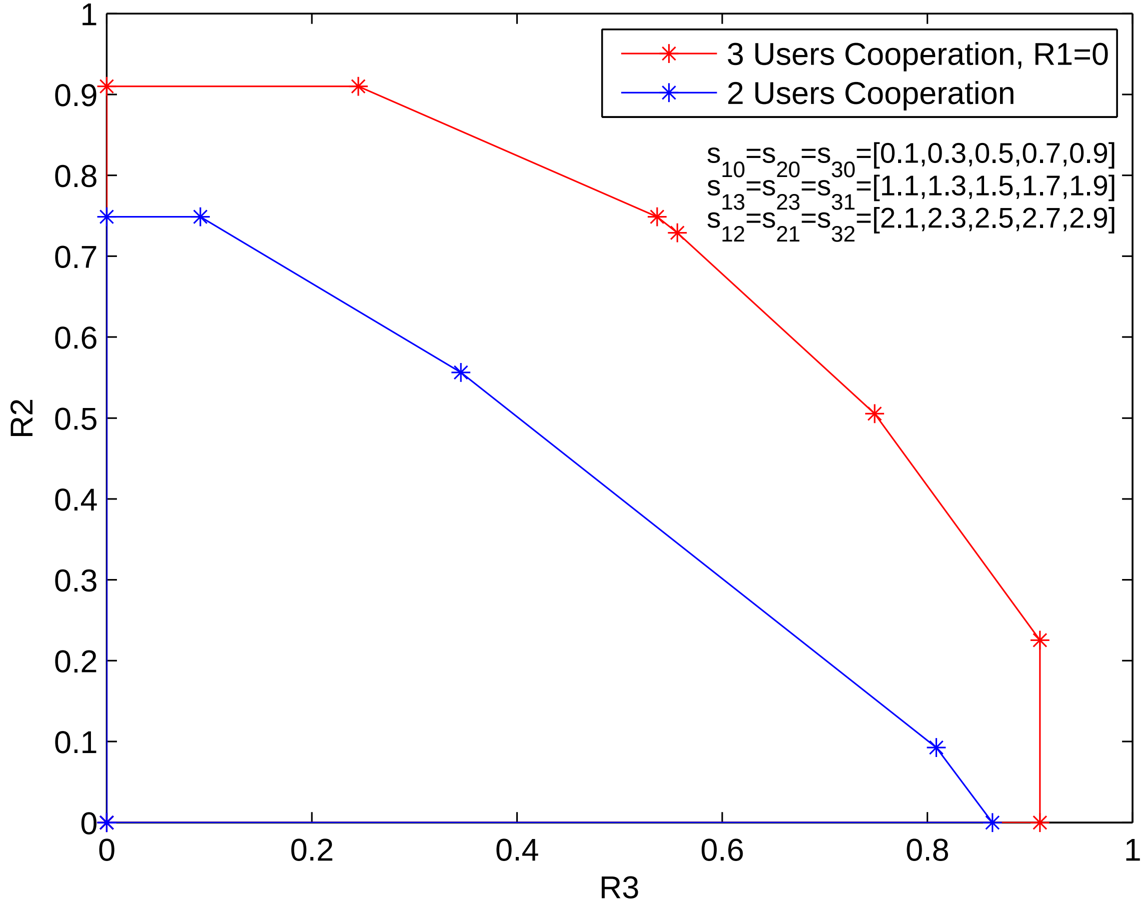
Achievable Rates for Three User Cooperation



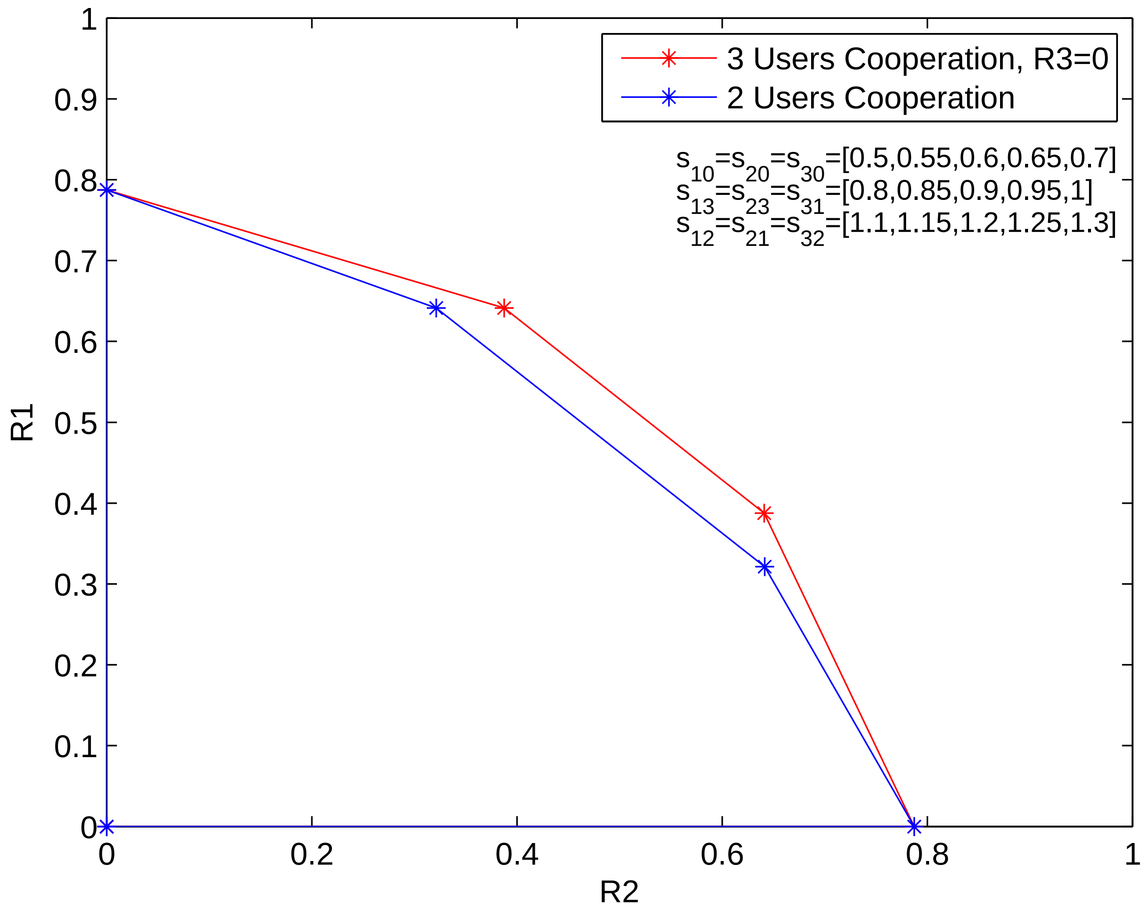
Achievable Rates for Three User Cooperation



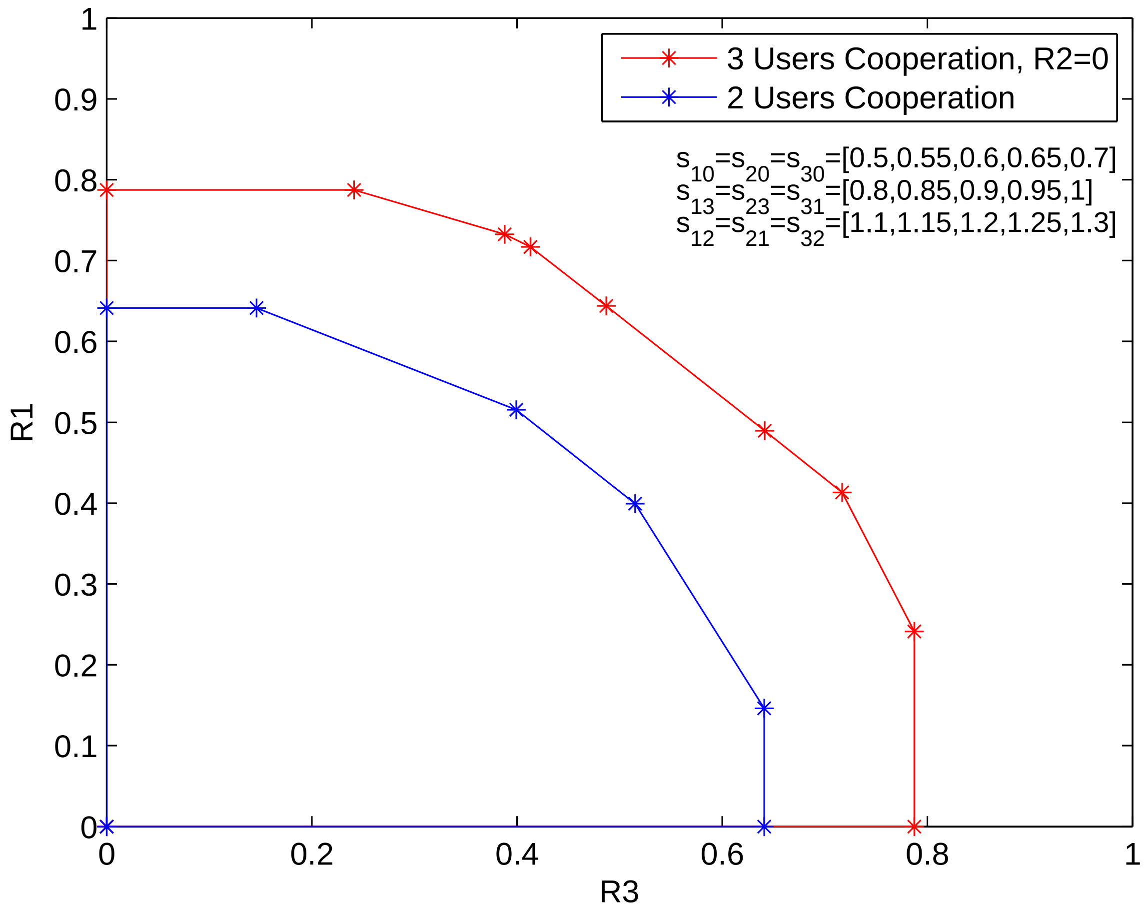
Achievable Rates for Three User Cooperation



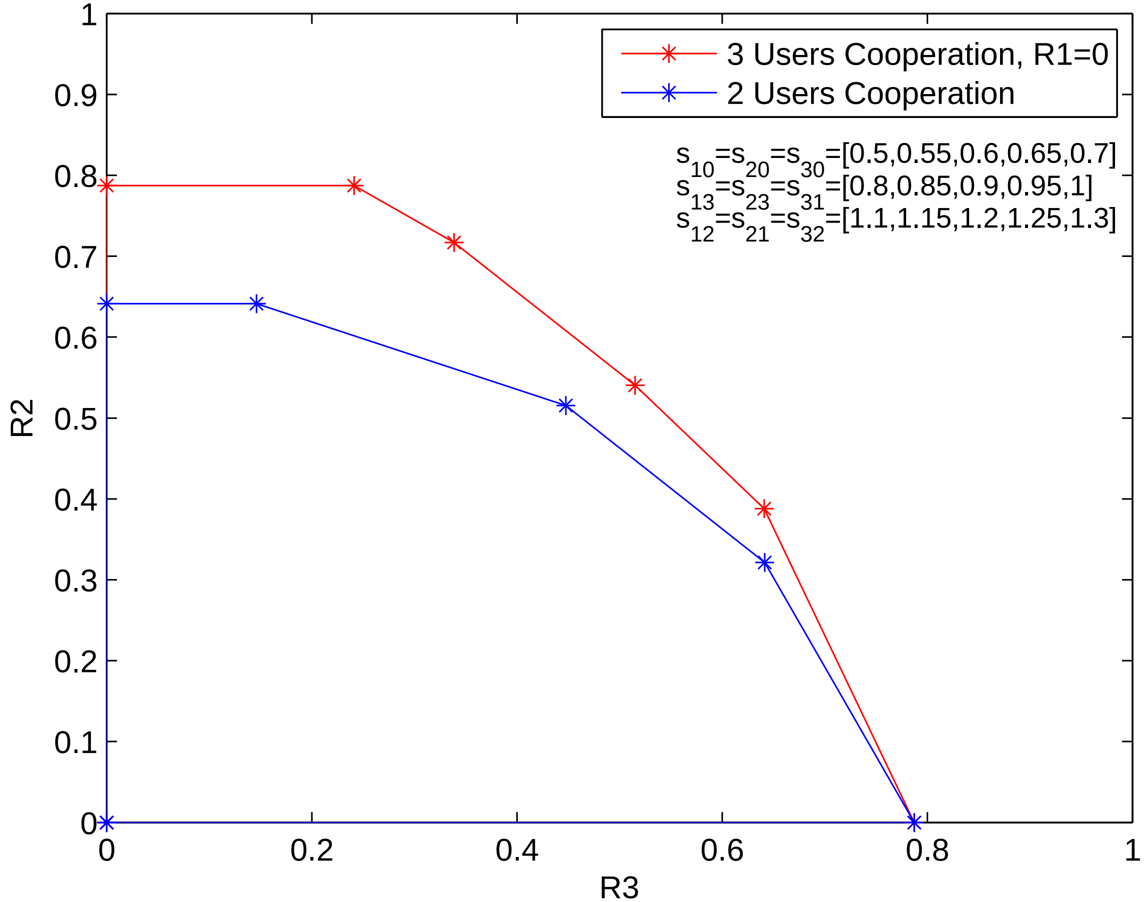
Achievable Rates for Three User Cooperation



Achievable Rates for Three User Cooperation



Achievable Rates for Three User Cooperation



Channel Ordering Assumption Revisited

- Considered channel orderings of type

$$s_{ij} > s_{ik}, \quad s_{ji} > s_{jk}, \quad s_{kj} > s_{ki}, \quad i \neq j \neq k$$

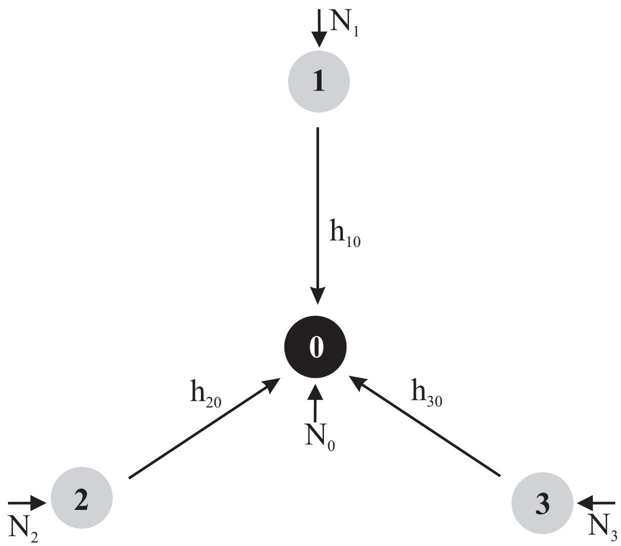
  - Concept of ”strong” and ”weak” users

- Other channel orderings possible

$$s_{ij} > s_{ik}, \quad s_{jk} > s_{ji}, \quad s_{ki} > s_{kj}, \quad i \neq j \neq k$$

- We need to update the encoding/decoding accordingly.
- Not asymmetric as before, no particular ”strong user”: strategy becomes somewhat different.
- Idea: choose the right encoding strategy based on the instantaneous channel ordering.

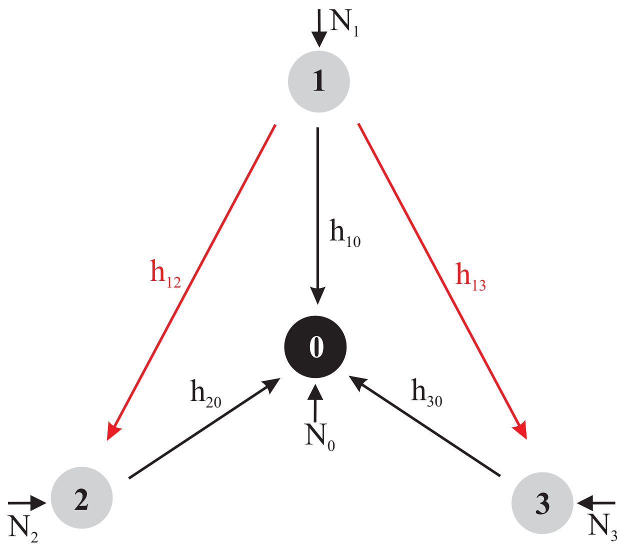
Users' Decoding Capability



- Normalization: $s_{ij} = h_{ij}/\sigma_j^2$
- Assumption:
  - $s_{ij} > s_{i0},\ \forall i, j \in \{1,2,3\},\ i \neq j$

User	Own Messages	Decoded Messages
1	w_{12}, w_{13}	
2	w_{21}, w_{23}	
3	w_{31}, w_{32}	

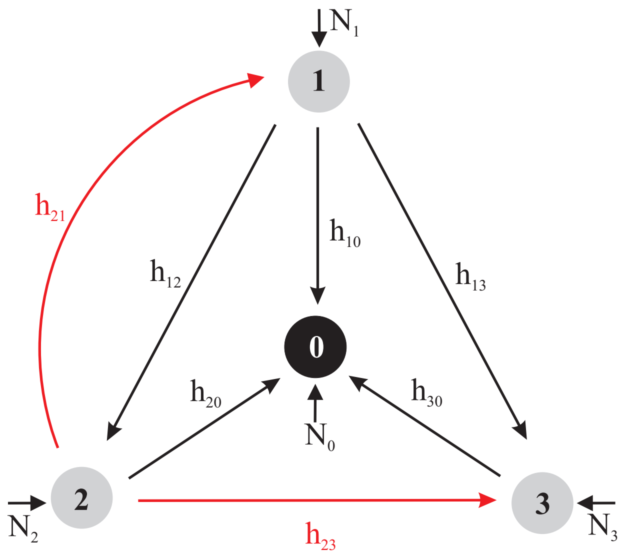
Users' Decoding Capability



- Normalization: $s_{ij} = h_{ij}/\sigma_j^2$
- Assumption:
  - $s_{ij} > s_{i0},\ \forall i, j \in \{1,2,3\},\ i \neq j$
  - $s_{12} > s_{13}$.

User	Own Messages	Decoded Messages
1	w_{12}, w_{13}	
2	w_{21}, w_{23}	w_{12}, w_{13}
3	w_{31}, w_{32}	w_{13}

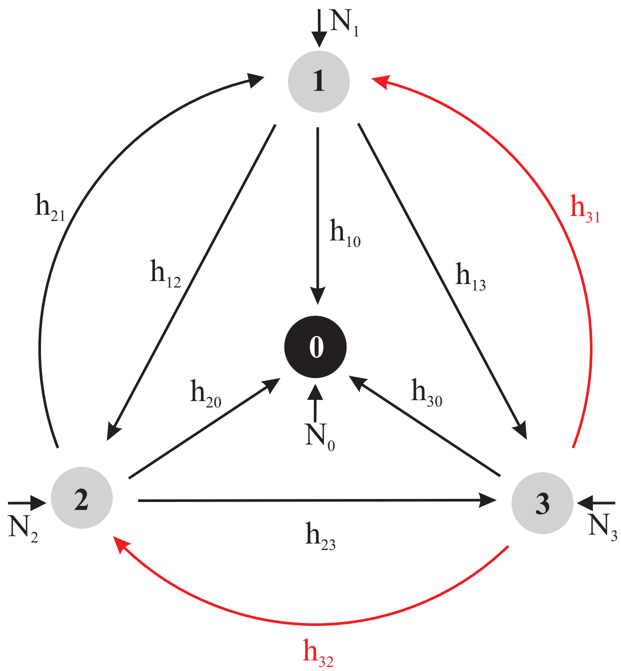
Users' Decoding Capability



- Normalization: $s_{ij} = h_{ij}/\sigma_j^2$
- Assumption:
  - $s_{ij} > s_{i0}, \ \forall i, j \in \{1, 2, 3\}, \ i \neq j$
  - $s_{12} > s_{13}$,
  - $s_{23} > s_{21}$

User	Own Messages	Decoded Messages
1	w_{12}, w_{13}	w_{21}
2	w_{21}, w_{23}	w_{12}, w_{13}
3	w_{31}, w_{32}	w_{13}, w_{23}, w_{21}

Users' Decoding Capability



- Normalization: $s_{ij} = h_{ij}/\sigma_j^2$
- Assumption:
  - $s_{ij} > s_{i0}$, $\forall i, j \in \{1, 2, 3\}$, $i \neq j$
  - $s_{12} > s_{13}$,
  - $s_{23} > s_{21}$
  - $s_{31} > s_{32}$

User	Own Messages	Decoded Messages
1	w_{12}, w_{13}	w_{21}, w_{31}, w_{32}
2	w_{21}, w_{23}	w_{12}, w_{13}, w_{32}
3	w_{31}, w_{32}	w_{13}, w_{23}, w_{21}

Block Markov Coding

User	Transmitted Codeword
1	$U(w'_{13}, w'_{21}, w'_{32})$, $U_1(w'_{12}, U)$, $U_3(w'_{31}, U)$, $X_{12}(w_{12}, U_1, U)$, $X_{13}(w_{13}, U_3, U)$, $X_{10}(w_{10}, X_{12}, X_{13}, U_1, U_3, U)$
2	$U(w'_{13}, w'_{21}, w'_{32})$, $U_1(w'_{12}, U)$, $U_2(w'_{23}, U)$, $X_{21}(w_{21}, U_1, U)$, $X_{23}(w_{23}, U_2, U)$, $X_{20}(w_{20}, X_{21}, X_{23}, U_1, U_2, U)$
3	$U(w'_{13}, w'_{21}, w'_{32})$, $U_3(w'_{31}, U)$, $U_2(w'_{23}, U)$, $X_{31}(w_{31}, U_3, U)$, $X_{32}(w_{32}, U_2, U)$, $X_{30}(w_{30}, X_{31}, X_{32}, U_2, U_3, U)$

$$X_1 = \sqrt{P_{10}}X_{10} + \sqrt{P_{12}}X_{12} + \sqrt{P_{13}}X_{13} + \sqrt{P_{1U_1}}U_1 + \sqrt{P_{1U_3}}U_3 + \sqrt{P_{1U}}U$$

$$X_2 = \sqrt{P_{20}}X_{20} + \sqrt{P_{21}}X_{21} + \sqrt{P_{23}}X_{23} + \sqrt{P_{2U_1}}U_1 + \sqrt{P_{2U_2}}U_2 + \sqrt{P_{1U}}U$$

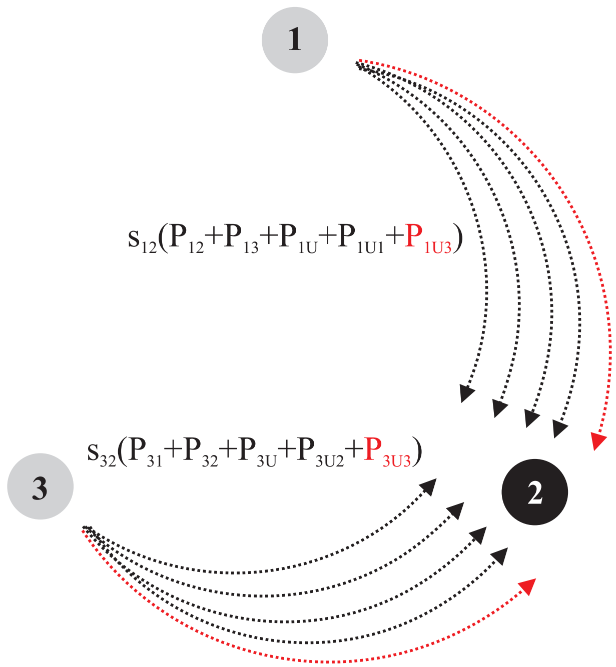
$$X_3 = \sqrt{P_{30}}X_{30} + \sqrt{P_{31}}X_{31} + \sqrt{P_{32}}X_{32} + \sqrt{P_{3U_2}}U_2 + \sqrt{P_{3U_3}}U_3 + \sqrt{P_{3U}}U$$

$$P_{10} + P_{12} + P_{13} + P_{1U_1} + P_{1U_3} + P_{1U} \leq P_1$$

$$P_{20} + P_{21} + P_{23} + P_{2U_1} + P_{2U_2} + P_{2U} \leq P_2$$

$$P_{30} + P_{31} + P_{32} + P_{3U_2} + P_{3U_3} + P_{3U} \leq P_3$$

Rate Constraints for Error Free Decoding at User 1

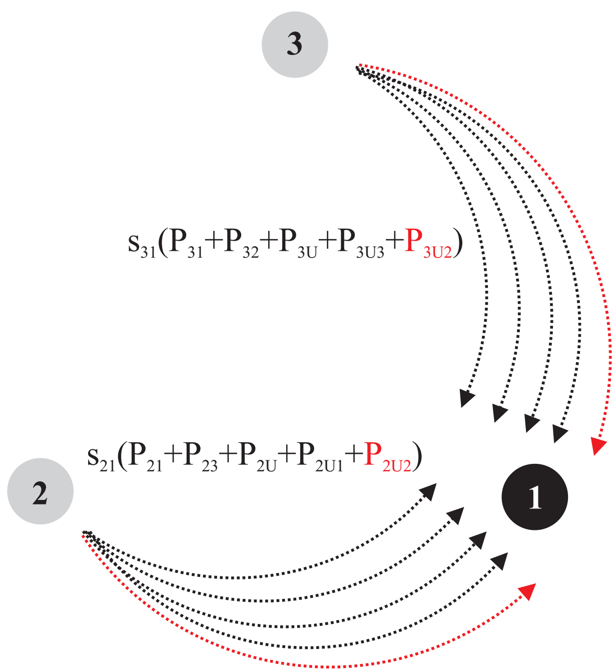


$$\sum_{\{i,j\}\in\Gamma_1} R_{ij} < E\left[\log\left(1+\frac{\sum_{\{i,j\}\in\Gamma_1} s_{i1}P_{ij}}{A}\right)\right]$$

$$\forall\Gamma_1 \subset \{\{2,1\},\{3,1\},\{3,2\}\}$$

- User 1 can decode w_{21}, w_{31} and w_{32} without error
- $U_2(w'_{23})$ is treated as noise at User 1
  - $A = 1 + s_{21}P_{2U2} + s_{31}P_{3U2} + 2\sqrt{s_{21}s_{31}P_{2U2}P_{3U2}}$

Rate Constraints for Error Free Decoding at User 2

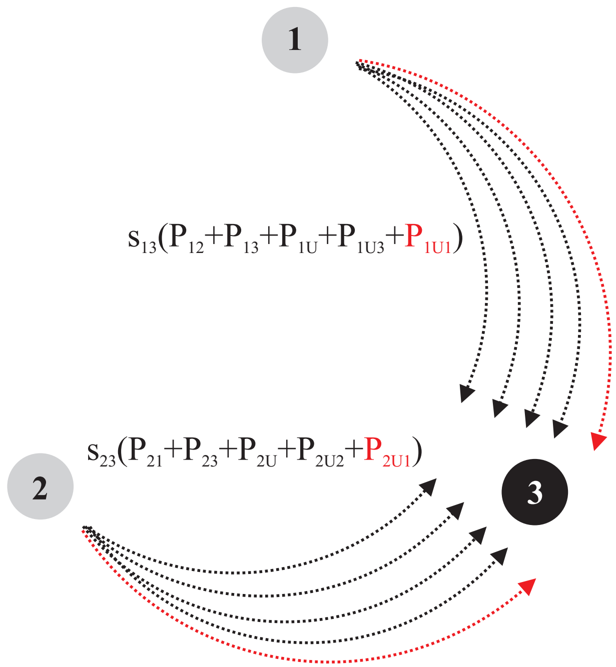


$$\sum_{\{i,j\}\in\Gamma_2} R_{ij} < E\left[\log\left(1+\frac{\sum_{\{i,j\}\in\Gamma_2} s_{i2}P_{ij}}{B}\right)\right]$$

$$\forall\Gamma_2 \subset \{\{1,2\},\{1,3\},\{3,2\}\}$$

- User 2 can decode w_{12}, w_{13} and w_{32} without error
- $U_3(w'_{31})$ is treated as noise at User 2
  - $B = 1 + s_{12}P_{1U3} + s_{32}P_{3U3} + 2\sqrt{s_{12}s_{32}P_{1U3}P_{3U3}}$

Rate Constraints for Error Free Decoding at User 3

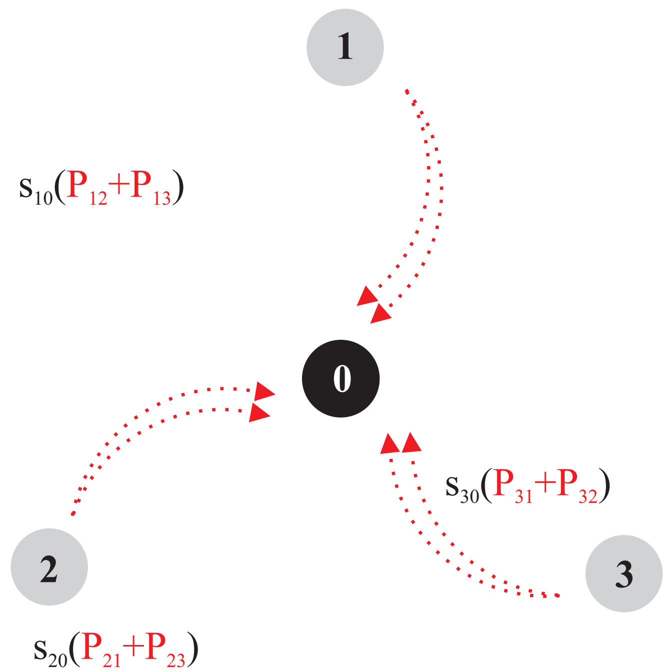


$$\sum_{\{i,j\}\in\Gamma_3} R_{ij} < E\left[\log\left(1+\frac{\sum_{\{i,j\}\in\Gamma_3} s_{i3}P_{ij}}{C}\right)\right]$$

$$\forall\Gamma_3 \subset \{\{1,3\},\{2,1\},\{2,3\}\}$$

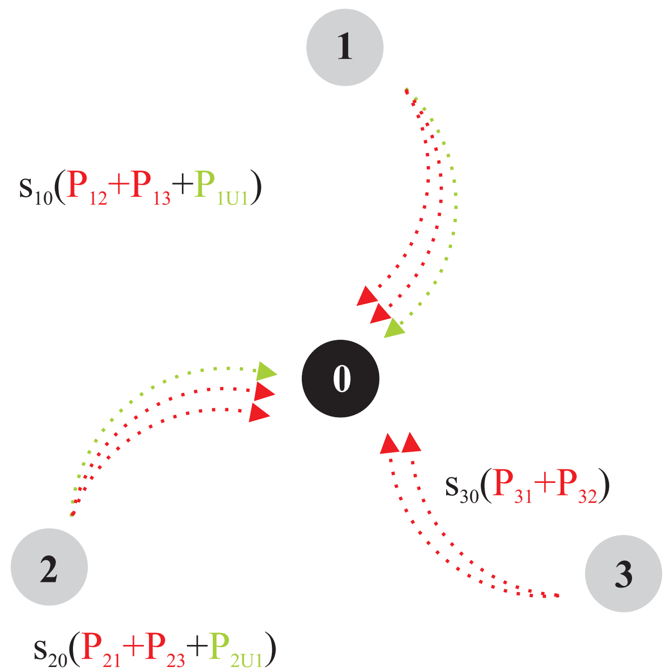
- User 3 can decode w_{13}, w_{21} and w_{23} without error
- $U_1(w'_{12})$ is treated as noise at User 3
  - $C = 1 + s_{13}P_{1U1} + s_{23}P_{2U1} + 2\sqrt{s_{13}s_{23}P_{1U1}P_{2U1}}$

MAC Rate Constraints at Destination Side



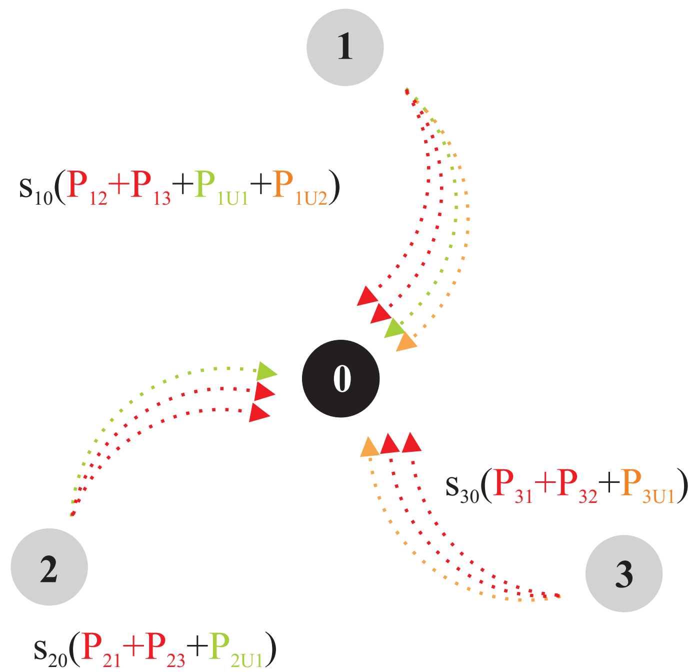
- The destination can decode all transmitted signals using backward decoding.

MAC Rate Constraints at Destination Side



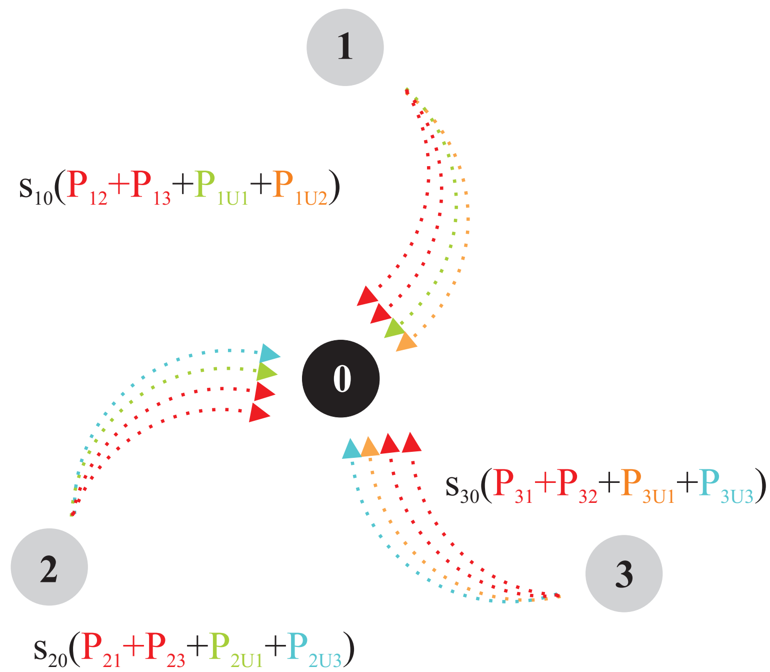
- The destination can decode all transmitted signals using backward decoding.

MAC Rate Constraints at Destination Side



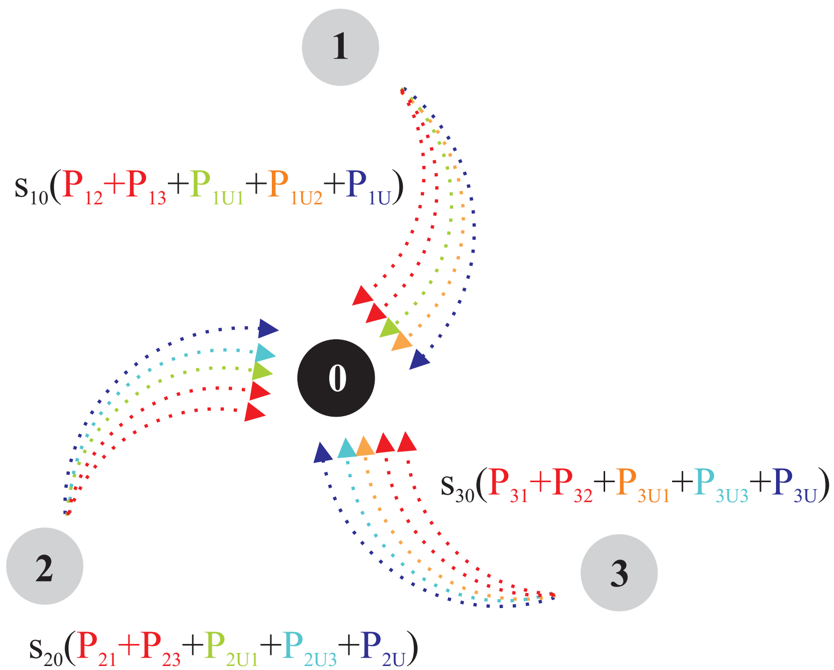
- The destination can decode all transmitted signals using backward decoding.

MAC Rate Constraints at Destination Side



- The destination can decode all transmitted signals using backward decoding.

MAC Rate Constraints at Destination Side



- The destination can decode all transmitted signals using backward decoding.

MAC Rate Constraints at Destination Side

$$R_{12} < E\left[\log\left(1+s_{10}(P_{12}+P_{1U1})+s_{20}(P_{21}+P_{2U1})+2\sqrt{s_{10}s_{20}P_{1U1}P_{2U1}}\right)\right]$$

$$R_{31} < E\left[\log\left(1+s_{10}(P_{13}+P_{1U3})+s_{30}(P_{31}+P_{3U3})+2\sqrt{s_{10}s_{30}P_{1U3}P_{3U3}}\right)\right]$$

$$R_{23} < E\left[\log\left(1+s_{20}(P_{23}+P_{2U2})+s_{30}(P_{32}+P_{3U2})+2\sqrt{s_{20}s_{30}P_{2U2}P_{3U2}}\right)\right]$$

$$R_{12}+R_{23} < E\left[\log\left(1+s_{10}(P_{12}+P_{1U1})+s_{20}(P_{21}+P_{23}+P_{2U1}+P_{2U2})+s_{30}(P_{32}+P_{3U2}) +2\sqrt{s_{10}s_{20}P_{1U1}P_{2U1}}+2\sqrt{s_{20}s_{30}P_{2U2}P_{3U2}}\right)\right]$$

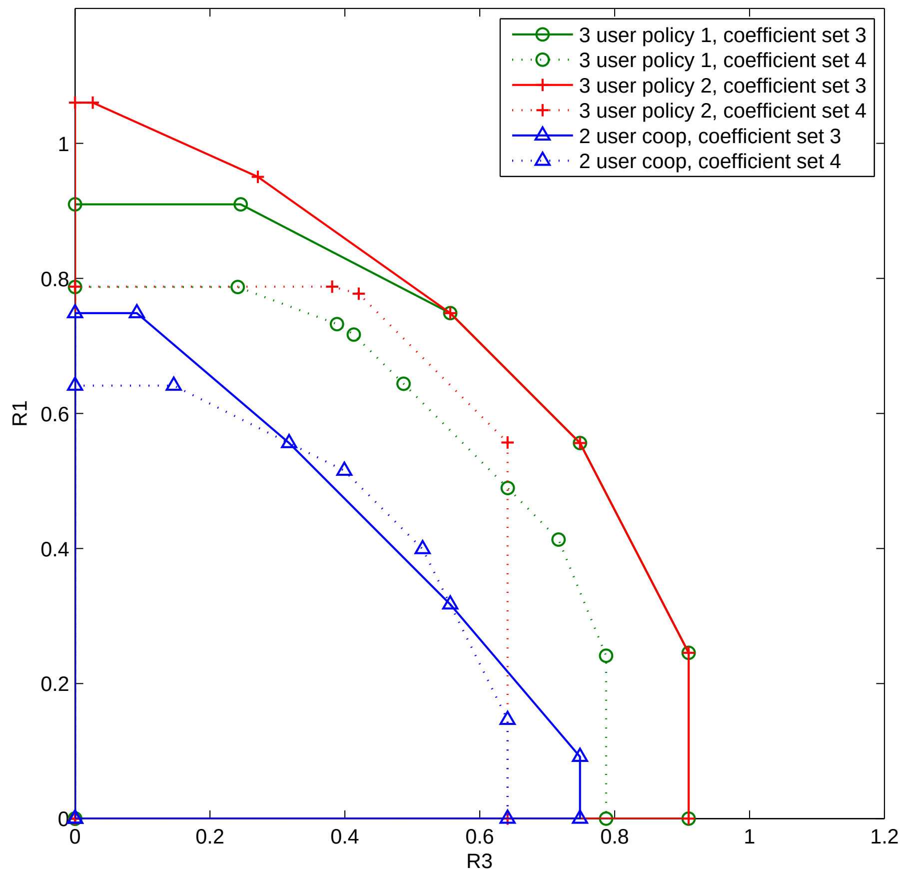
$$R_{12}+R_{31} < E\left[\log\left(1+s_{10}(P_{12}+P_{13}+P_{1U1}+P_{1U3})+s_{20}(P_{21}+P_{2U1})+s_{30}(P_{31}+P_{3U3}) +2\sqrt{s_{10}s_{20}P_{1U1}P_{2U1}}+2\sqrt{s_{10}s_{30}P_{1U3}P_{3U3}}\right)\right]$$

$$R_{23}+R_{31} < E\left[\log\left(1+s_{10}(P_{13}+P_{1U3})+s_{20}(P_{23}+P_{2U2})+s_{30}(P_{31}+P_{32}+P_{3U2}+P_{3U3}) +2\sqrt{s_{10}s_{30}P_{1U3}P_{3U3}}+2\sqrt{s_{20}s_{30}P_{2U2}P_{3U2}}\right)\right]$$

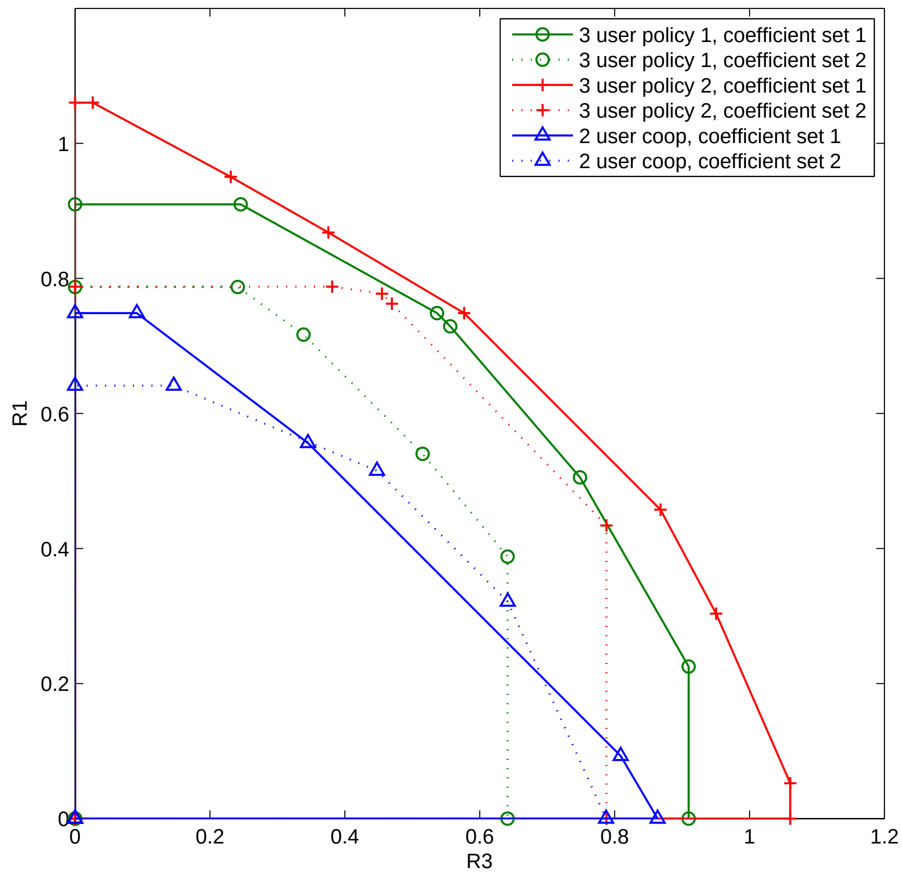
$$R_{12}+R_{23}+R_{31} < E\left[\log\left(1+s_{10}(P_{12}+P_{13}+P_{1U1}+P_{1U3})+s_{20}(P_{21}+P_{23}+P_{2U1}+P_{2U2})+s_{30}(P_{31}+P_{32}+P_{3U2}+P_{3U3}) +2\sqrt{s_{10}s_{20}P_{1U1}P_{2U1}}+2\sqrt{s_{10}s_{30}P_{1U3}P_{3U3}}+2\sqrt{s_{20}s_{30}P_{2U2}P_{3U2}}\right)\right]$$

$$R_1+R_2+R_3 < E\left[\log\left(1+s_{10}P_1+s_{20}P_2+s_{30}P_3+2\sqrt{s_{10}s_{20}P_{1U1}P_{2U1}} +2\sqrt{s_{10}s_{30}P_{1U3}P_{3U3}}+2\sqrt{s_{20}s_{30}P_{2U2}P_{3U2}}+2(\sqrt{s_{10}s_{20}P_{1U}P_{2U}}+\sqrt{s_{10}s_{30}P_{1U}P_{3U}}+\sqrt{s_{20}s_{30}P_{2U}P_{3U}})\right)\right]$$

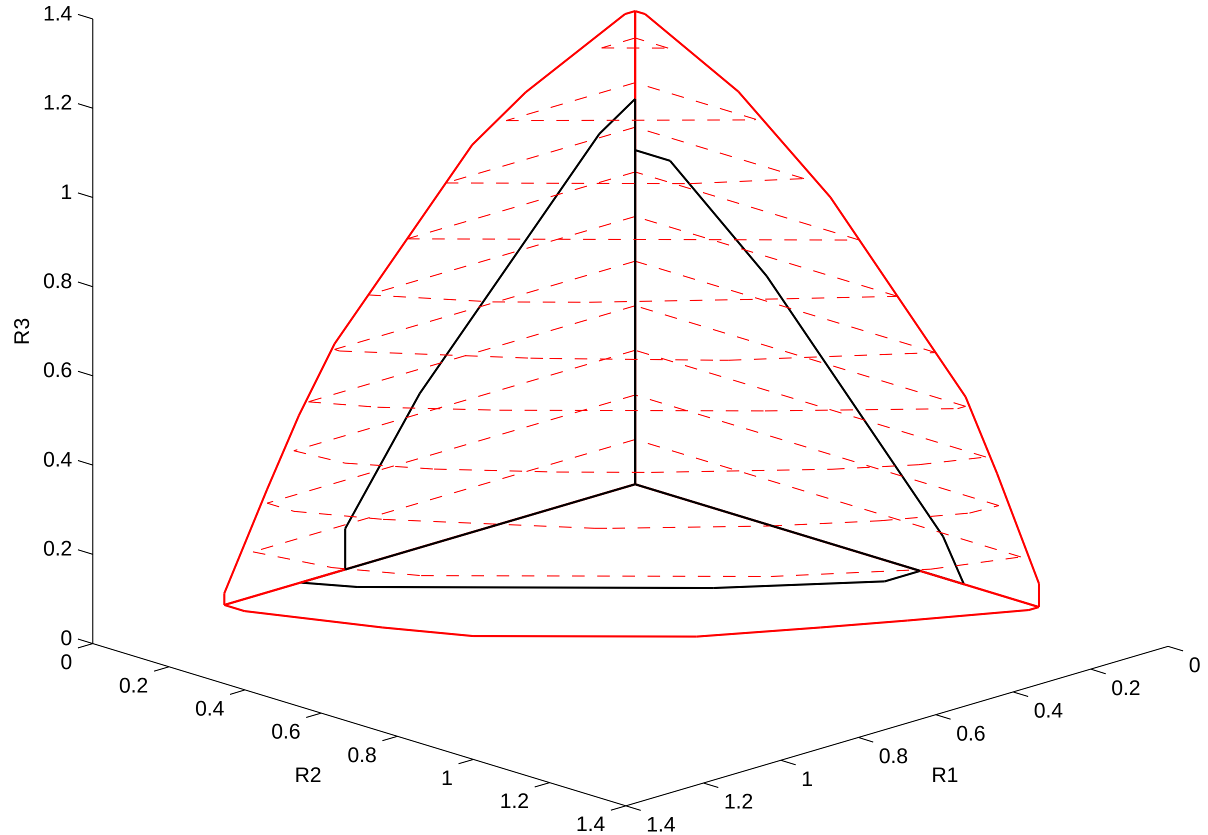
Achievable Rates for Three User Cooperation - Channel Ordering I



Achievable Rates for Three User Cooperation - Channel Ordering II



Achievable Rates for Three User Cooperation (3D)



Summary and Conclusions

- Proposed a new block Markov superposition type encoding policy for the three user MAC
  - Non-trivial generalization of the two user policy
  - Channel adaptive
- Obtained the achievable rate regions
- Significant improvement with respect to two user cooperation
  - Multi-user cooperation quite promising as a means of improving diversity.
- Adapting the encoding, decoding and transmit strategies to the channel in cooperative networks is a key approach for improving achievable rates.