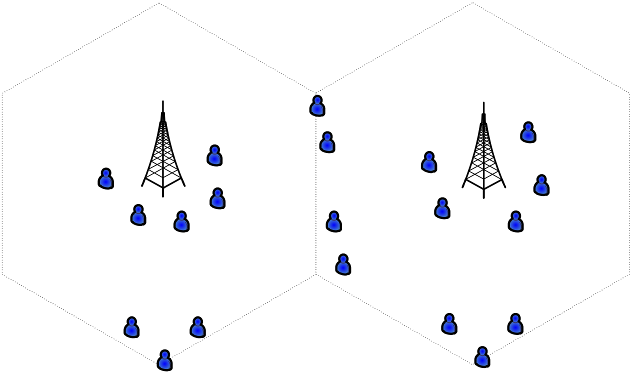
Cooperation in multiuser wireless networks: advocating the relays' rights

Onur Kaya[1]

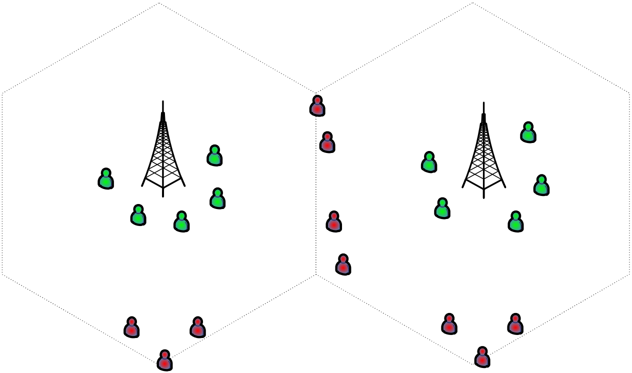
Ankara Seminars on Communication, METU, May 28th 2014.

[1] Joint work with S. Baksi, T. Biyikoglu, S. Bakim. Supported by Tubitak Grants 108E258, 111E108

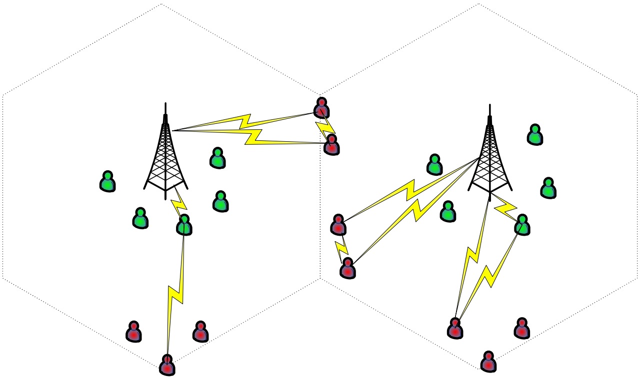
Motivation



Motivation



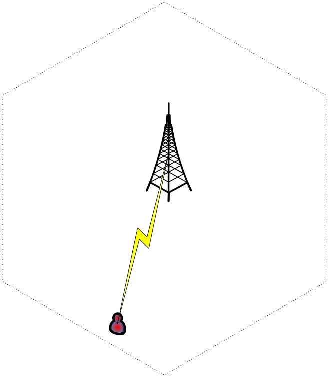
Motivation



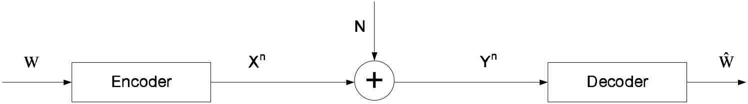
Context

- Information theoretic approach – capacity, achievable rates
  - Provides a benchmark for system performance – fundamental limits
  - Though strictly theoretical, gives insight to practical algorithms and applications
- We will combine several theoretical and practical concepts:
  - Achievable rates (block Markov encoding, backwards decoding),
  - Resource (power) allocation in fading, to maximize average rates,
  - Multiple access (OFDMA),
  - Single cell and multi-cell cooperation. (partnering, receiver selection)
  - Frequency planning (complementary fractional frequency reuse),
  - Cognitive radio (cooperative secondary users)

Single User Channel



Single User Gaussian Channel



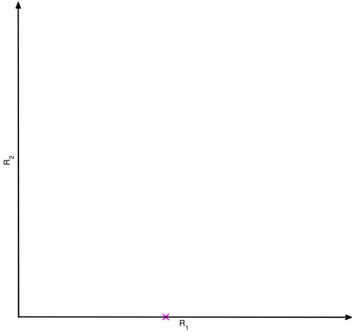
$$Y[i] = X[i] + N[i], \qquad i = 1, \ldots, N$$

- For Gaussian channels with signal power P and noise variance σ^2, the capacity is given by

$$C = \frac{1}{2} \log \left(1 + \frac{P}{\sigma^2} \right)$$

- To achieve capacity, the codeword X^n is taken from a codebook generated randomly,
  - Each symbol in the sequence X^n is i.i.d Gaussian, i.e., $X \sim \mathcal{N}(0, P)$.
- The capacity is achieved as the codeword length $n \rightarrow \infty$
- Decoding is performed based on jointly typical sequences.

Gaussian Channel Capacity



Fading Gaussian Channel (Goldsmith-Varaiya 1994)

- Channel capacity for single user

$$C = \frac{1}{2}\log\left(1+\frac{p}{\sigma^2}\right)$$

- In the presence of fading, for a fixed channel state h

$$y = \sqrt{p(h)h}x + n$$
$$C(h) = \frac{1}{2}\log\left(1+\frac{p(h)h}{\sigma^2}\right)$$

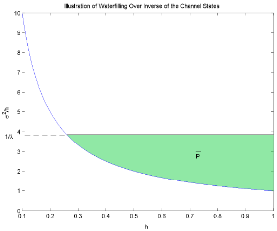
- Maximize the ergodic capacity, given an average power constraint

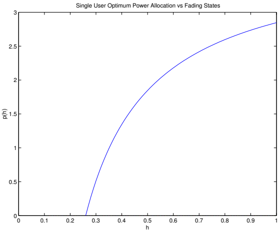
$$\begin{aligned} \max_{\{p(h)\}} \quad & E_h\left[\log\left(1+\frac{p(h)h}{\sigma^2}\right)\right] \\ \text{s.t.} \quad & E_h\left[p(h)\right] \leq \bar{p}, \qquad p(h) \geq 0 \end{aligned}$$

Single User Solution-Waterfilling

- Optimal power allocation: waterfilling of power over time

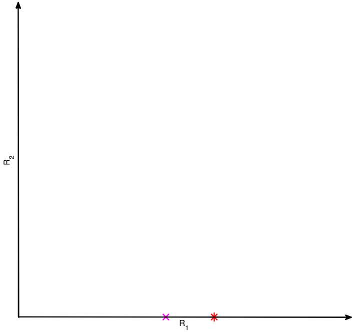
$$p(h) = \left(\frac{1}{\lambda} - \frac{\sigma^2}{h}\right)^+$$



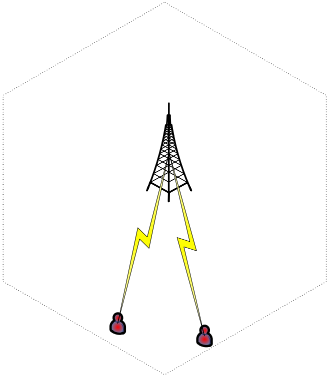


- More power to better channel states; no power to very poor channel states

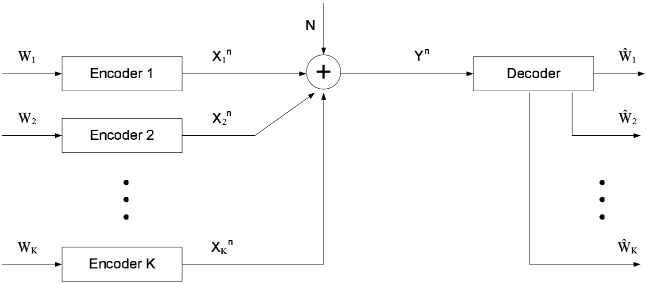
Fading Gaussian Channel Capacity with Power Control



Multiple Access Channel

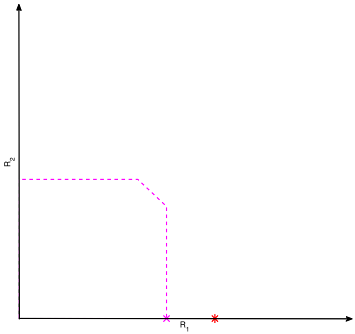


Gaussian Multiple Access Channel



- Multiple sources convey independent messages to the same receiver.
  - E.g., uplink of a cellular system, all mobiles send data to the base station.
- The rates achievable by users is worse than their single user performance due to interference.

Capacity Region of the Gaussian MAC



$$R_1 < \frac{1}{2}\log\left(1+\frac{P_1}{\sigma^2}\right)$$
$$R_2 < \frac{1}{2}\log\left(1+\frac{P_2}{\sigma^2}\right)$$
$$R_1+R_2 < \frac{1}{2}\log\left(1+\frac{P_1+P_2}{\sigma^2}\right)$$

- Region of achievable rates rather than a single rate value.
- The capacity region is a pentagon
- The rate of a user can be increased up to its single user limit, in expense of rate of other user.
- Corners of the boundary can be achieved by successive decoding.

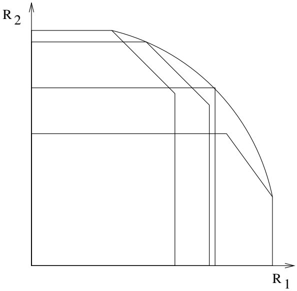
Fading Gaussian MAC (Knopp-Humblet 1995), (Hanly-Tse 98)

- The received signal

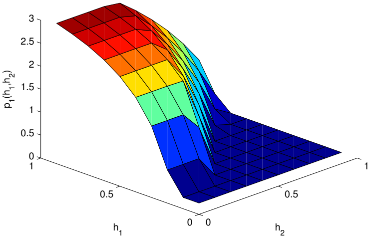
$$y = \sum_{i=1}^{K} \sqrt{p_i(\mathbf{h}) h_i} x_i + n$$

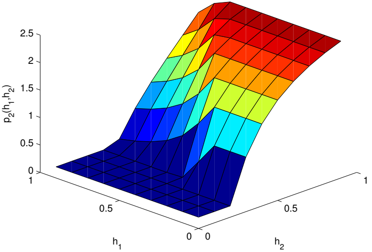
- Union of rate regions (polymatroids) achievable by all valid power control policies.

$$\bigcup_{\{\mathbf{p}(\mathbf{h}):\, E_{\mathbf{h}}[p_i(\mathbf{h})] \leq \bar{p}_i,\, \forall i\}} \left\{ \mathbf{R} : \sum_{i \in \Gamma} R_i \leq E_{\mathbf{h}} \left[\frac{1}{2} \log \left(1 + \sigma^{-2} \sum_{i \in \Gamma} h_i p_i(\mathbf{h}) \right) \right], \quad \forall \Gamma \subset \{1, \cdots, K\} \right\}$$



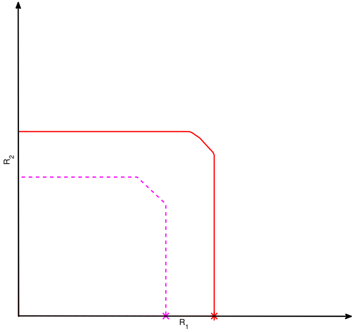
Optimal Power Allocation – Simultaneous Waterfilling



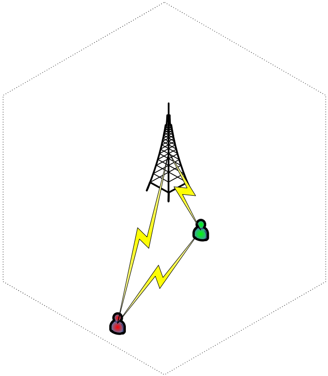


- Can be obtained by a greedy algorithm [Hanly-Tse 98], or by using generalized iterative waterfilling [Kaya-Ulukus 2006].
- Has a simultaneous waterfilling nature.

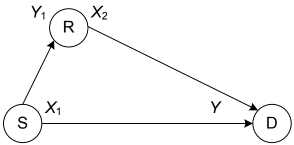
Fading Gaussian MAC Capacity Region



Gaussian Relay Channel



Gaussian Relay Channel (van der Meulen 71, Cover El-Gamal 79)



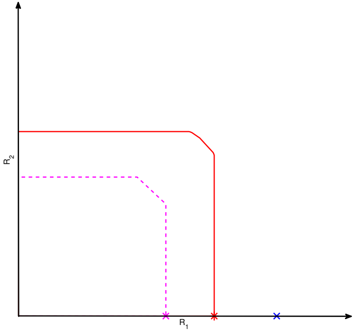
$$Y = \sqrt{h_{sd}} X_1 + \sqrt{h_{rd}} X_2 + N$$
$$Y_1 = \sqrt{h_{sr}} X_1 + N_1$$

- Capacity not known in general, achievable rates instead.
- Many cooperation schemes possible. AF, DF or CF ...
- An achievable rate with DF, using block Markov encoding

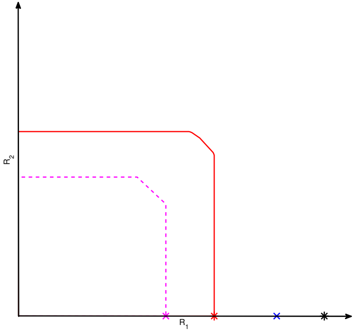
$$R_{\text{ach}} = \max_{0 \le \alpha \le 1} \min \left\{ \frac{1}{2} \log \left(1 + \frac{h_{sd} P + h_{rd} P_1 + 2\sqrt{h_{sd} h_{rd} (1-\alpha) P P_1}}{\sigma^2} \right), \frac{1}{2} \log \left(1 + \frac{h_{sr} \alpha P}{\sigma_1^2} \right) \right\}$$

- Coherent combining gain.

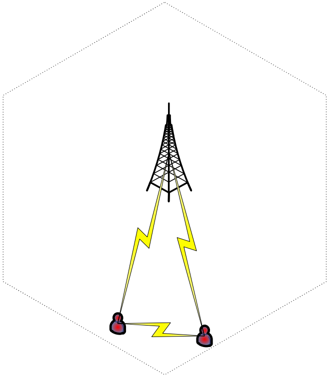
Relay Channel – Achievable Rate



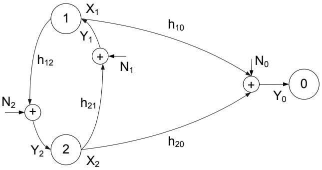
Fading Relay Channel – Achievable Rate with Power Control



Cooperative Multiple Access Channel



Two User Cooperation Model [WVS 83], [SEA 03]



$$Y_0 = \sqrt{h_{10}} X_1 + \sqrt{h_{20}} X_2 + N_0$$
$$Y_1 = \sqrt{h_{21}} X_2 + N_1$$
$$Y_2 = \sqrt{h_{12}} X_1 + N_2$$

Block Markov superposition coding

- Build common information (X_{12}, X_{21})
- Cooperatively send (U)
- Inject new information (X_{10}, X_{20})

$$X_1 = \sqrt{p_{10}} X_{10} + \sqrt{p_{12}} X_{12} + \sqrt{p_{u1}} U$$
$$X_2 = \sqrt{p_{20}} X_{20} + \sqrt{p_{21}} X_{21} + \sqrt{p_{u2}} U$$

Two User Cooperation MAC – Achievable Rate Region

- Rate constraints for reliable decoding at users:

$$R_{12} < E\left[\log\left(1 + \frac{h_{12}p_{12}}{h_{12}p_{10}+1}\right)\right]$$

$$R_{21} < E\left[\log\left(1 + \frac{h_{21}p_{21}}{h_{21}p_{20}+1}\right)\right]$$

- Rate constraints for reliable decoding at receiver:

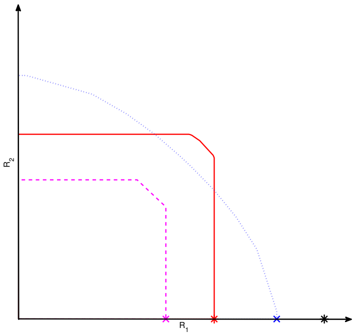
$$R_{10} < E\left[\log\left(1 + h_{10}p_{10}\right)\right]$$

$$R_{20} < E\left[\log\left(1 + h_{20}p_{20}\right)\right]$$

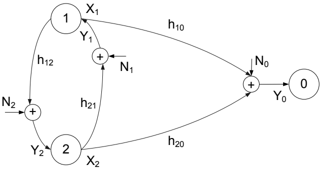
$$R_{10} + R_{20} < E\left[\log\left(1 + h_{10}p_{10} + h_{20}p_{20}\right)\right]$$

$$R_{12} + R_{21} + R_{10} + R_{20} < C_s \triangleq E\left[\log\left(1 + h_{10}p_1 + h_{20}p_2 + 2\sqrt{h_{10}h_{20}p_{u_1}p_{u_2}}\right)\right]$$

Two User Cooperation MAC – Achievable Rate Region



Power Control for Fading Two User Cooperative MAC [KU 07]



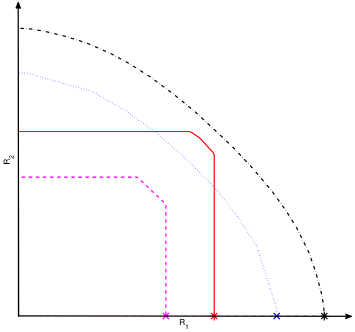
$$Y_0 = \sqrt{h_{10}}X_1 + \sqrt{h_{20}}X_2 + N_0$$
$$Y_1 = \sqrt{h_{21}}X_2 + N_1$$
$$Y_2 = \sqrt{h_{12}}X_1 + N_2$$

Block Markov superposition coding

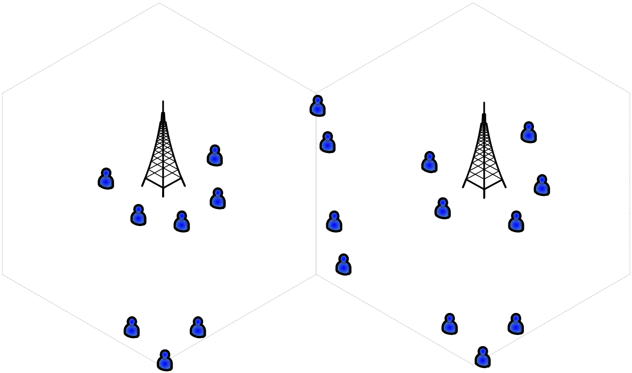
- Build common information (X_{12}, X_{21})
- Cooperatively send (U)
- Inject new information (X_{10}, X_{20})

$$X_1 = \sqrt{p_{10}(\mathbf{h})}X_{10} + \sqrt{p_{12}(\mathbf{h})}X_{12} + \sqrt{p_{u1}(\mathbf{h})}U$$
$$X_2 = \sqrt{p_{20}(\mathbf{h})}X_{20} + \sqrt{p_{21}(\mathbf{h})}X_{21} + \sqrt{p_{u2}(\mathbf{h})}U$$

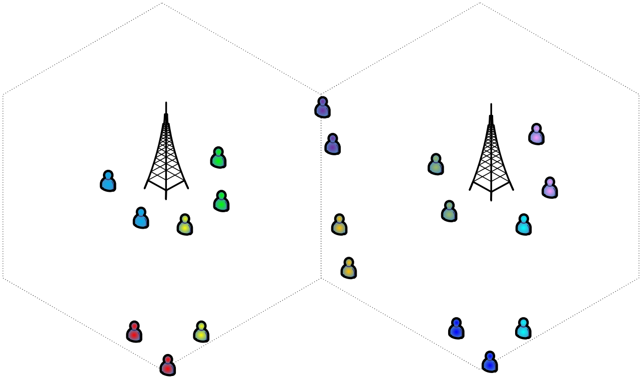
Two User Cooperation MAC – Achievable Rate Region with Power Control



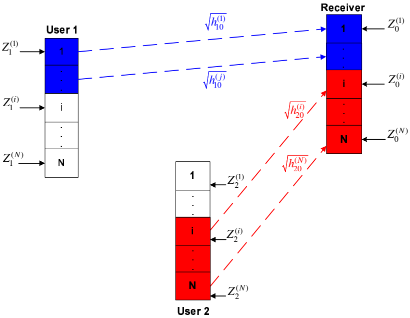
From 2 Users to N Users

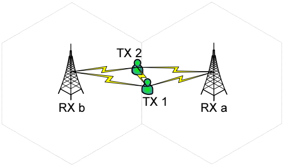


From 2 Users to N Users



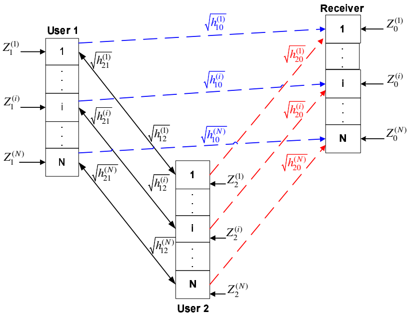
Cooperative OFDMA–Motivation





- OFDMA: Divides the entire transmission bandwidth into N orthogonal subchannels.
- Converts a frequency selective fading channel into parallel flat fading subchannels.
- Creates diversity across subchannels.
- Avoids interference, but incurs rate penalty due to orthogonalization of transmissions.

Two User Cooperative OFDMA Channel Model



$$Y_0^{(i)} = \sqrt{h_{10}^{(i)}}X_1^{(i)} + \sqrt{h_{20}^{(i)}}X_2^{(i)} + Z_0^{(i)}$$
$$Y_1^{(i)} = \sqrt{h_{21}^{(i)}}X_2^{(i)} + Z_1^{(i)}$$
$$Y_2^{(i)} = \sqrt{h_{12}^{(i)}}X_1^{(i)} + Z_2^{(i)}$$
$$i = 1, \ldots, N.$$

- Equivalent to N orthogonal cooperative MACs.
- Both users may TX & RX on the same subchannel: makes use of overheard information.
- May cooperate independently over each subchannel (intra-subchannel cooperation),
- May cooperate across subchannels (inter-subchannel cooperation).

OFDMA: Intra/Inter Subchannel Cooperative Encoding [Bakım, Kaya 2010]

- Block Markov superposition coding [Willems et al., 83][Sendonaris et al., 03]
- Each user's message is divided into two sub-messages: $w_k = (w_{k0}, w_{kj})$
- OFDMA: These two sub-messages are further divided into N submessages each
  - $w_{k0} = \left\{ w_{k0}^{(1)}, \dots, w_{k0}^{(N)} \right\}, \quad w_{kj} = \left\{ w_{kj}^{(1)}, \dots, w_{kj}^{(N)} \right\}$
- Two extensions to OFDMA: One trivial, one non-trivial.

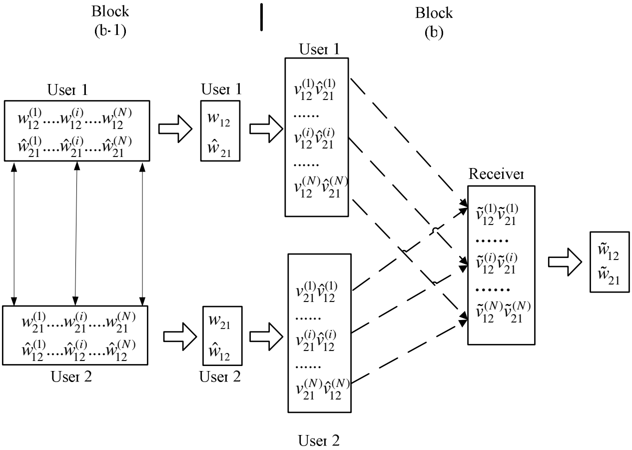
Purpose	Intra Subchannel CE	Inter Subchannel CE
Inject new information	$X_{k0}^{(i)}\left(w_{k0}^{(i)}[b], w_{kj}^{(i)}[b-1], \hat{w}_{jk}^{(i)}[b-1]\right)$	$X_{k0}^{(i)}\left(w_{k0}^{(i)}[b], v_{kj}^{(i)}[b-1], \hat{v}_{jk}^{(i)}[b-1]\right)$
Build common information	$X_{kj}^{(i)}\left(w_{kj}^{(i)}[b], w_{kj}^{(i)}[b-1], \hat{w}_{jk}^{(i)}[b-1]\right)$	$X_{kj}^{(i)}\left(w_{kj}^{(i)}[b], v_{kj}^{(i)}[b-1], \hat{v}_{jk}^{(i)}[b-1]\right)$
Cooperatively send	$U_k^{(i)}\left(w_{kj}^{(i)}[b-1], \hat{w}_{jk}^{(i)}[b-1]\right)$	$U_k^{(i)}\left(v_{kj}^{(i)}[b-1], \hat{v}_{jk}^{(i)}[b-1]\right)$

$$X_1^{(i)} = \sqrt{p_{10}^{(i)}} X_{10}^{(i)} + \sqrt{p_{12}^{(i)}} X_{12}^{(i)} + \sqrt{p_{U_1}^{(i)}} U_1^{(i)}$$

$$X_2^{(i)} = \sqrt{p_{20}^{(i)}} X_{20}^{(i)} + \sqrt{p_{21}^{(i)}} X_{21}^{(i)} + \sqrt{p_{U_2}^{(i)}} U_2^{(i)}$$

$$\sum_{i=1}^{N} p_{k0}^{(i)} + p_{kj}^{(i)} + p_{Uk}^{(i)} = P_k$$

Inter-Subchannel Cooperative Encoding



Channel Adaptive Cooperative Encoding over OFDMA with full CSI

- For full benefit from diversity, need to adapt our encoding according to the channel state.
- Assume users and receiver have full CSI of both cooperative links and direct link.
- Channel adaptive powers can be assigned to codewords used in each subchannel:

$$X_1^{(i)} = \sqrt{p_{10}^{(i)}(\mathbf{h})} X_{10}^{(i)} + \sqrt{p_{12}^{(i)}(\mathbf{h})} X_{12}^{(i)} + \sqrt{p_{U_1}^{(i)}(\mathbf{h})} U_1^{(i)}$$
$$X_2^{(i)} = \sqrt{p_{20}^{(i)}(\mathbf{h})} X_{20}^{(i)} + \sqrt{p_{21}^{(i)}(\mathbf{h})} X_{21}^{(i)} + \sqrt{p_{U_2}^{(i)}(\mathbf{h})} U_2^{(i)}$$

$$\sum_{i=1}^{N} E\left[p_{k0}^{(i)}(\mathbf{h}) + p_{kj}^{(i)}(\mathbf{h}) + p_{Uk}^{(i)}(\mathbf{h})\right] = P_k$$

- A [Hanly-Tse 98] like approach: do the adaptation by only varying the powers assigned to each fixed codeword, as opposed to using variable rate codebooks.

Achievable Rates – Intra-Subchannel Cooperative Encoding

- Achievable rate region is equivalent to the closure of the convex hull of all rate pairs:

$$R_1 < \sum_i \min\left\{ E\left[\frac{1}{2}\log\left(1+\frac{h_{12}^{(i)}p_{12}^{(i)}(\mathbf{h})}{h_{12}^{(i)}p_{10}^{(i)}(\mathbf{h})+1}\right)\right] + E\left[\frac{1}{2}\log\left(1+h_{10}^{(i)}p_{10}^{(i)}(\mathbf{h})\right)\right], R_s^{(i)} \right\}$$

$$R_2 < \sum_i \min\left\{ E\left[\frac{1}{2}\log\left(1+\frac{h_{21}^{(i)}p_{21}^{(i)}(\mathbf{h})}{h_{21}^{(i)}p_{20}^{(i)}(\mathbf{h})+1}\right)\right] + E\left[\frac{1}{2}\log\left(1+h_{20}^{(i)}p_{20}^{(i)}(\mathbf{h})\right)\right], R_s^{(i)} \right\}$$

$$R_1+R_2 < \sum_i \min\left\{ R_s^{(i)}, E\left[\frac{1}{2}\log\left(1+\frac{h_{12}^{(i)}p_{12}^{(i)}(\mathbf{h})}{h_{12}^{(i)}p_{10}^{(i)}(\mathbf{h})+1}\right)\right] + E\left[\frac{1}{2}\log\left(1+\frac{h_{21}^{(i)}p_{21}^{(i)}(\mathbf{h})}{h_{21}^{(i)}p_{20}^{(i)}(\mathbf{h})+1}\right)\right] + E\left[\frac{1}{2}\log\left(1+h_{10}^{(i)}p_{10}^{(i)}(\mathbf{h})+h_{20}^{(i)}p_{20}^{(i)}(\mathbf{h})\right)\right] \right\}$$

- $R_s^{(i)} \triangleq E\left[\frac{1}{2}\log\left(1+h_{10}^{(i)}p_1^{(i)}(\mathbf{h})+h_{20}^{(i)}p_2^{(i)}(\mathbf{h})+2\sqrt{h_{10}^{(i)}h_{20}^{(i)}p_{U_1}^{(i)}(\mathbf{h})p_{U_2}^{(i)}(\mathbf{h})}\right)\right]$

Achievable Rates – Inter-Subchannel Cooperative Encoding

- Achievable rate region is equivalent to the closure of the convex hull of all rate pairs:

$$R_1 < \sum_i E\left[\frac{1}{2}\log\left(1+\frac{h_{12}^{(i)}p_{12}^{(i)}(\mathbf{h})}{h_{12}^{(i)}p_{10}^{(i)}(\mathbf{h})+1}\right)\right] + E\left[\frac{1}{2}\log\left(1+h_{10}^{(i)}p_{10}^{(i)}(\mathbf{h})\right)\right]$$

$$R_2 < \sum_i E\left[\frac{1}{2}\log\left(1+\frac{h_{21}^{(i)}p_{21}^{(i)}(\mathbf{h})}{h_{21}^{(i)}p_{20}^{(i)}(\mathbf{h})+1}\right)\right] + E\left[\frac{1}{2}\log\left(1+h_{20}^{(i)}p_{20}^{(i)}(\mathbf{h})\right)\right]$$

$$R_1+R_2 < \min\left\{\sum_i R_s^{(i)}, \sum_i E\left[\frac{1}{2}\log\left(1+\frac{h_{12}^{(i)}p_{12}^{(i)}(\mathbf{h})}{h_{12}^{(i)}p_{10}^{(i)}(\mathbf{h})+1}\right)\right] + E\left[\frac{1}{2}\log\left(1+\frac{h_{21}^{(i)}p_{21}^{(i)}(\mathbf{h})}{h_{21}^{(i)}p_{20}^{(i)}(\mathbf{h})+1}\right)\right]\right.$$
$$\left. + E\left[\frac{1}{2}\log\left(1+h_{10}^{(i)}p_{10}^{(i)}(\mathbf{h})+h_{20}^{(i)}p_{20}^{(i)}(\mathbf{h})\right)\right]\right\}$$

- $R_s^{(i)} \triangleq E\left[\frac{1}{2}\log\left(1+h_{10}^{(i)}p_1^{(i)}(\mathbf{h})+h_{20}^{(i)}p_2^{(i)}(\mathbf{h})+2\sqrt{h_{10}^{(i)}h_{20}^{(i)}p_{U_1}^{(i)}(\mathbf{h})p_{U_2}^{(i)}(\mathbf{h})}\right)\right]$

Sum Rate Optimal Power Allocation

$p_{kj}^{(i)*}(\mathbf{h})$: Power control policy that maximizes sum rate of cooperative OFDMA system for both intra-subchannel and inter-subchannel cooperative encoding:

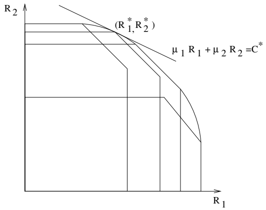
Theorem

- **Case 1:** if $h_{12}^{(i)} > h_{10}^{(i)}$ and $h_{21}^{(i)} > h_{20}^{(i)} \Rightarrow p_{10}^{(i)*}(\mathbf{h}) = p_{20}^{(i)*}(\mathbf{h}) = 0$
- **Case 2:** if $h_{12}^{(i)} > h_{10}^{(i)}$ and $h_{21}^{(i)} \leq h_{20}^{(i)} \Rightarrow p_{10}^{(i)*}(\mathbf{h}) = p_{21}^{(i)*}(\mathbf{h}) = 0$
- **Case 3:** if $h_{12}^{(i)} \leq h_{10}^{(i)}$ and $h_{21}^{(i)} > h_{20}^{(i)} \Rightarrow p_{12}^{(i)*}(\mathbf{h}) = p_{20}^{(i)*}(\mathbf{h}) = 0$
- **Case 4:** if $h_{12}^{(i)} \leq h_{10}^{(i)}$ and $h_{21}^{(i)} \leq h_{20}^{(i)} \Rightarrow p_{12}^{(i)*}(\mathbf{h}) = p_{21}^{(i)*}(\mathbf{h}) = 0$ or $p_{10}^{(i)*}(\mathbf{h}) = p_{21}^{(i)*}(\mathbf{h}) = 0$ or $p_{12}^{(i)*}(\mathbf{h}) = p_{20}^{(i)*}(\mathbf{h}) = 0$

Very similar result to [Kaya-Ulukus 07] for scalar MAC, despite the inter-subchannel encoding, and power constraint.

Key Properties of Sum-rate Optimal Power Allocation

- Each transmitter either sends a cooperation signal or fresh information, but not both!
- This simplified coding policy not only maximizes the sum rate, but also the individual rate constrains on R_1 and R_2, and is optimal in terms of the entire rate region.
- Bounds describing the achievable rate region are greatly simplified, and are now concave!
- Can obtain points on the rate region boundary by maximizing $R_{\boldsymbol{\mu}} = \mu_1 R_1 + \mu_2 R_2$, as rate region is convex.
- Any weighted sum $\mu_1 R_1 + \mu_2 R_2$ at the corners is also concave in powers.



Power control for cooperative OFDMA

$$
\begin{aligned}
\max_{\mathbf{p}(\mathbf{h})} \Bigg(& (\mu_1-\mu_2)\sum_i E_{S_1,S_2}\left[C\left(h_{12}^{(i)}p_{12}^{(i)}(\mathbf{h})\right)\right] + E_{S_3,S_4}\left[C\left(h_{10}^{(i)}p_{10}^{(i)}(\mathbf{h})\right)\right] \\
& + \mu_2 \min\Bigg\{\sum_i^N E\left[C\left(h_{10}^{(i)}p_1^{(i)}(\mathbf{h}) + h_{20}^{(i)}p_2^{(i)}(\mathbf{h}) + 2\sqrt{h_{10}^{(i)}h_{20}^{(i)}p_{U_1}^{(i)}(\mathbf{h})p_{U_2}^{(i)}(\mathbf{h})}\right)\right], \\
& \sum_i^N E_{S_1}\left[C\left(h_{12}^{(i)}p_{12}^{(i)}(\mathbf{h})\right) + C\left(h_{21}^{(i)}p_{21}^{(i)}(\mathbf{h})\right)\right] + E_{S_2}\left[C\left(h_{12}^{(i)}p_{12}^{(i)}(\mathbf{h})\right) + C\left(h_{20}^{(i)}p_{20}^{(i)}(\mathbf{h})\right)\right] \\
& + E_{S_3}\left[C\left(h_{10}^{(i)}p_{10}^{(i)}(\mathbf{h})\right) + C\left(h_{21}^{(i)}p_{21}^{(i)}(\mathbf{h})\right)\right] + E_{S_4}\left[C\left(h_{10}^{(i)}p_{10}^{(i)}(\mathbf{h}) + h_{20}^{(i)}p_{20}^{(i)}(\mathbf{h})\right)\right]\Bigg\}\Bigg) \\
\text{s.t. } & \sum_i E\left[p_{k0}^{(i)}(\mathbf{h}) + p_{kj}^{(i)}(\mathbf{h}) + p_{U_k}^{(i)}(\mathbf{h})\right] \le \bar{p}_k \\
& p_{k0}^{(i)}(\mathbf{h}), p_{kj}^{(i)}(\mathbf{h}), p_{U_k}^{(i)}(\mathbf{h}) \ge 0,\ k,j \in \{1,2\},\ k \ne j
\end{aligned}
$$

- $C(x) = \frac{1}{2}\log(1+x)$
- Concave optimization problem over a convex constraint set, but non-differentiable.

Differentiable Reformulation of the Optimization Problem

$$\max_{\mathbf{p}(\mathbf{h})} \quad R_\mu$$

$$s.t.\ R_\mu \leq (\mu_1 - \mu_2) \sum_i^N E_{S_1,S_2}\left[C\left(h_{12}^{(i)} p_{12}^{(i)}(\mathbf{h})\right)\right] + E_{S_3,S_4}\left[C\left(h_{10}^{(i)} p_{10}^{(i)}(\mathbf{h})\right)\right]$$

$$+\mu_2 \sum_i^N E\left[C\left(h_{10}^{(i)} p_1^{(i)}(\mathbf{h}) h_{20}^{(i)} p_2^{(i)}(\mathbf{h}) + 2\sqrt{h_{10}^{(i)} h_{20}^{(i)} p_{U_1}^{(i)}(\mathbf{h}) p_{U_2}^{(i)}(\mathbf{h})}\right)\right]$$

$$R_\mu \leq (\mu_1 - \mu_2) \sum_i^N E_{S_1,S_2}\left[C\left(h_{12}^{(i)} p_{12}^{(i)}(\mathbf{h})\right)\right] + E_{S_3,S_4}\left[C\left(h_{10}^{(i)} p_{10}^{(i)}(\mathbf{h})\right)\right]$$

$$+\mu_2 \left(\sum_i^N E_{S_1}\left[C\left(h_{12}^{(i)} p_{12}^{(i)}(\mathbf{h})\right) + C\left(h_{21}^{(i)} p_{21}^{(i)}(\mathbf{h})\right)\right] + E_{S_2}\left[C\left(h_{12}^{(i)} p_{12}^{(i)}(\mathbf{h})\right) + C\left(h_{20}^{(i)} p_{20}^{(i)}(\mathbf{h})\right)\right]\right.$$

$$\left. + E_{S_3}\left[C\left(h_{10}^{(i)} p_{10}^{(i)}(\mathbf{h})\right) + C\left(h_{21}^{(i)} p_{21}^{(i)}(\mathbf{h})\right)\right] E_{S_4}\left[C\left(h_{10}^{(i)} p_{10}^{(i)}(\mathbf{h}) + h_{20}^{(i)} p_{20}^{(i)}(\mathbf{h})\right)\right]\right)$$

$$\sum_i E\left[p_{k0}^{(i)}(\mathbf{h}) + p_{kj}^{(i)}(\mathbf{h}) + p_{U_k}^{(i)}(\mathbf{h})\right] \leq \bar{p}_k$$

$$p_{k0}^{(i)}(\mathbf{h}), p_{kj}^{(i)}(\mathbf{h}), p_{U_k}^{(i)}(\mathbf{h}) \geq 0,\ k,j \in \{1,2\},\ k \neq j$$

Simple Example: Sum-Rate Maximization Problem

- Set $\mu_1 = \mu_2$, and assume operation in case 1, for simplicity.

$$\max_{\mathbf{p}(\mathbf{h})} \quad R_s$$

$$\text{s.t.} \quad R_s \leq \sum_i E\left[\log\left(1 + h_{10}^{(i)}\left(p_{12}^{(i)}(\mathbf{h}) + p_{U_1}^{(i)}(\mathbf{h})\right) + h_{20}^{(i)}\left(p_{21}^{(i)}(\mathbf{h}) + p_{U_2}^{(i)}(\mathbf{h})\right) + 2\sqrt{h_{10}^{(i)} h_{20}^{(i)} p_{U_1}^{(i)}(\mathbf{h}) p_{U_2}^{(i)}(\mathbf{h})}\right)\right]$$

$$R_s \leq \sum_i E\left[\log(1 + p_{12}^{(i)}(\mathbf{h}) h_{12}^{(i)}) + \log(1 + p_{21}^{(i)}(\mathbf{h}) h_{21}^{(i)})\right]$$

$$\sum_i \left(E\left[p_{12}^{(i)}(\mathbf{h})\right] + E\left[p_{U_1}^{(i)}(\mathbf{h})\right]\right) \leq \bar{p}_1$$

$$\sum_i \left(E\left[p_{21}^{(i)}(\mathbf{h})\right] + E\left[p_{U_2}^{(i)}(\mathbf{h})\right]\right) \leq \bar{p}_2$$

$$p_{12}^{(i)}(\mathbf{h}), p_{U_1}^{(i)}(\mathbf{h}), p_{21}^{(i)}(\mathbf{h}), p_{U_2}^{(i)}(\mathbf{h}) \geq 0, \quad \forall \mathbf{h}$$

Lagrangian Approach

$$
\begin{aligned}
L = R_s + \gamma_1 \Bigg(\sum_i \Big(E \Big[\log(1 + h_{12}^{(i)} p_{12}^{(i)}(\mathbf{h})) + \log(1 + h_{21}^{(i)} p_{21}^{(i)}(\mathbf{h})) \Big] \Big) - R_s \Bigg) \\
+ \gamma_2 \Bigg(\sum_i E \Big[\log \Big(1 + h_{10}^{(i)} (p_{12}^{(i)}(\mathbf{h}) + p_{U_1}^{(i)}(\mathbf{h})) + h_{20}^{(i)} (p_{21}^{(i)}(\mathbf{h}) + p_{U_2}^{(i)}(\mathbf{h})) \\
+ 2\sqrt{h_{10}^{(i)} h_{20}^{(i)} p_{U_1}^{(i)}(\mathbf{h}) p_{U_2}^{(i)}(\mathbf{h})} \Big) \Big] - R_s \Bigg) + \lambda_1 \Bigg(\bar{p}_1 - \sum_i \Big(E \Big[p_{12}^{(i)}(\mathbf{h}) + p_{U_1}^{(i)}(\mathbf{h}) \Big] \Big) \Bigg) \\
+ \lambda_2 \Bigg(\bar{p}_2 - \sum_i \Big(E \Big[p_{21}^{(i)}(\mathbf{h}) + p_{U_2}^{(i)}(\mathbf{h}) \Big] \Big) \Bigg) + \varepsilon_1^{(i)}(\mathbf{h}) p_{12}^{(i)}(\mathbf{h}) + \varepsilon_2^{(i)}(\mathbf{h}) p_{U_1}^{(i)}(\mathbf{h}) \\
+ \varepsilon_3^{(i)}(\mathbf{h}) p_{21}^{(i)}(\mathbf{h}) + \varepsilon_4^{(i)}(\mathbf{h}) p_{U_2}^{(i)}(\mathbf{h}).
\end{aligned}
$$

Karush-Kuhn-Tucker Conditions

$$\gamma_1 \frac{h_{12}^{(i)}}{1+h_{12}^{(i)}p_{12}^{(i)}(\mathbf{h})} + \gamma_2 \frac{h_{10}^{(i)}}{D^{(i)}} \leq \lambda_1$$

$$\gamma_1 \frac{h_{21}^{(i)}}{1+h_{21}^{(i)}p_{21}^{(i)}(\mathbf{h})} + \gamma_2 \frac{h_{20}^{(i)}}{D^{(i)}} \leq \lambda_2$$

$$\gamma_2 \frac{\sqrt{h_{10}^{(i)}h_{20}^{(i)}p_{U_2}^{(i)}(\mathbf{h})} + h_{10}^{(i)}\sqrt{p_{U_1}^{(i)}(\mathbf{h})}}{D^{(i)}\sqrt{p_{U_1}^{(i)}(\mathbf{h})}} \leq \lambda_1$$

$$\gamma_2 \frac{\sqrt{h_{10}^{(i)}h_{20}^{(i)}p_{U_1}^{(i)}(\mathbf{h})} + h_{20}^{(i)}\sqrt{p_{U_2}^{(i)}(\mathbf{h})}}{D^{(i)}\sqrt{p_{U_2}^{(i)}(\mathbf{h})}} \leq \lambda_2$$

- $D = 1 + h_{10}^{(i)}p_1^{(i)}(\mathbf{h}) + h_{20}^{(i)}p_2^{(i)}(\mathbf{h}) + 2\sqrt{h_{10}^{(i)}h_{20}^{(i)}p_{U_1}^{(i)}(\mathbf{h})p_{U_2}^{(i)}(\mathbf{h})}$
- $\gamma_1 + \gamma_2 = 1$
- Each condition satisfied with strict equality, if the corresponding power is positive.
- All we need to do is find λ_i and γ_1

Structure of Optimal Power Allocation

- When p_{U_1} and p_{U_2} are both positive,

$$p_{12}^{(i)}(\mathbf{h}) = \left(\frac{\gamma_1 \left(\lambda_2 h_{10}^{(i)} + \lambda_1 h_{20}^{(i)} \right)}{\lambda_1^2 h_{20}^{(i)}} - \frac{1}{h_{12}^{(i)}} \right)^+$$

$$p_{21}^{(i)}(\mathbf{h}) = \left(\frac{\gamma_1 \left(\lambda_2 h_{10}^{(i)} + \lambda_1 h_{20}^{(i)} \right)}{\lambda_2^2 h_{10}^{(i)}} - \frac{1}{h_{21}^{(i)}} \right)^+$$

- When both are zero, p_{12} and p_{21} solved from,

$$\gamma_1 \frac{h_{12}^{(i)}}{1 + h_{12}^{(i)} p_{12}^{(i)}(\mathbf{h})} + \gamma_2 \frac{h_{10}^{(i)}}{1 + h_{10}^{(i)} p_{12}^{(i)}(\mathbf{h}) + h_{20}^{(i)} p_{21}^{(i)}(\mathbf{h})} \leq \lambda_1$$

$$\gamma_1 \frac{h_{21}^{(i)}}{1 + h_{21}^{(i)} p_{21}^{(i)}(\mathbf{h})} + \gamma_2 \frac{h_{20}^{(i)}}{1 + h_{10}^{(i)} p_{12}^{(i)}(\mathbf{h}) + h_{20}^{(i)} p_{21}^{(i)}(\mathbf{h})} \leq \lambda_2$$

$$p_{U_k}^{(i)}(\mathbf{h}) = \left[\left((1-\gamma_1)(h_{k0}^{(i)} + (\lambda_k/\lambda_j) h_{j0}^{(i)})/\lambda_k - (1 + h_{k0}^{(i)} p_{kj}^{(i)}(\mathbf{h}) + h_{j0}^{(i)} p_{jk}^{(i)}(\mathbf{h})) \right) h_{k0}^{(i)} / (h_{k0}^{(i)} + h_{j0}^{(i)})^2 \right]^+$$

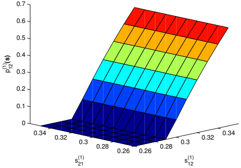
Iterative Solution of the KKT Conditions

- Not exactly closed form: p_{U_1} and p_{U_2}'s depend on p_{12} and p_{21}, and vice versa.
- All powers can be computed using KKT conditions, by iteratively searching for Lagrange multipliers.
- Objective function concave, constraints strictly convex, Cartesian nature across users:
  - Can solve the users' powers iteratively – one user at a time.
  - Start by assuming p_U's positive, and iterate. Converges to optimum.

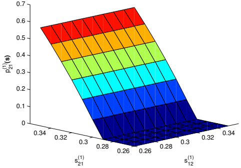
Iterative Power Allocation Algorithm

- Fix the Lagrange multipliers λ_1, λ_2 and γ_1.
- For each subchannel, i and each TX, k:
  - Calculate $p_{kj}^{(i)}(\mathbf{h})$ and then $p_{U_k}^{(i)}(\mathbf{h})$ assuming $p_{U_k}^{(i)}(\mathbf{h}) > 0$
  - If $p_{U_k}^{(i)}(\mathbf{h}) < 0$, then for those $\mathbf{h}$, set $p_{U_k}^{(i)}(\mathbf{h}) = 0$ and re-calculate $p_{kj}^{(i)}(\mathbf{h})$ using alternate expression, and $p_{U_k}^{(i)}(\mathbf{h})$.
  - Iterate this procedure across TXs, until all KKT conditions are satisfied for given λ_1, λ_2 and γ_1.
- Iteratively update λ_1, λ_2, γ_1, until average power constraints and rate constraints are satisfied.

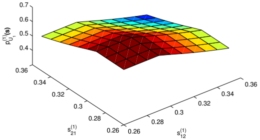
Optimal Power Allocation over Fading States– U-D links high



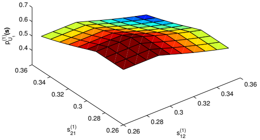
(a) Power levels, $p_{12}^{(1)}$



(b) Power level, $p_{21}^{(1)}$

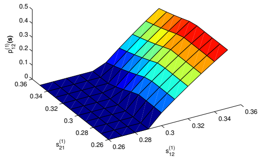


(c) Power level, $p_{U1}^{(1)}$

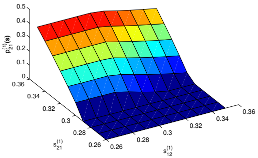


(d) Power level, $p_{U2}^{(1)}$

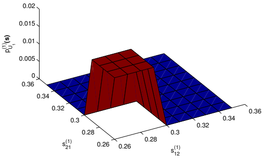
Optimal Power Allocation over Fading States– U-D moderate



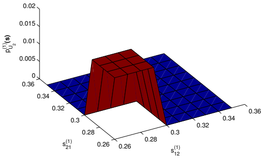
(e) Power levels, $p_{12}^{(1)}$



(f) Power levels, $p_{21}^{(1)}$

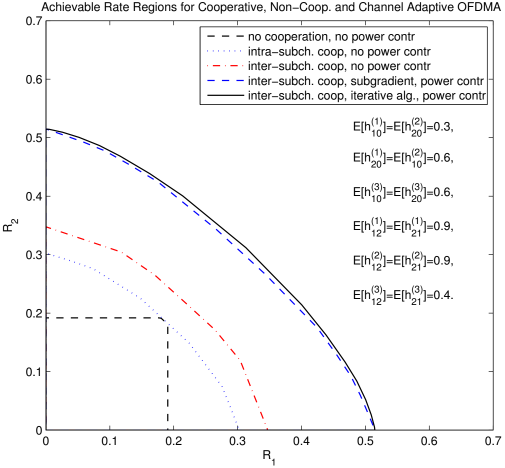


(g) Power levels, $p_{U1}^{(1)}$



(h) Power levels, $p_{U2}^{(1)}$

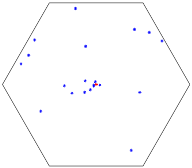
Cooperative OFDMA – Achievable Rates with Power Control



Summary - Cooperative OFDMA

- Obtained optimal power allocation strategy, and resulting rate region for mutually cooperative OFDMA.
- Pooling resources and allowing for cooperation instead of avoiding interference in OFDMA may lead to significant rate / user capacity improvements.
- Power control is key in exploiting the diversity created by cooperative OFDMA.
    - Also may give the optimum way to utilize the subchannels in cooperation.

Single Cell Setup [BKB 12]



- Idea: combine OFDMA with cooperation, in a MAC with many users
    - Orthogonolize separate user pairs, but share subchannels within each pair.
- N users, grouped into orthogonal pairs by OFDMA: $N/2$ cooperative MACs.
- Pair $\{i, j\}$, assigned M orthogonal subchannels $S_{ij} \subset \{1, \ldots, NM/2\}$.
- Cooperation done using intersubchannel cooperative encoding for OFDMA [BK 11].
- Subchannel allocation is done once and fixed throughout the transmission.
- Rayleigh fading and simplified path loss model.
- Goal: maximize system's sum rate, by optimal partnering and power control.

Received Signal Model

For each pair i, j:

$$Y_i = \sqrt{h_{ji}^{(s)} d_{ij}^{-\alpha}} X_j^{(s)} + N_i^{(s)},$$
$$Y_j = \sqrt{h_{ij}^{(s)} d_{ij}^{-\alpha}} X_i^{(s)} + N_j^{(s)},$$
$$Y_0 = \sqrt{h_{i0}^{(s)} d_{i0}^{-\alpha}} X_i^{(s)} + \sqrt{h_{j0}^{(s)} d_{j0}^{-\alpha}} X_j^{(s)} + N_0^{(s)}.$$

- Noise components $N_i^{(s)}$, $N_j^{(s)}$ and $N_0^{(s)}$ over each subchannel are independent, zero mean white Gaussian with variances ${\sigma_i^{(s)}}^2$, ${\sigma_j^{(s)}}^2$, ${\sigma_0^{(s)}}^2$.
- Instantaneous power fading coefficients $h_{ij}^{(s)}$, $h_{ji}^{(s)}$, $h_{i0}^{(s)}$, $h_{j0}^{(s)}$ i.i.d. exponential.
- Assume full CSI, denoted **h**, is available at each user pair and the receiver.
- CSI of other pairs not needed, once pairing is done based on channel statistics.
- Simplified path loss model used with path loss exponent α; d_{ij}, d_{i0} and d_{j0} denote user i-user j, user i-receiver and user j-receiver distances respectively.

Channel Adaptive Encoding

$$X_i^{(s)} = \sqrt{p_{i0}^{(s)}(\mathbf{h})} X_{i0}^{(s)} + \sqrt{p_{ij}^{(s)}(\mathbf{h})} X_{ij}^{(s)} + \sqrt{p_{U_i}^{(s)}(\mathbf{h})} U^{(s)},$$

$$X_j^{(s)} = \sqrt{p_{j0}^{(s)}(\mathbf{h})} X_{j0}^{(s)} + \sqrt{p_{ji}^{(s)}(\mathbf{h})} X_{ji}^{(s)} + \sqrt{p_{U_j}^{(s)}(\mathbf{h})} U^{(s)},$$

$$s \in S_{ij}, \quad \forall\{i,j\} \in \Gamma_l, \quad \Gamma_l \in \Gamma.$$

- Γ: set of all two user partitions of $\{1,\ldots,N\}$, Γ_l: some valid partition.
- The component codewords $X_{i0}^{(s)}$, $X_{ij}^{(s)}$ and $U^{(s)}$ are used for direct transmission, common message generation, and cooperation purposes respectively.
- $p_{i0}^{(s)}(\mathbf{h})$, $p_{ij}^{(s)}(\mathbf{h})$ and $p_{U_i}^{(s)}(\mathbf{h})$: channel adaptive powers assigned to these codewords.
- The definitions for user j follow similarly.
- Rate regions obtained using intersubchannel cooperative encoding [BK '11].

Joint Optimization Problem

$$
\begin{aligned}
\max_{\substack{\Gamma_l \in \Gamma, \\ \mathbf{p}(\mathbf{h})}} \quad & \sum_{\{i,j\}\in\Gamma_l} R_i + R_j \\
\text{s.t.} \quad & \sum_{s\in S_{ij}} E\left[p_{i0}^{(s)}(\mathbf{h}) + p_{ij}^{(s)}(\mathbf{h}) + p_{U_i}^{(s)}(\mathbf{h})\right] \triangleq \sum_{s\in S_{ij}} E\left[p_i^{(s)}(\mathbf{h})\right] \le \bar{p}_i, \\
& \sum_{s\in S_{ij}} E\left[p_{j0}^{(s)}(\mathbf{h}) + p_{ji}^{(s)}(\mathbf{h}) + p_{U_j}^{(s)}(\mathbf{h})\right] \triangleq \sum_{s\in S_{ij}} E\left[p_j^{(s)}(\mathbf{h})\right] \le \bar{p}_j.
\end{aligned}
$$

$$
\begin{aligned}
R_i + R_j \le \min \Bigg\{ & \sum_{s\in S_{ij}} E\left[\log\left(1 + \frac{h_{i0}^{(s)} d_{i0}^{-\alpha} p_i^{(s)}(\mathbf{h}) + h_{j0}^{(s)} d_{j0}^{-\alpha} p_j^{(s)}(\mathbf{h}) + 2\sqrt{h_{i0}^{(s)} d_{i0}^{-\alpha} h_{j0}^{(s)} d_{j0}^{-\alpha} p_{u_i}^{(s)}(\mathbf{h}) p_{u_j}^{(s)}(\mathbf{h})}}{{\sigma_0^{(s)}}^2}\right)\right], \\
& \sum_{s\in S_{ij}} E\left[\log\left(1 + \frac{h_{i0}^{(s)} d_{i0}^{-\alpha} p_{i0}^{(s)}(\mathbf{h}) + h_{j0}^{(s)} d_{j0}^{-\alpha} p_{j0}^{(s)}(\mathbf{h})}{{\sigma_0^{(s)}}^2}\right)\right] \\
& + \sum_{s\in S_{ij}} E\left[\log\left(1 + \frac{h_{ij}^{(s)} d_{ij}^{-\alpha} p_{ij}^{(s)}(\mathbf{h})}{h_{ij}^{(s)} d_{ij}^{-\alpha} p_{i0}^{(s)}(\mathbf{h}) + {\sigma_j^{(s)}}^2}\right) + \log\left(1 + \frac{h_{ji}^{(s)} d_{ji}^{-\alpha} p_{ji}^{(s)}(\mathbf{h})}{h_{ji}^{(s)} d_{ji}^{-\alpha} p_{j0}^{(s)}(\mathbf{h}) + {\sigma_i^{(s)}}^2}\right)\right] \Bigg\}
\end{aligned}
$$

Two Stage Decomposition of the Problem

- Due to the orthogonal nature of OFDMA,
  - each pair's adaptive power allocation (PA) computable independently of others,
  - each pair's sum rate computable independently, once partnering is fixed.

$$\begin{aligned}
\max_{\Gamma_l \in \Gamma,} \quad & \sum_{\{i,j\}\in\Gamma_l} \max_{\mathbf{p}_i(\mathbf{h}),\mathbf{p}_j(\mathbf{h})} (R_i + R_j), \\
\text{s.t.} \quad & \sum_{s\in S_{ij}} E\left[p_{i0}^{(s)}(\mathbf{h}) + p_{ij}^{(s)}(\mathbf{h}) + p_{U_i}^{(s)}(\mathbf{h})\right] \le \bar{p}_i, \ \ \forall\{i,j\}\in\Gamma_l \\
& R_i + R_j \text{ achievable}, \quad \forall\{i,j\}\in\Gamma_l.
\end{aligned}$$

- Inner problem solved using the iterative approach [BK '11] for all possible pairs.
- Problem further reduces to

$$\max_{\Gamma_l \in \Gamma,} \quad \sum_{\{i,j\}\in\Gamma_l} (R_i + R_j)^*,$$

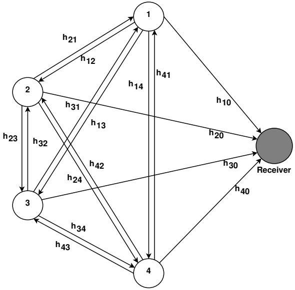
- Can exhaustively search for the best 2-user partition to maximize the sum rate.
- However, the number of all possible 2-user partitions is combinatorically large:

$$L = \prod_{n=1}^{N/2} (N - 2n + 1).$$

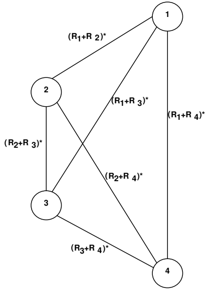
- Key observation: choosing the best 2-user partitions is equivalent to a matching problem in graph theory, which has an optimum solution.

Equivalent Graph Representation

- Convert the cooperative OFDMA model into an undirected graph.
- Users are nodes, weights are pairwise optimized rates.



(i) 4-user OFDMA system model, with pairwise cooperation.



(j) 4-node undirected graph equivalent of the system in Figure (a)

Figure: Graph representations of the system

- Can use Edmonds' Maximum Weighted Matching (MWM) algorithm
- Optimal partnering is achieved by MWM, complexity is $O(N^3)$

Practical Suboptimal Partnering Algorithms

In a practical wireless cellular system

- There are moderate number of users in a cell
- Users are generally not stationary
- Therefore locations should be updated and partnering algorithm should be reapplied regularly
- The path loss has bigger impact on sum rates in comparison to Rayleigh fading
- Location based low-complexity faster algorithms can be developed

Practical Suboptimal Partnering Algorithms

Several location based algorithms are developed:

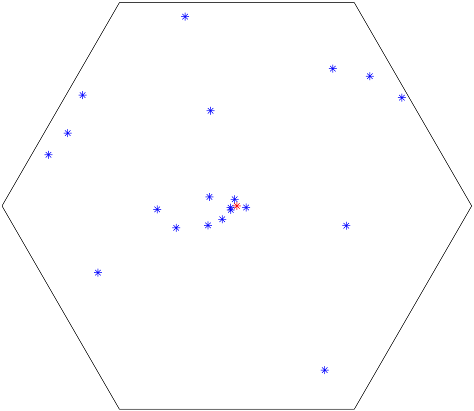
- A Select Nearest Two to Receiver
- B Select Farthest Two from Receiver
- C Maximum Matching on Nearest Four to Receiver
- D Maximum Matching on Farthest Four from Receiver
- E Select Nearest and Farthest Ones to Receiver

Simulation Results

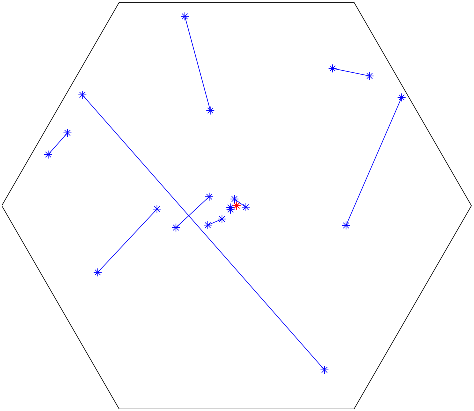
Table: Transmission rates of cooperating pairs obtained by a sample run of proposed algorithms

Pair	MWM	AlgoA	AlgoB	AlgoC	AlgoD	AlgoE
1	17.084	21.045	19.439	21.045	17.926	17.078
2	16.618	19.596	18.133	18.062	17.731	16.621
3	16.414	13.073	16.649	15.336	16.727	16.410
4	14.924	10.064	13.073	11.534	16.417	14.911
5	10.683	4.833	5.484	4.833	7.164	10.683
6	8.716	3.906	4.388	3.798	3.906	8.657
7	7.938	3.451	3.906	3.496	3.451	7.760
8	7.164	3.074	3.496	2.793	3.074	5.111
9	3.906	2.841	2.841	2.642	2.865	4.833
10	3.596	2.329	2.793	2.706	2.858	4.429
Total	107.043	84.211	90.202	86.245	92.117	106.494

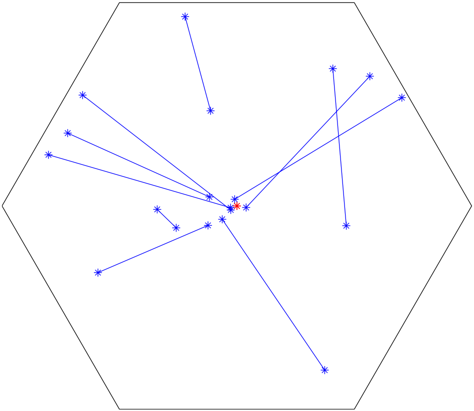
Single Cell Optimal Partnering



Single Cell Optimal Partnering



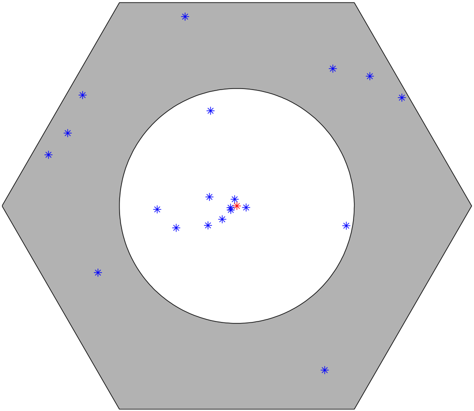
Single Cell Optimal Partnering



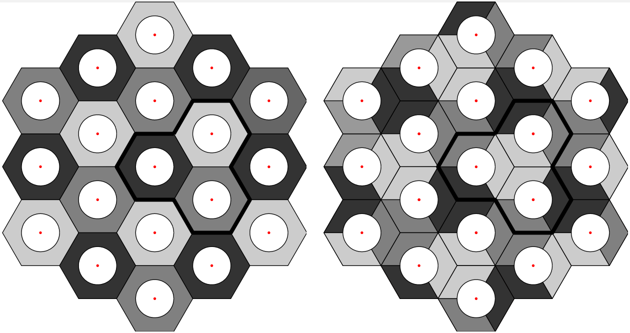
Single Cell Setup – Summary and Discussion

- Obtained the jointly optimal power allocation and partnering algorithms for a single cell cooperative OFDMA setup.
- Partnering problem equivalent to MWM, polynomial time solution.
- Cell center users paired with cell edge users, awkward.
- Cell center users snatch cell edge users' channels! User cooperation turns into adaptive frequency resource allocation.
- Need to protect cell edge users, especially in multicell scenario.

Isolating Inner and Outer Users



Proposed Cooperation and FFR Model



(a) Strict FFR Scheme. (b) Complementary FFR Scheme.

Figure: Illustration of complementary FFR scheme, compared to strict FFR

- Do not protect adjacent cell users from each other, let them cooperate.

Complementary Fractional Frequency Reuse

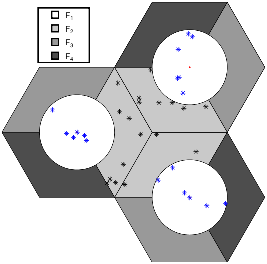


Figure: Complementary FFR: cluster of interest

- Cells sectorized, each cell is divided into 1 inner, 3 outer sectors.
- Each sector uses one of 4 different frequency bands F_1, F_2, F_3 and F_4
- Adjacent sectors of three neighboring cells use a common frequency band and create *pseudo-cell*
- Assume hexagonal cell's radius is r, the inner-cell sector's radius is $r_{in} = r/2$
- Assume N randomly distributed users per sector: $12N$ users per cluster.

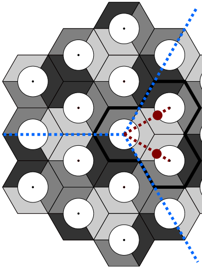
Received Signal Model

- Unlike the single-cell scenario, intercell interference added to received signals.

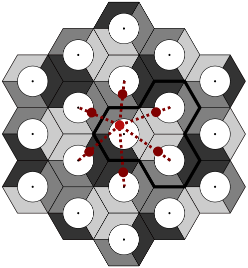
$$
\begin{aligned}
Y_i &= \sqrt{h_{ji}^{(s)} d_{ij}^{-\alpha}} X_j^{(s)} + I_i^{(s)} + N_i^{(s)}, \\
Y_j &= \sqrt{h_{ij}^{(s)} d_{ij}^{-\alpha}} X_i^{(s)} + I_j^{(s)} + N_j^{(s)}, \\
Y_b &= \sqrt{h_{ib}^{(s)} d_{ib}^{-\alpha}} X_i^{(s)} + \sqrt{h_{jb}^{(s)} d_{jb}^{-\alpha}} X_j^{(s)} + I_b^{(s)} + N_b^{(s)},
\end{aligned}
$$

- $h_{ij}^{(s)}$: fading coefficient, d_{ij}: distance, α: path loss exponent, s: subchannel index.
- Consider interference from first tier only.
- Due to the sectorization, directional antennas at BS decrease interference $I_b^{(s)}$.
- However, the users still need to use omnidirectional antennas, $I_i^{(s)}$ larger.

Interference Model for Inner Users



(a) Calculation of $I_{b,in}$



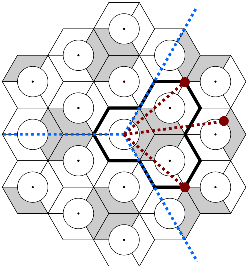
(b) Calculation of $I_{in,j}$

Figure: Interferer locations in complementary FFR

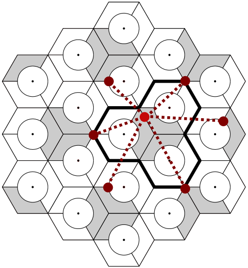
$$I_{b,in} = 2 \times \bar{P}/(r\sqrt{3} - r_{in})^{\alpha}$$

$$I_{in,j} = \sum_{m=1}^{6} \bar{P}/d_{jm,in}^{\alpha}$$

Interference Model for Outer Users



(a) Calculation of $I_{b,out}$



(b) Calculation of $I_{out,j}$

Figure: Interferer locations in complementary FFR

$$I_{b,out} = 2 \times \bar{P}/(r\sqrt{7})^{\alpha} + \bar{P}/(r\sqrt{10})^{\alpha}$$

$$I_{out,j} = \sum_{m=1}^{6} \bar{P}/d_{jm,out}^{\alpha}$$

Encoding and Decoding Strategy

- Block Markov encoding (intersubchannel cooperative encoding) [BK 11]

$$X_i^{(s)} = \sqrt{p_{i0}^{(s)}(\mathbf{h})} X_{i0}^{(s)} + \sqrt{p_{ij}^{(s)}(\mathbf{h})} X_{ij}^{(s)} + \sqrt{p_{U_i}^{(s)}(\mathbf{h})} U^{(s)}$$

$$X_j^{(s)} = \sqrt{p_{j0}^{(s)}(\mathbf{h})} X_{j0}^{(s)} + \sqrt{p_{ji}^{(s)}(\mathbf{h})} X_{ji}^{(s)} + \sqrt{p_{U_j}^{(s)}(\mathbf{h})} U^{(s)},$$

- Codewords are subject to average power constraints:

$$\sum_{s \in S_{ij}} E\left[p_{i0}^{(s)}(\mathbf{h}) + p_{ij}^{(s)}(\mathbf{h}) + p_{U_i}^{(s)}(\mathbf{h})\right] \triangleq \sum_{s \in S_{ij}} E\left[p_i^{(s)}(\mathbf{h})\right] \leq \bar{p}_i,$$

$$\sum_{s \in S_{ij}} E\left[p_{j0}^{(s)}(\mathbf{h}) + p_{ji}^{(s)}(\mathbf{h}) + p_{U_j}^{(s)}(\mathbf{h})\right] \triangleq \sum_{s \in S_{ij}} E\left[p_j^{(s)}(\mathbf{h})\right] \leq \bar{p}_j.$$

- Decoding at the receiver is performed using backwards decoding.

Achievable Rates

- The sum rates of inner users very similar to single-cell scenario [BKB 12].
- The sum rate per outer users pair more interesting: need to select BS!

$$
\begin{aligned}
(R_i+R_j)_b \leq \min \Bigg\{ & \sum_{s\in S_{ij}} E\left[\log\left(1+\frac{h_{ib}^{(s)}d_{ib}^{-\alpha}p_i^{(s)}(\mathbf{h})+h_{jb}^{(s)}d_{jb}^{-\alpha}p_j^{(s)}(\mathbf{h})+2\sqrt{h_{ib}^{(s)}d_{ib}^{-\alpha}h_{jb}^{(s)}d_{jb}^{-\alpha}p_{u_i}^{(s)}(\mathbf{h})p_{u_j}^{(s)}(\mathbf{h})}}{{\sigma_b^{(s)}}^2+I_b}\right)\right], \\
& \sum_{s\in S_{ij}} E\left[\log\left(1+\frac{h_{ij}^{(s)}d_{ij}^{-\alpha}p_{ij}^{(s)}(\mathbf{h})}{h_{ij}^{(s)}d_{ij}^{-\alpha}p_{ib}^{(s)}(\mathbf{h})+{\sigma_j^{(s)}}^2+I_j}\right)+\log\left(1+\frac{h_{ji}^{(s)}d_{ji}^{-\alpha}p_{ji}^{(s)}(\mathbf{h})}{h_{ji}^{(s)}d_{ji}^{-\alpha}p_{jb}^{(s)}(\mathbf{h})+{\sigma_i^{(s)}}^2+I_i}\right)\right] \\
& +\sum_{s\in S_{ij}} E\left[\log\left(1+\frac{h_{ib}^{(s)}d_{ib}^{-\alpha}p_{ib}^{(s)}(\mathbf{h})+h_{jb}^{(s)}d_{jb}^{-\alpha}p_{jb}^{(s)}(\mathbf{h})}{{\sigma_b^{(s)}}^2+I_b}\right)\right]\Bigg\} \qquad (1)
\end{aligned}
$$

Problem Statement

- Goal: maximize the system's sum rate.
- CFFR model: enough to focus on one cluster.
- Inner and outer users can be treated separately.
- Sum rate maximization for inner cell users is the same as single-cell case
- For outer users, sum rate maximization problem includes receiver selection:

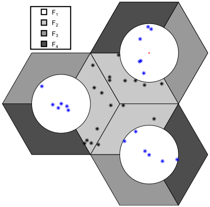
$$\max_{\substack{\Gamma_l \in \Gamma, \\ b_{ij} \in \{1,2,3\}, \\ \mathbf{p}(\mathbf{h})}} \sum_{\{i,j\} \in \Gamma_l} (R_i + R_j)_{b_{ij}}$$

$$\text{s.t.} \quad \sum_{s \in S_{ij}} E\left[p_{ib_{ij}}^{(s)}(\mathbf{h}) + p_{ij}^{(s)}(\mathbf{h}) + p_{U_i}^{(s)}(\mathbf{h})\right] \leq \bar{P}_i,$$

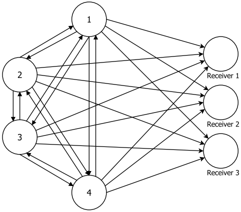
$$(R_i + R_j)_{b_{ij}} \text{ satisfies } (1), \quad \forall \{i,j\} \in \Gamma_l,$$

- Γ_l: two user partition of the outer user set in pseudo-cell of interest.
- Γ : the set of all partitions Γ_l,
- b_{ij} : the receiver selected by $\{i,j\}$
- $\mathbf{p}(\mathbf{h})$: the vector of all power variables at all channel states.

Decomposition of Problem – Power Allocation



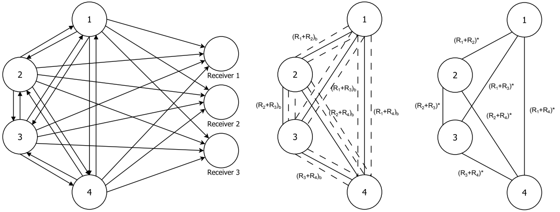
(a) Simple 4 user example



(b) Directed graph with all possible connections

- Orthogonality of OFDMA: each pair can apply power control independently.
- Each pair's sum rate maximization is independent of others, once partnering and BS assignment fixed.
- First compute optimal powers for any fixed partnering and BS assignment.
  - PC problem for cooperative OFDMA is convex, solved using KKT conditions [BK 13].
- Then search for optimum partnering and BS assignment.

Decomposition of Problem – Partnering and Receiver Selection



(c) 3 receiver system model

(d) Weighted graph, $b = 1,2,3$ for each pair

(e) Reduced graph, after selecting b^* for each pair

Figure: Determination of weights for edge cell users

- The problem is mapped into a graph theory problem,
- Each pair has three different rates as weights coming from power allocation
- Three edges per pair, due to different receivers.
- Pooling of resources: all users in pseudo-cell could be served by the same BS.
- BS selections can be made independently: select largest weight edge, dismiss others.

Equivalent Multistage Optimization Problem

- The jointly optimal partnering, receiver selection and power allocation problem can be stated as an equivalent three stage problem,

$$\max_{\Gamma_l \in \Gamma,} \quad \sum_{\{i,j\}\in\Gamma_l} \max_{b} \max_{\mathbf{p}_i(\mathbf{h}),\mathbf{p}_j(\mathbf{h})} (R_i + R_j)_b,$$
$$\text{s.t.} \quad \sum_{s\in S_{ij}} E\left[p_{ib}^{(s)}(\mathbf{h})+p_{ij}^{(s)}(\mathbf{h})+p_{U_i}^{(s)}(\mathbf{h})\right] \leq \bar{P}_i,$$
$$(R_i + R_j)_b \text{ satisfies } (1), \quad \forall\{i,j\} \in \Gamma_l.$$

- The outermost maximization is equivalent to a maximum weighted matching problem.

$$\max_{\Gamma_l \in \Gamma,} \quad \sum_{\{i,j\}\in\Gamma_l} (R_i + R_j)^*,$$

- The MWM on a complete graph: use Edmond's algorithm $(O(n^3))$ to find partnering.

Suboptimal Heuristic Partner Selection

- Location based heuristic algorithm can be used.
- For inner users, nearest and farthest to receiver are paired.
- Paired users taken off the user pool and procedure is repeated.
- For outer users, the distances among each pair of outer users in each pseudo-cell are computed and sorted.
- The users closest to each other are matched, removed from the list of users, then the same procedure is applied to the remaining users.

Simulation Results

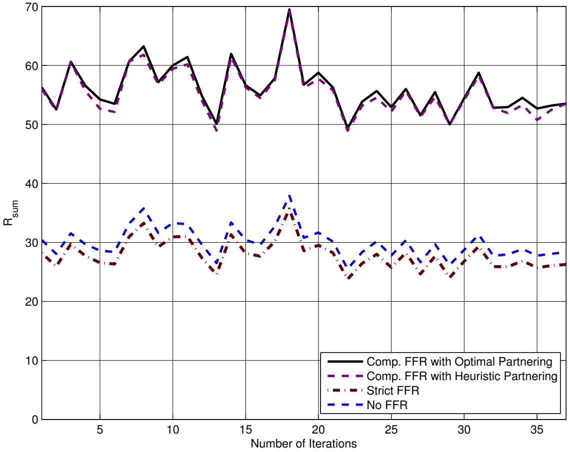


Figure: Sum rate comparison of proposed model with non-cooperative models.

Simulation Results

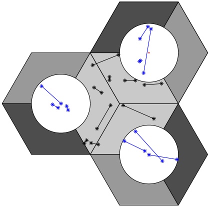


Figure: Sample optimal partnering strategy obtained by MWM.

User Pair	No FFR User Rates	Strict FFR User Rates	Comp. FFR Sum rate
12-16	1.20 - 1.14	1.20 - 1.14	4.92
6-17	0.99 - 1.21	0.99 - 1.21	4.29
2-5	0.98 - 0.84	0.98 - 0.84	4.24
3-10	0.75 - 0.78	0.75 - 0.78	3.88
4-14	0.71 - 0.75	0.71 - 0.75	3.76
8-11	1.01 - 1.10	1.01 - 1.10	3.44
1-9	0.69 - x	0.69 - 0.65	3.37
15-18	x - x	0.68 - 0.59	3.13
7-13	0.80 - x	0.80 - 0.68	3.07

Table: Comparison of user rates for cooperative vs. noncooperative protocols.

Multi-Cell Setup – Summary and Discussion

- We developed a comprehensive cooperation model, which combines
  - User cooperation
  - Frequency planning
  - Partner selection
  - Receiver selection
  - Power optimization
  - OFDMA
- We introduced the concept of complementary FFR.
  - Allows efficient use of frequency resources
  - Encourages cooperation among cell edge users, and across cells.
  - Naturally incorporates sectoring, decreases interference.
  - Enables receiver selection
- We solved the jointly optimal power allocation, receiver selection and partner selection problem.

Cognitive Radio–Motivation

- Original idea: efficient usage of the previously assigned but unused spectrum.
- Broader definition for cognitive radios: all around devices, which are aware of their environment
  - messages, codewords and channel states of the other users sharing the medium
- They can use this awareness to increase their capacity/rates
  - Constraint: should not adversely affect the communication quality of the existing primary users in the network.
- Three modes: interweave, underlay, overlay.
- In overlay: either cancel additional interference by cognitive user, or compensate for it by additional cooperation.

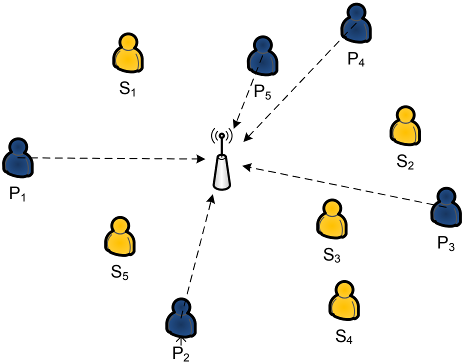
Cognitive Radio–Related work

- Information theoretic approach to overlay cognitive radio
  - Usually simplistic models, mostly interference channel.
- Interference channel model, 1 primary and 1 secondary TX-RX pair
- Non-causal side information
  - [Jovicic and Viswanath ’09], [Wei, Vishwanath and Arapostathis ’07], [Jiang and Xin ’09], [Maric, Goldsmith, Kramer and Shamai ’10].
  - Superposition, dirty paper coding, rate splitting, Gel’fand Pinsker binning.
- Causal side information
  - [Devroye, Mitran and Tarokh ’06], [Tuninetti ’07], [Seyedmehdi, Jiang, Xin and Wang ’09], [Cao, Chen and Zhang ’07], [Rini, Tuninetti and Devroye ’12]
  - Combination of similar techniques as above, taking into account message generation.

Cognitive Radio–Resource allocation

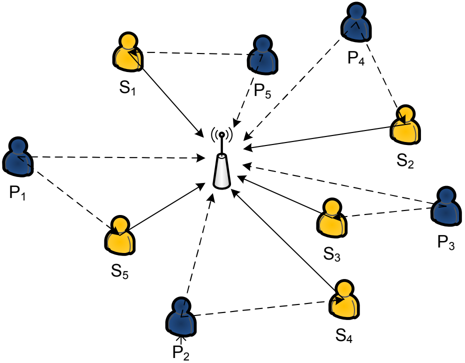
- Not many works on resource allocation in the causal overlay scenario.
- Rate constaints usually complicated in the interference model.
- Underlay model: [Zhang, Cui and Liang '09]
- Interference channel model not very friendly for resource optimized cognitive transmissions
  - Channel state information distribution an issue
  - With more and more advanced techniques, require too much collaboration from the primary.
- This paper: resource allocation for a cognitive cooperative MAC, cooperation causal.
- Receiver common, hence can distribute the CSI, more realistic.

Cognitive Cooperative MAC Model



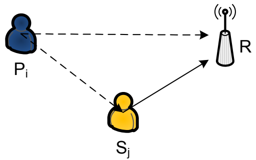
- Multiple access channel model, primary users already assigned resources.
- Assume orthogonal transmissions by all primary users: OFDMA
- Primary users able to optimally allocate powers: single user waterfilling.

Cognitive Cooperative MAC Model



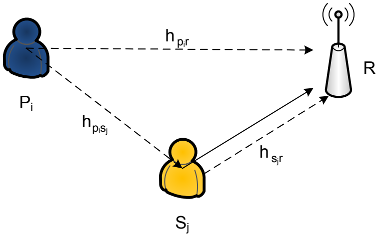
- SU's share resources with PU's, each SU shares one PU channel.
- SU's overhear PU transmissions and cooperate to buy 'interference rights'.
- How to cooperate? Sum rate optimal power allocation? Optimal partnering?

Cognitive Cooperative MAC Model



- SU's share resources with PU's, each SU shares one PU channel.
- SU's overhear PU transmissions and cooperate to buy 'interference rights'.
- How to cooperate? Sum rate optimal power allocation? Optimal partnering?

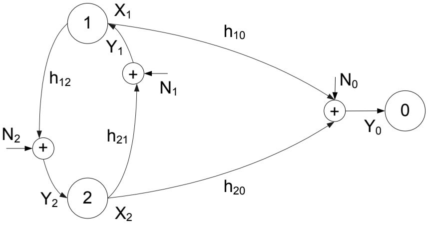
2-user Cognitive Cooperative MAC Model



$$Y_{r_{ij}} = \sqrt{q_{p_i r} d_{p_i r}^{-\beta}} X_{p_i} + \sqrt{q_{s_j r} d_{s_j r}^{-\beta}} X_{s_j} + N_r,$$

$$Y_{s_j} = \sqrt{q_{p_i s_j} d_{p_i s_j}^{-\beta}} X_{p_i} + N_{s_j},$$

Two User Cooperation Model [WVS 83], [SEA 03], [KU 07]

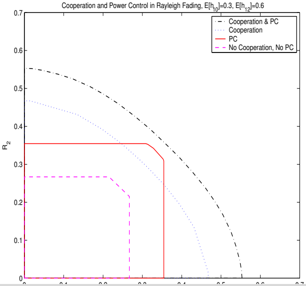


$$Y_0 = \sqrt{h_{10}}X_1 + \sqrt{h_{20}}X_2 + N_0$$
$$Y_1 = \sqrt{h_{21}}X_2 + N_1$$
$$Y_2 = \sqrt{h_{12}}X_1 + N_2$$

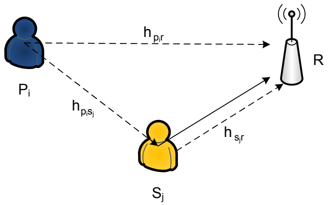
Block Markov superposition coding

- Build common information (X_{12}, X_{21})
- Cooperatively send (U)
- Inject new information (X_{10}, X_{20})

$$X_1 = \sqrt{p_{10}(\mathbf{h})}X_{10} + \sqrt{p_{12}(\mathbf{h})}X_{12} + \sqrt{p_{u1}(\mathbf{h})}U$$
$$X_2 = \sqrt{p_{20}(\mathbf{h})}X_{20} + \sqrt{p_{21}(\mathbf{h})}X_{21} + \sqrt{p_{u2}(\mathbf{h})}U$$



Two User Cognitive Cooperative MAC with Overlay [KI 12]



$$Y_{r_{ij}} = \sqrt{h_{p_ir}}X_{p_i} + \sqrt{h_{s_jr}}X_{s_j} + N_r,$$

$$Y_{s_j} = \sqrt{h_{p_is_j}}X_{p_i} + N_{s_j},$$

Block Markov superposition coding

$$\begin{aligned} X_{p_i} &= \sqrt{P_{p_ir}(\mathbf{h})}X_{p_ir}(W_{p_ir}(b), W_{p_is_j}(b-1)) \\ &\quad + \sqrt{P_{p_is_j}(\mathbf{h})}X_{p_is_j}(W_{p_is_j}(b), W_{p_is_j}(b-1)) + \sqrt{P_{p_ic}(\mathbf{h})}C(W_{p_is_j}(b-1)), \\ X_{s_j} &= \sqrt{P_{s_jr}(\mathbf{h})}X_{s_jr}(W_{s_j}(b), W_{p_is_j}(b-1)) + \sqrt{P_{s_jc}(\mathbf{h})}C(W_{p_is_j}(b-1)). \end{aligned}$$

Powers Assigned to Cooperative Codewords

$$P_{p_i}(\mathbf{h}) = P_{p_i r}(\mathbf{h}) + P_{p_i s_j}(\mathbf{h}) + P_{p_i c}(\mathbf{h})$$
$$P_{s_j}(\mathbf{h}) = P_{s_j r}(\mathbf{h}) + P_{s_j c}(\mathbf{h})$$
$$E\left[P_n(\mathbf{h})\right] \leq \bar{P}_n \quad \text{where} \quad n \in \{p_i, s_j\}.$$

- Can be shown that some of the codewords should always be assigned 0 power based on channel conditions [KU 07]:
  - Define: $\phi_1 \triangleq \{\mathbf{h} \ : \ h_{p_i s_j} \geq h_{p_i r}\}$, and $\phi_2 \triangleq \{\mathbf{h} \ : \ h_{p_i s_j} < h_{p_i r}\}$.
  - Then, we have either cooperative message or direct message transmitted by the PU, but not both:

$$P_{p_i r}(\mathbf{h}) = 0, \qquad \text{if } \mathbf{h} \in \phi_1$$
$$P_{p_i s_j}(\mathbf{h}) = 0, \qquad \text{if } \mathbf{h} \in \phi_2$$

Achievable Rates for Each User Pair - Overlay Mode

$$R_{p_i} \leq E\left\{\log(1+h_{p_ir}P_{p_ir}(\mathbf{h}))|\phi_2\right\}Pr[\phi_2] + E\left\{\log(1+h_{p_is_j}P_{p_is_j}(\mathbf{h}))|\phi_1\right\}Pr[\phi_1]$$

$$R_{s_j} < E\left\{\log\left[1+h_{s_jr}P_{s_jr}(\mathbf{h})\right]\right\}$$

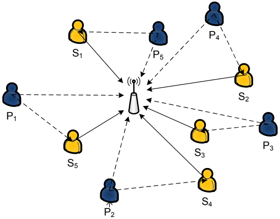
$$R_{p_i}+R_{s_j} \leq \min\left\{E\left\{\log(A)\right\}, E\left\{\log\left[1+h_{p_ir}P_{p_ir}(\mathbf{h})+h_{s_jr}P_{s_jr}(\mathbf{h})\right]|\phi_2\right\}Pr[\phi_2]\right.$$
$$\left. + E\left\{\log\left[1+h_{p_is_j}P_{p_is_j}(\mathbf{h})\right]|\phi_1\right\}Pr[\phi_1] + E\left\{\log\left[1+h_{s_jr}P_{s_jr}(\mathbf{h})\right]|\phi_1\right\}Pr[\phi_1]\right\}$$

- $A = 1 + h_{p_ir}P_{p_i}(\mathbf{h}) + h_{s_jr}P_{s_j}(\mathbf{h}) + 2\sqrt{h_{p_ir}h_{s_jr}P_{p_ic}P_{s_jc}}$.
- Additional minimum rate constraint should be imposed on the primary user rate,

$$R_{p_i} \geq E\left\{\log\left[1+P^{\star}_{p_i}(\mathbf{h})h_{p_ir}\right]\right\} \triangleq B^*$$

- $P^{\star}_{p_i}(\mathbf{h})$: optimal power level for single user transmission, obtained by waterfilling,
- B^*: resulting single user capacity of the primary user, without cooperation.

Sum-Rate Maximization Problem



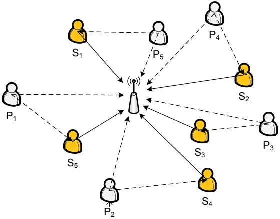
$$\max_{\substack{\Gamma_l \in \Gamma,\\ \mathbf{P}(\mathbf{h})}} \sum_{(i,j)\in\Gamma_l} R_{p_i} + R_{s_j}$$

$$\text{s.t.} \quad \{R_{p_i}, R_{s_j}\} \in R_{feasible}, \quad \forall (i,j) \in \Gamma_l$$

$$E[P_{p_i}] \leq \bar{P}_{p_i}, E[P_{s_j}] \leq \bar{P}_{s_j}$$

- Γ_l: admissible partnering strategy, $l \in \{1, \ldots, K!\}$.

Secondary User Total Rate Maximization Problem



$$\begin{aligned}\max_{\substack{\Gamma_l \in \Gamma,\\ \mathbf{P}(\mathbf{h})}} \quad & \sum_{(i,j)\in\Gamma_l} R_{s_j} \\ \text{s.t.} \quad & \{R_{p_i}, R_{s_j}\} \in R_{feasible}, \quad \forall (i,j) \in \Gamma_l \\ & E[P_{p_i}] \le \bar{P}_{p_i}, E[P_{s_j}] \le \bar{P}_{s_j}\end{aligned}$$

- Γ_l: admissible partnering strategy, $l \in \{1, \ldots, K!\}$.

Two Stage Decomposition of the Problem

- Once partnering is fixed, achievable rates of a pair does not affect any other pair.
- Power optimization problem is independent for each pair: compute optimal per pair rates for each fixed partnering policy.
- Then search for the optimal partnering to maximize the total rate.
- For sum rate maximization, can solve

$$\begin{aligned} \max_{\Gamma_l \in \Gamma}, \sum_{(i,j)\in\Gamma_l} \max_{\mathbf{P}_i(\mathbf{h}),\mathbf{P}_j(\mathbf{h})} \quad & R_{p_i} + R_{s_j} \\ \text{s.t.} \quad & \{R_{p_i}, R_{s_j}\} \in R_{feasible}, \quad \forall (i,j) \in \Gamma_l \\ & E[P_{p_i}] \leq \bar{P}_{p_i}, E[P_{s_j}] \leq \bar{P}_{s_j} \end{aligned}$$

- For secondary user total rate maximization, can solve

$$\begin{aligned} \max_{\Gamma_l \in \Gamma}, \sum_{(i,j)\in\Gamma_l} \max_{\mathbf{P}_i(\mathbf{h}),\mathbf{P}_j(\mathbf{h})} \quad & R_{s_j} \\ \text{s.t.} \quad & \{R_{p_i}, R_{s_j}\} \in R_{feasible}, \quad \forall (i,j) \in \Gamma_l \\ & E[P_{p_i}] \leq \bar{P}_{p_i}, E[P_{s_j}] \leq \bar{P}_{s_j} \end{aligned}$$

Inner Problem – Optimal Power Allocation

- Two optimization problems can be solved simultaneously.
  - $\alpha = 0$: secondary user rate maximization
  - $\alpha = 1$: sum rate maximization;

$$\sum_{\{i,j\}\in\Gamma_l} \max_{\mathbf{P}_i(\mathbf{h}),\mathbf{P}_j(\mathbf{h})} \alpha R_{p_i} + R_{s_j}$$

$$\text{s.t.}\quad R_{p_i} \le E\Big\{\log(1+h_{p_ir}P_{p_ir}(\mathbf{h}))|\phi_2\Big\}Pr[\phi_2] + E\Big\{\log(1+h_{p_is_j}P_{p_is_j}(\mathbf{h}))|\phi_1\Big\}Pr[\phi_1]$$

$$R_{s_j} < E\Big\{\log\Big[1+h_{s_jr}P_{s_jr}(\mathbf{h})\Big]\Big\}$$

$$R_{p_i}+R_{s_j} \le \min\Bigg\{E\Big\{\log(A)\Big\}, E\Big\{\log\Big[1+h_{p_ir}P_{p_ir}(\mathbf{h})+h_{s_jr}P_{s_jr}(\mathbf{h})\Big]|\phi_2\Big\}Pr[\phi_2]$$

$$+E\Big\{\log\Big[1+h_{p_is_j}P_{p_is_j}(\mathbf{h})\Big]|\phi_1\Big\}Pr[\phi_1] + E\Big\{\log\Big[1+h_{s_jr}P_{s_jr}(\mathbf{h})\Big]|\phi_1\Big\}Pr[\phi_1]\Bigg\}$$

$$R_{p_i} \ge B^\star$$

$$E\Big[P_{p_ir}(\mathbf{h})+P_{p_is_j}(\mathbf{h})+P_{pc}(\mathbf{h})\Big] \le \bar{P}_{p_i}, \quad E\Big[P_{s_jr}(\mathbf{h})+P_{sc}(\mathbf{h})\Big] \le \bar{P}_{s_j}$$

$$P_{p_is_j}(\mathbf{h}), P_{p_ic}(\mathbf{h}), P_{s_jr}(\mathbf{h}), P_{s_jc}(\mathbf{h}) \ge 0$$

Optimal Power Allocation: KKT Conditions

$$\lambda_1 \geq (\gamma_1 + \gamma_4) \frac{h_{p_i s_j}}{1 + h_{p_i s_j} P_{p_i s_j}(\mathbf{h})} + \gamma_3 \frac{h_{p_i r}}{A}, \qquad \mathbf{h} \in \phi_1$$

$$\lambda_2 \geq (\gamma_2 + \gamma_4) \frac{h_{s_j r}}{1 + h_{s_j r} P_{s_j r}(\mathbf{h})} + \gamma_3 \frac{h_{s_j r}}{A}, \qquad \mathbf{h} \in \phi_1$$

$$\lambda_1 \geq (\gamma_1 + \gamma_4) \frac{h_{p_i r}}{1 + h_{p_i r} P_{p_i r}(\mathbf{h})} + \gamma_3 \frac{h_{p_i r}}{1 + h_{p_i r} P_{p_i r} + h_{s_j r} P_{s_j r}}, \qquad \mathbf{h} \in \phi_2$$

$$\lambda_2 \geq (\gamma_2 + \gamma_4) \frac{h_{s_j r}}{1 + h_{s_j r} P_{s_j r}(\mathbf{h})} + \gamma_3 \frac{h_{s_j r}}{1 + h_{p_i r} P_{p_i r} + h_{s_j r} P_{s_j r}}, \qquad \mathbf{h} \in \phi_2$$

$$\lambda_1 \geq \gamma_3 \frac{h_{p_i r} \sqrt{P_{p_i c}(\mathbf{h})} + \sqrt{h_{p_i r} h_{s_j r} P_{s_j c}(\mathbf{h})}}{A \sqrt{P_{p_i c}(\mathbf{h})}}, \qquad \mathbf{h} \in \phi_1 \cup \phi_2$$

$$\lambda_2 \geq \gamma_3 \frac{h_{s_j r} \sqrt{P_{s_j c}(\mathbf{h})} + \sqrt{h_{p_i r} h_{s_j r} P_{p_i c}(\mathbf{h})}}{A \sqrt{P_{s_j c}(\mathbf{h})}}, \qquad \mathbf{h} \in \phi_1 \cup \phi_2$$

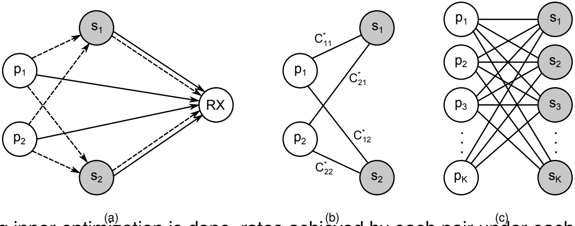
$$1 = \gamma_2 + \gamma_3 + \gamma_4, \qquad \mathbf{h} \in \phi_1 \cup \phi_2$$

$$\alpha + \gamma_5 = \gamma_1 + \gamma_3 + \gamma_4, \qquad \mathbf{h} \in \phi_1 \cup \phi_2$$

Optimal Power Allocation: Solution of KKT Conditions

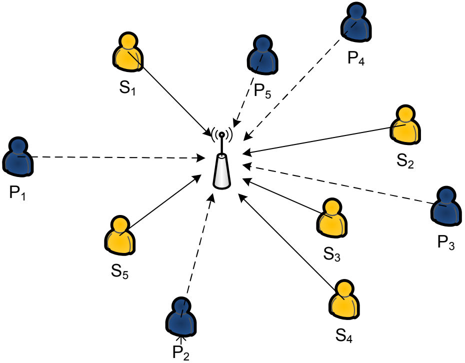
- KKT conditions coupled.
- Powers of each user can be reduced to a function of its partner's powers only.
- Concave optimization over convex power constraints, Cartesian nature across users.
- Iterative solution possible. One-user-at-a-time updates.
- Equivalent to a search over at most two Lagrange multipliers.
- Since partnering is based on statistics (offline), each PU-SU pair only require their own CSI.
- Resulting rates form the weights on the bipartite graph.

Outer Problem – Optimal Partner Selection



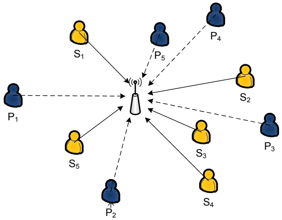
- Assuming inner optimization is done, rates achieved by each pair under each partnering strategy is known.
- Each primary can be paired by only one secondary.
- View the rates for each pair (sum or secondary user rate) as weights on a graph connecting each pair.
- Problem becomes maximum weighted matching on a bipartite graph: can be solved in polynomial time using Edmonds' algorithm.

Benchmark Underlay System



- Each SU still shares resources with PU's, partnering still important.
- PU transmits using single user optimal policy, rate fixed.
- Receiver performs 'onion peeling' while decoding, decodes SU first.

Secondary User Total Rate Maximization – Underlay System



$$\max_{\substack{\Gamma_l \in \Gamma, \\ \mathbf{P}_s(\mathbf{h})}} \sum_{(i,j)\in\Gamma_l} E\left[\log\left(1+\frac{h_{s_j r} P_{s_j r}(\mathbf{h})}{1+h_{p_i r} P^*_{p_i}(\mathbf{h})}\right)\right]$$

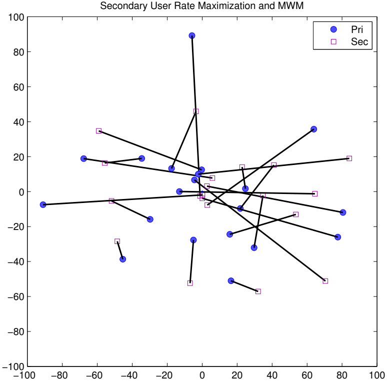
$$\text{s.t.} \quad E[P_{s_j}] \le \bar{P}_{s_j}$$

- Primary user constraint already satisfied, secondary user power allocation obeys multiuser waterfilling:

$$P_{s_j}(\mathbf{h}) = \left(\frac{1}{\lambda_{s_j}} - \frac{1+h_{p_i r} P^*_{p_i}(\mathbf{h})}{h_{s_j r}}\right)^+$$

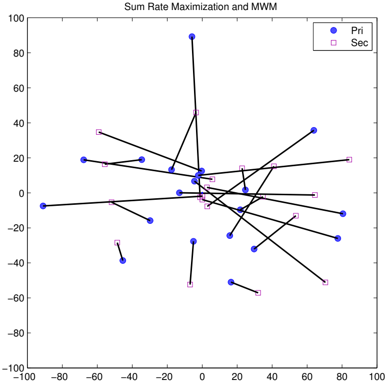
- Partnering same as in overlay setup, using MWM. Aim: minimize interference.

Simulation Results - Total Secondary User Rate Maximizing Partnering



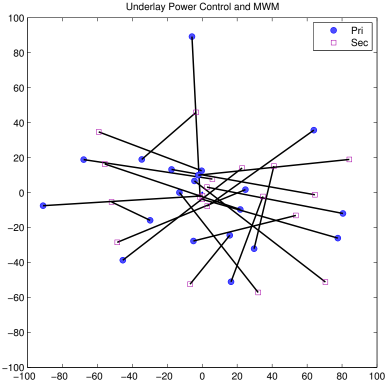
- $\sum R_s = 25.92, \quad \sum R_p + R_s = 42.82$

Simulation Results - Sum Rate Maximizing Partnering



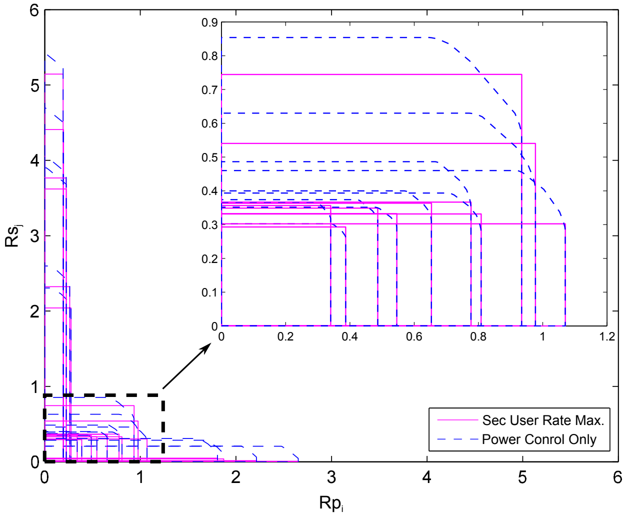
- $\sum R_s = 24.66, \quad \sum R_p + R_s = 43.52.$

Simulation Results - Secondary User Rate Maximizing Partnering in Underlay



- $\sum R_s = 25.55, \quad \sum R_p + R_s = 42.46.$

Simulation Results - Achievable rate regions



- $\sum R$ 25.55, $\sum R + R$ 42.46

Cognitive Cooperation – Summary and Discussion

- Solved the joint power control and partnering problem for cognitive cooperative multiple access channels.
- Obtained sum-rate and sum-of-secondary-user-rates optimal policies,
  - Convex optimization techniques
  - Techniques from graph theory, i.e., MWM.
- Cooperation is especially beneficial for mid-range primary-secondary user pairs,
- Overlay setup with cooperation promises higher rates than the underlay setup,
- Underlay with careful primary-secondary user pairing also promising.

Concluding Remarks

- Mutual user cooperation is quite natural in wireless networks.
    - More natural than relaying
- Joint use of cooperation, resource allocation, partnering possible with OFDMA.
- Many toy models from information theory can be extended to multiuser networks similarly.
- In multicell setups, frequency reuse schemes may need to be redesigned for cooperation.
- Cooperation is quite natural for cognitive radio, must be considered jointly with cognitive radio.
- We need to stop insisting on orthogonal transmissions, and make use of free side information.