Encoding/Decoding Strategies, Achievable Rates, and Resource Allocation for Cooperative Multiple Access Channels

Onur Kaya

Department of Electronics Engineering,

Işık University, Şile, Istanbul, Turkey

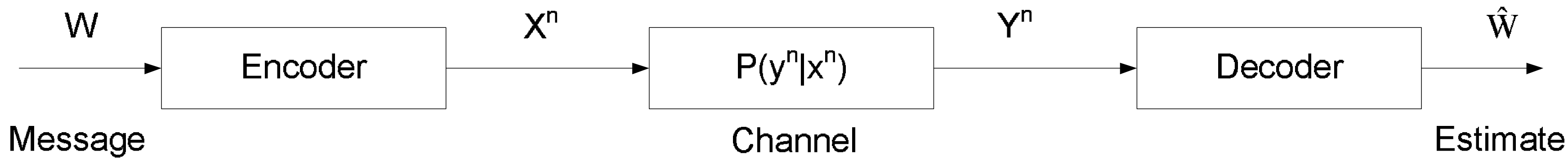
onurkaya@isikun.edu.tr

Introduction

Information theoretic analysis of communication systems

- Provides a benchmark for system performance – fundamental limits
  - E.g., “how close are we to channel capacity achievable by any scheme?”
- Gives direction to future research
  - It tells you what could still be achieved.
  - It suggests new ways to push your limits
    - E.g., use of multiple antennas, user cooperation, etc.
- Though strictly theoretical, gives insight to practical algorithms and applications
  - E.g., how to do encoding, decoding, resource allocation, medium access, etc.

Channel Capacity

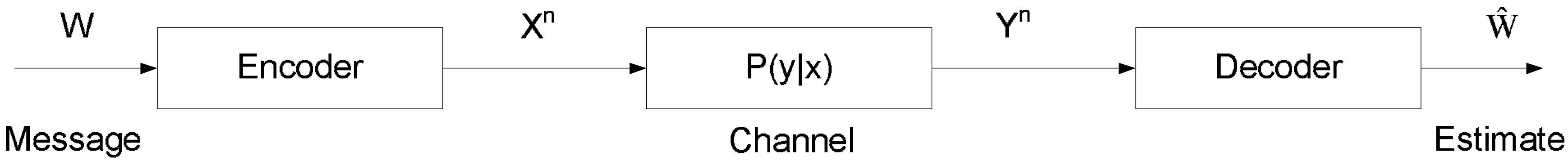


- The channel is memoryless if $P(Y^n|X^n) = \prod_{i=1}^{n} P(Y|X)$.
- A communication channel has capacity C, if
  - Any rate $R < C$ can be transmitted reliably (i.e., with arbitrarily low probability of error).
  - Any rate $R > C$ is guaranteed to have probability of error bounded away from zero.
- Achieved by using a random coding argument.

$$C = \max_{p(x)} I(X;Y)$$

- $p(x)$ is the marginal distribution of the random variable X.
- $I(X;Y)$ is the mutual information between X and Y, i.e., the reduction of uncertainty about X upon observing Y.

Channel Capacity

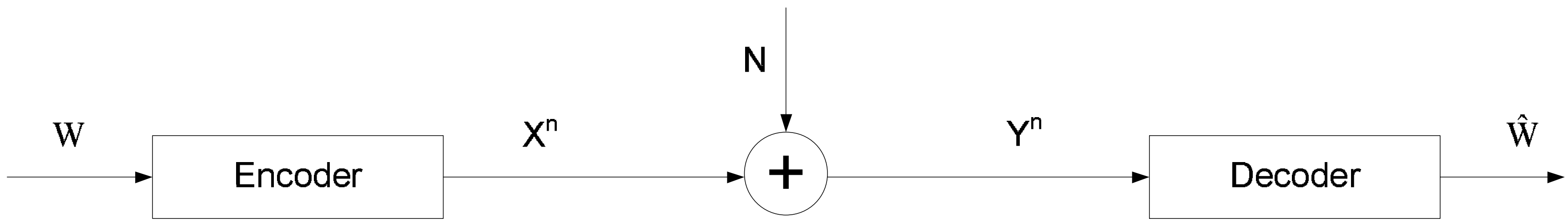


- The channel is memoryless if $P(Y^n|X^n) = \prod_{i=1}^{n} P(Y|X)$.
- A communication channel has capacity C, if
  - Any rate $R < C$ can be transmitted reliably (i.e., with arbitrarily low probability of error).
  - Any rate $R > C$ is guaranteed to have probability of error bounded away from zero.
- Achieved by using a random coding argument.

$$C = \max_{p(x)} I(X;Y)$$

- $p(x)$ is the marginal distribution of the random variable X.
- $I(X;Y)$ is the mutual information between X and Y, i.e., the reduction of uncertainty about X upon observing Y.

Gaussian Channel



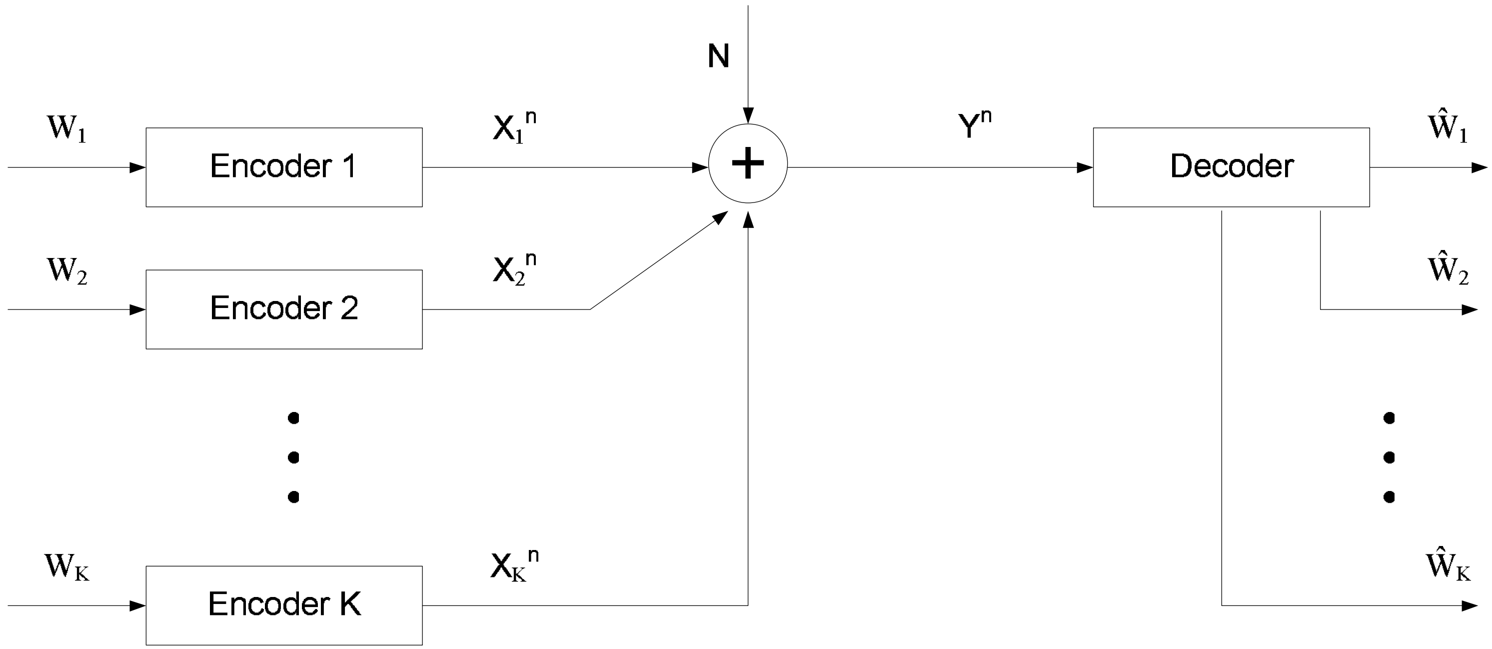
$$Y[i] = X[i] + N[i], \qquad i = 1, \ldots, N \tag{1}$$

- For Gaussian channels with signal power P and noise variance σ^2, the capacity is given by

$$C = \frac{1}{2} \log \left(1 + \frac{P}{\sigma^2} \right)$$

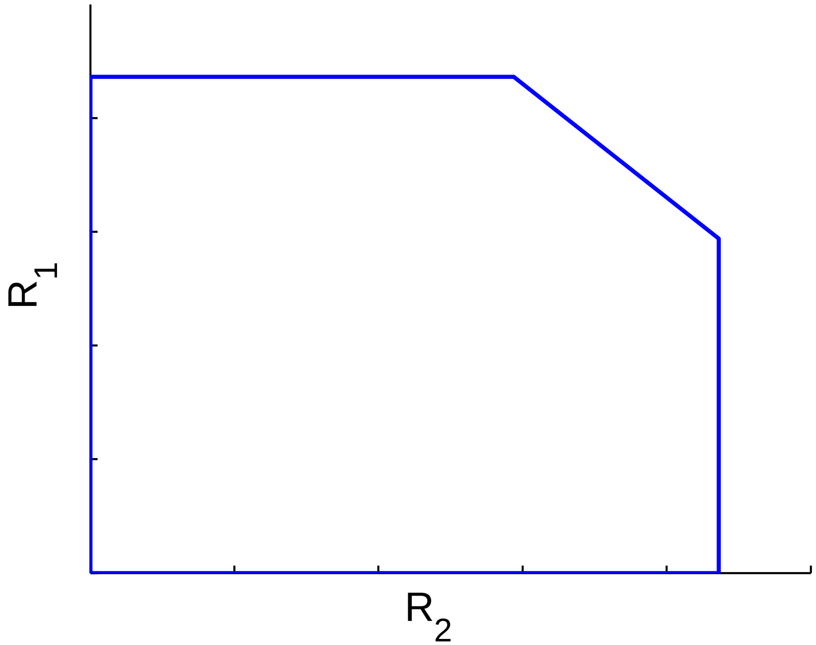
- To achieve capacity, the codeword X^n is taken from a codebook generated randomly,
  - Each symbol in the sequence X^n is i.i.d Gaussian, i.e., $X \sim \mathcal{N}(0, P)$.
- The capacity is achieved as the codeword length $n \rightarrow \infty$
- Decoding is performed based on jointly typical sequences.

Multiple Access Channels



- Multiple sources convey independent messages to the same receiver.
  - E.g., uplink of a cellular system, all mobiles send data to the base station.
- The rates achievable by users is worse than their single user performance due to interference.

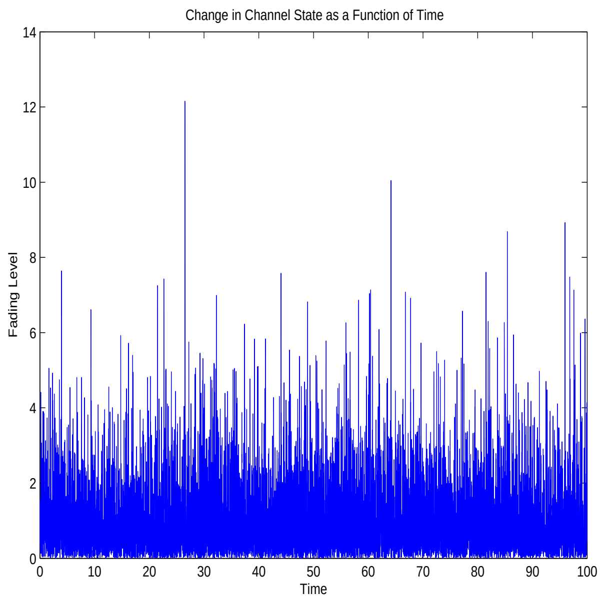
Capacity Region of the Gaussian MAC

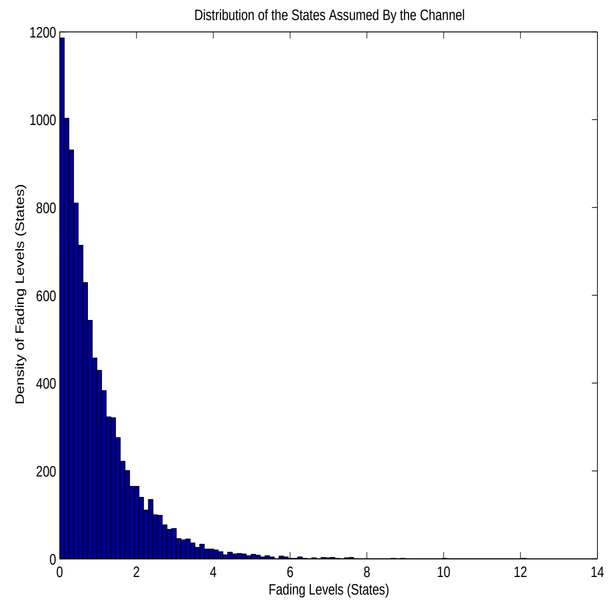


$$R_1 < \frac{1}{2}\log\left(1+\frac{P_1}{\sigma^2}\right)$$
$$R_2 < \frac{1}{2}\log\left(1+\frac{P_2}{\sigma^2}\right)$$
$$R_1+R_2 < \frac{1}{2}\log\left(1+\frac{P_1+P_2}{\sigma^2}\right)$$

- Region of achievable rates rather than a single rate value.
- The capacity region is a pentagon
- The rate of a user can be increased up to its single user limit, in expense of rate of other user.
- Corners of the boundary can be achieved by successive decoding.

Fading





- Fading: random fluctuations in the channel.
- Known statistics and the realization of the fading $\Rightarrow$ opportunistic resource allocation.
- Power control
  - Quality of service based (instantaneous requirements)
  - Capacity based (long term requirements)
- We are interested in long term capacities of systems with average power constraints.

Single User Channel (Goldsmith-Varaiya 1994)

- Channel capacity for single user

$$C = \frac{1}{2}\log\left(1+\frac{p}{\sigma^2}\right)$$

- In the presence of fading, for a fixed channel state h

$$y = \sqrt{p(h)h}x + n$$
$$C(h) = \frac{1}{2}\log\left(1+\frac{p(h)h}{\sigma^2}\right)$$

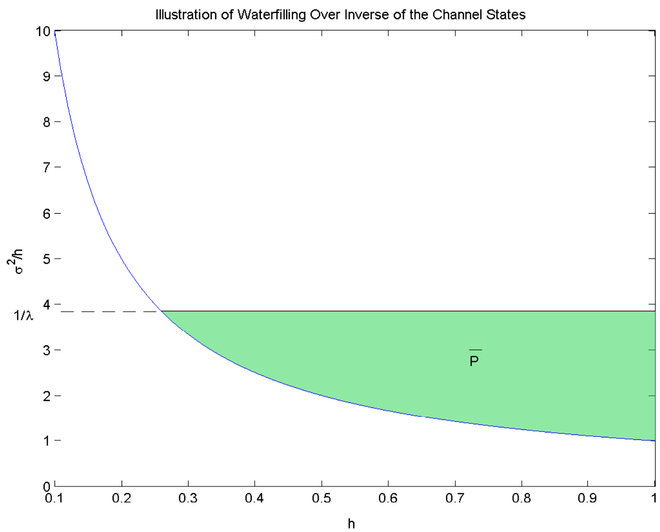
- Maximize the ergodic capacity, given an average power constraint

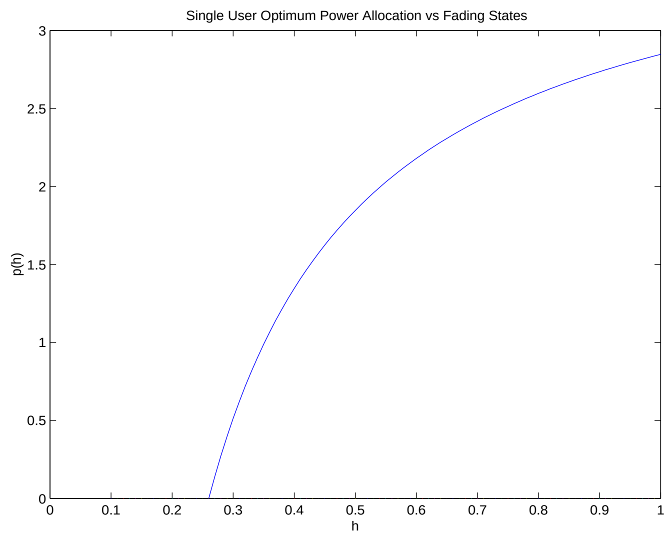
$$\max_{\{p(h)\}} \quad E_h\left[\log\left(1+\frac{p(h)h}{\sigma^2}\right)\right]$$
$$\text{s.t.} \quad E_h[p(h)] \leq \bar{p}, \qquad p(h) \geq 0$$

Single User Channel Solution-Waterfilling

- Optimal power allocation: waterfilling of power over time

$$p(h) = \left(\frac{1}{\lambda} - \frac{\sigma^2}{h}\right)^+$$





- More power to better channel states; no power to very poor channel states

Multiuser Scalar Gaussian Channel (Knopp-Humblet 1995)

- The received signal

$$y = \sum_{i=1}^{K} \sqrt{p_i(\mathbf{h}) h_i} x_i + n$$

- Region of achievable rates instead of a single capacity.
- Maximize ergodic sum capacity, given average power constraints

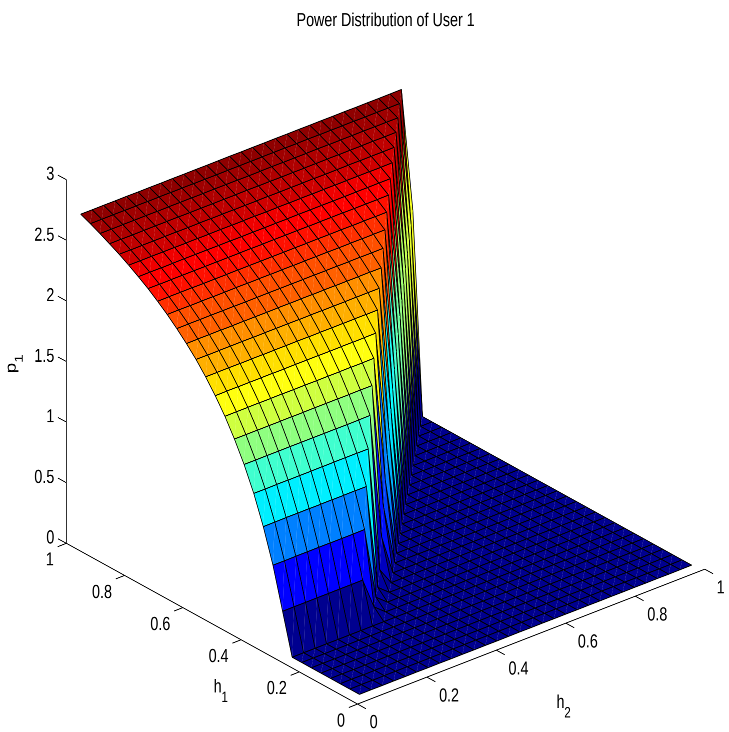
$$\begin{aligned} \max_{\{p_i(\mathbf{h})\}} \quad & E_{\mathbf{h}} \left[\frac{1}{2} \log \left(1 + \sigma^{-2} \sum_{i=1}^{K} h_i p_i(\mathbf{h}) \right) \right] \\ \text{s.t.} \quad & E_{\mathbf{h}} \left[p_i(\mathbf{h}) \right] \leq \bar{p}_i, \qquad p_i(\mathbf{h}) \geq 0, \qquad i = 1, \cdots, K \end{aligned}$$

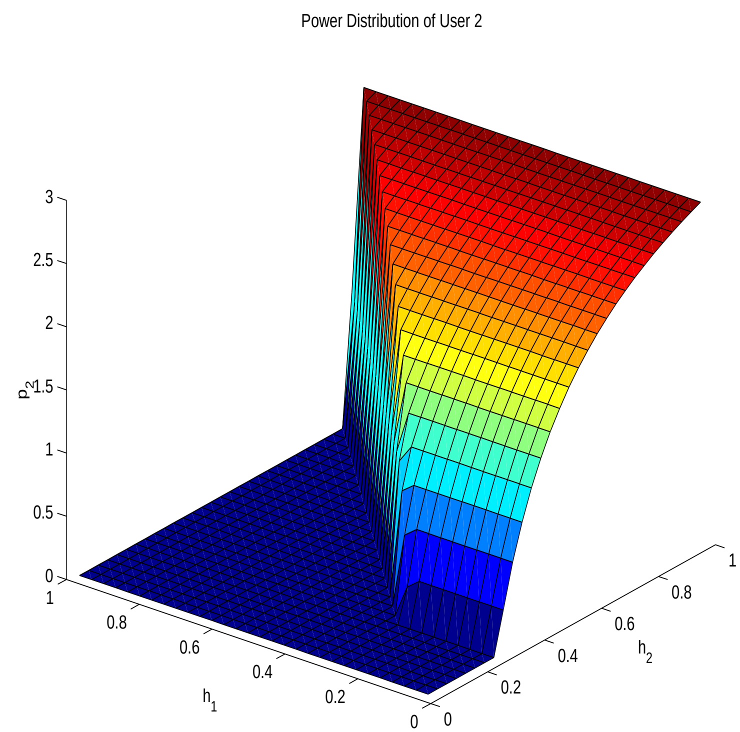
- Optimal power allocation: single user waterfilling on disjoint sets of channel states

$$p_k(\mathbf{h}) = \begin{cases} \left(\frac{1}{\lambda_k} - \frac{\sigma^2}{h_k} \right)^+, & \text{if} \quad h_k/\lambda_k > h_j/\lambda_j, \quad \forall j \neq k \\ 0, & \text{otherwise} \end{cases}$$

- Only the strongest user transmits at any given time. More than one user transmit w.p. 0.

Optimum Power Allocation: Scalar Multiuser Channel

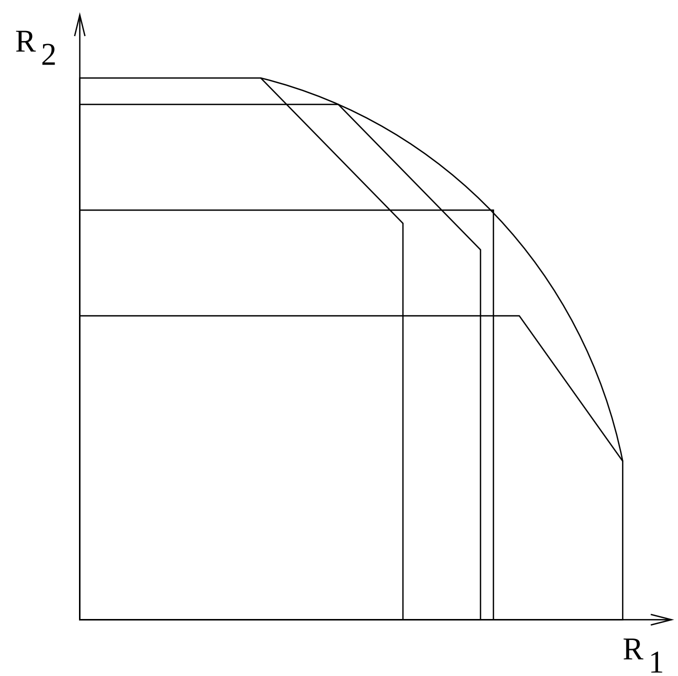




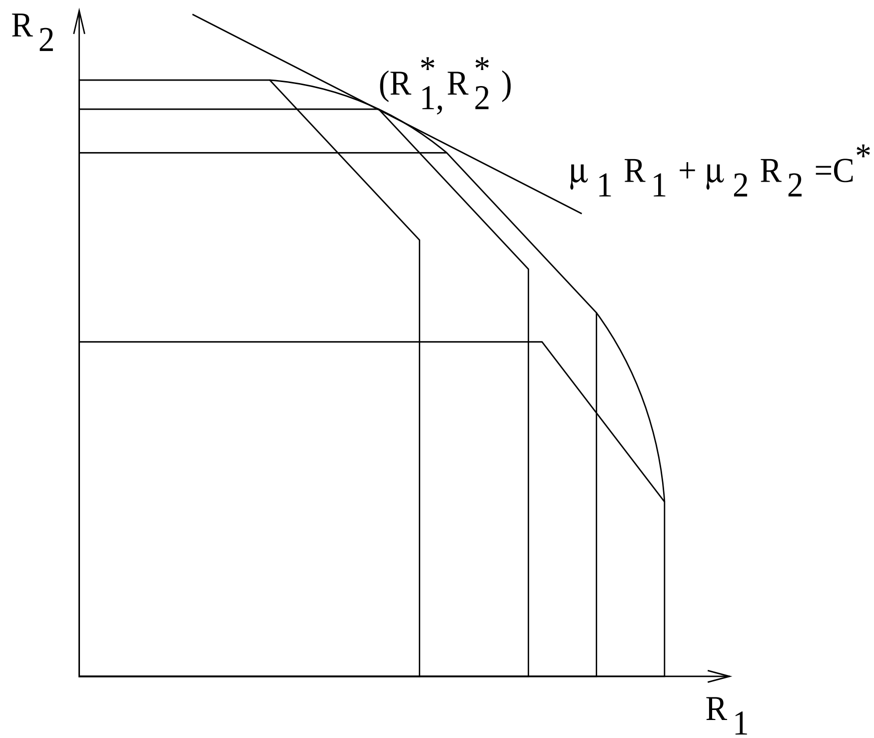
Capacity Region of Fading Scalar MAC with CSI (Hanly-Tse 98)

- Union of rate regions (polymatroids) achievable by all valid power control policies.

$$\bigcup_{\{\mathbf{p}(\mathbf{h}):\, E_{\mathbf{h}}[p_i(\mathbf{h})] \le \bar{p}_i,\, \forall i\}} \left\{ \mathbf{R} : \sum_{i\in\Gamma} R_i \le E_{\mathbf{h}}\left[\frac{1}{2}\log\left(1+\sigma^{-2}\sum_{i\in\Gamma} h_i p_i(\mathbf{h})\right)\right], \quad \forall \Gamma \subset \{1,\cdots,K\} \right\}$$

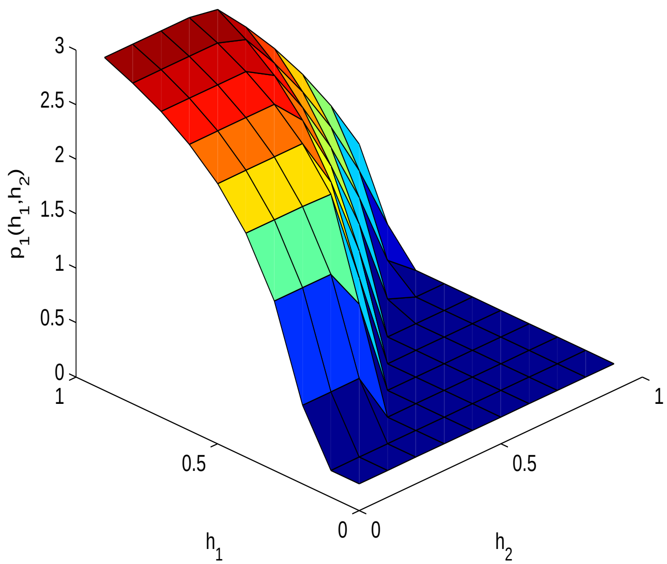


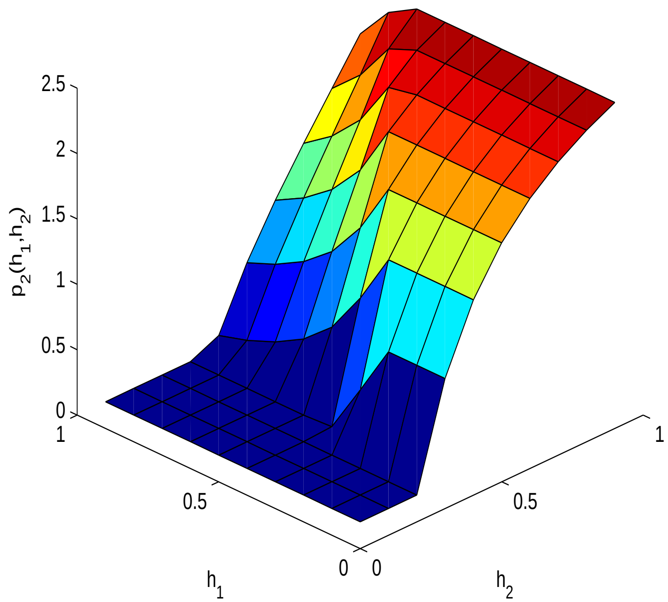
Properties of the Capacity Region



- The power control policy that corresponds to the rate pair (R_1^*, R_2^*) can be found by maximizing $\mu_1 R_1 + \mu_2 R_2$ subject to the average power constraints, for some μ_1, μ_2.
- Any (R_1^*, R_2^*) on the curved portion of the boundary is a corner of one of the pentagons.

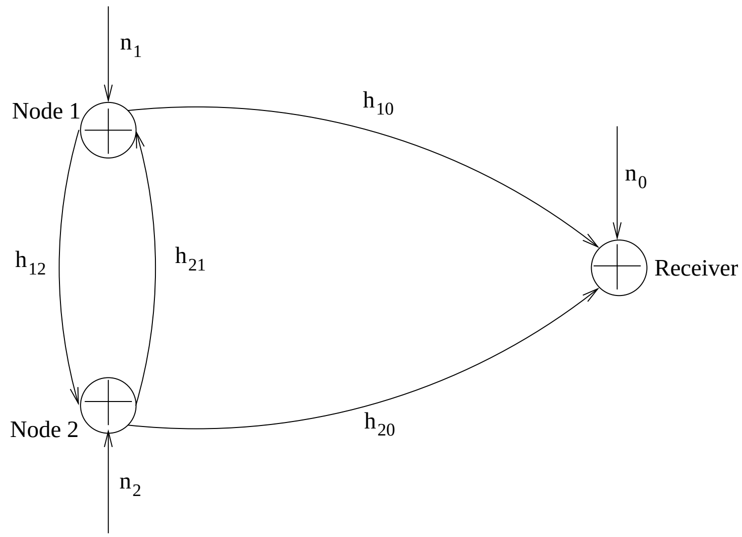
Optimum Power allocation





- Can be obtained by a greedy algorithm [Hanly-Tse 98], or by using generalized iterative waterfilling [Kaya-Ulukus 2006].
- Has a simultaneous waterfilling nature.

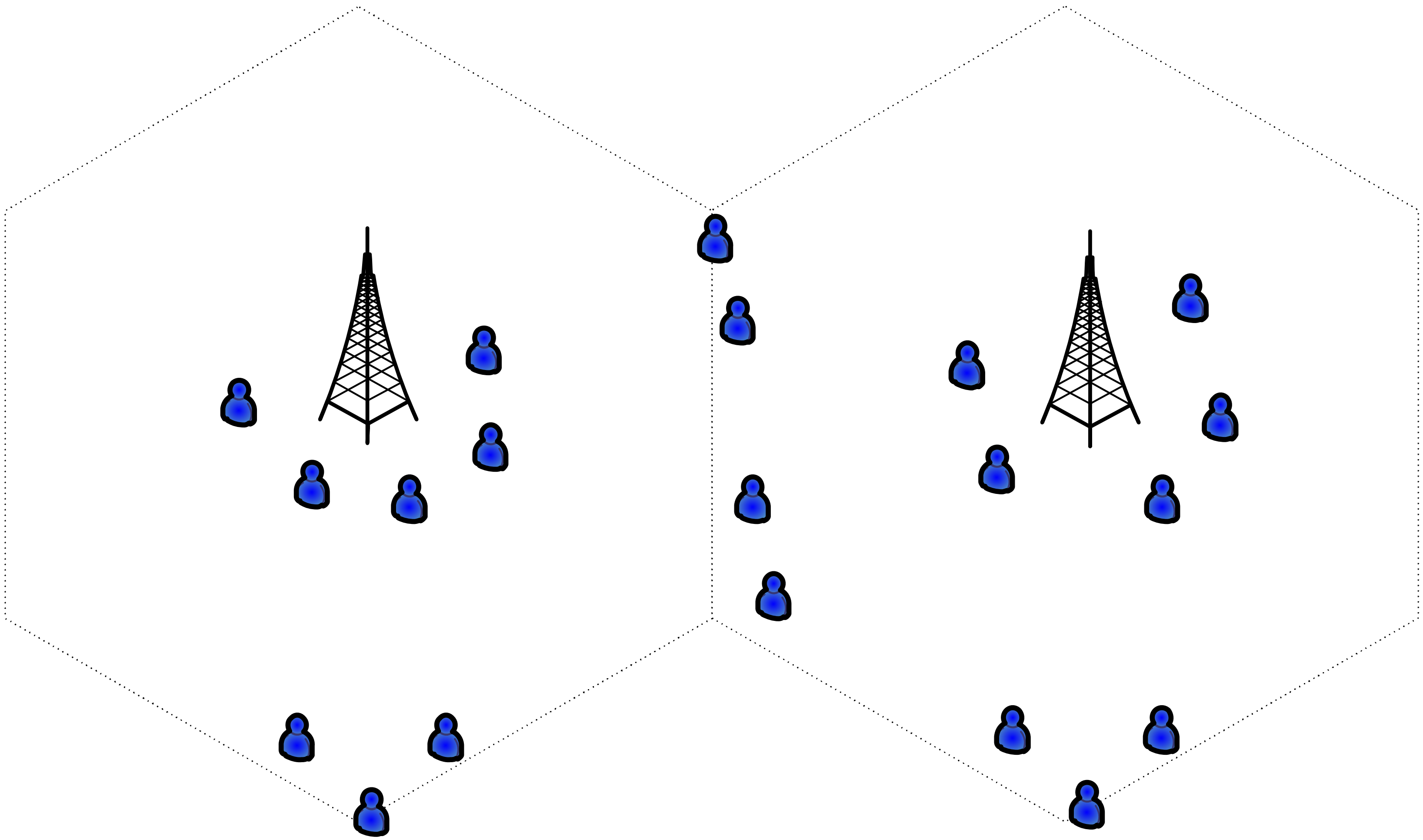
User Cooperation



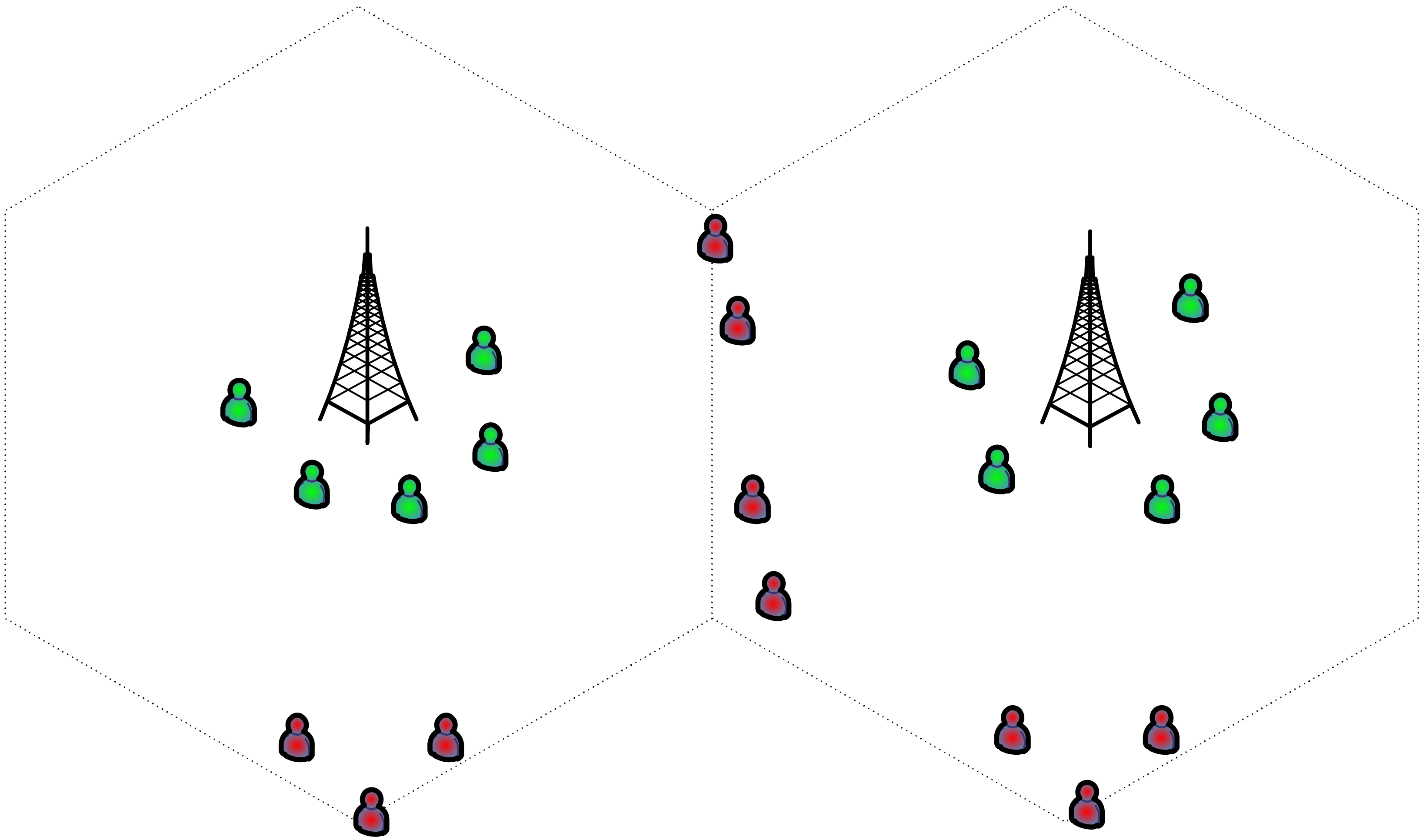
$$Y_0 = h_{10}X_1 + h_{20}X_2 + n_0$$
$$Y_1 = h_{21}X_2 + n_1$$
$$Y_2 = h_{12}X_1 + n_2$$

- Interference is information.
- Some versions of all transmitted signals are received by all nodes.
- User cooperation: exploit overheard information to jointly design encoding, transmit, routing policies.
- Building block towards the analysis of larger networks.

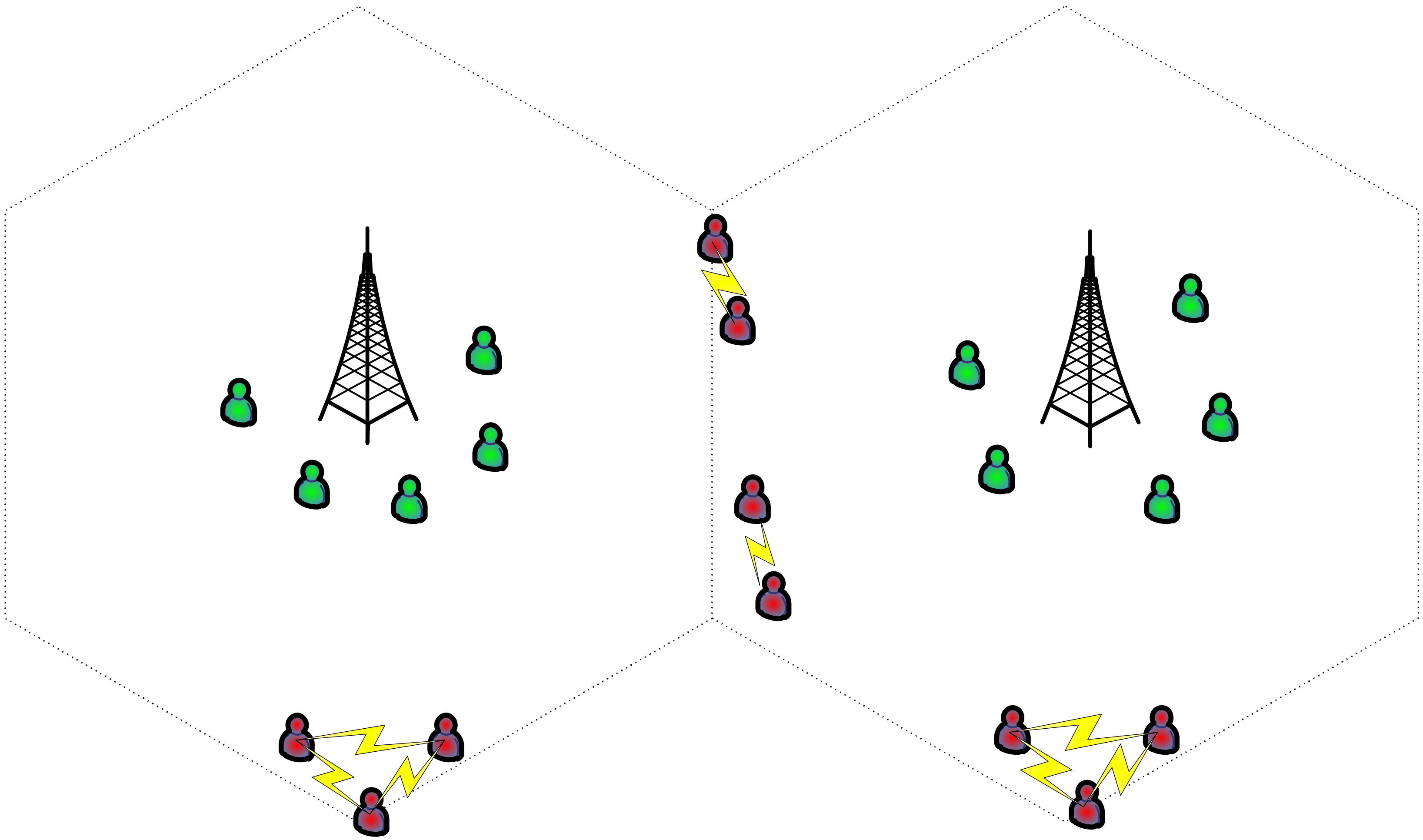
Motivation



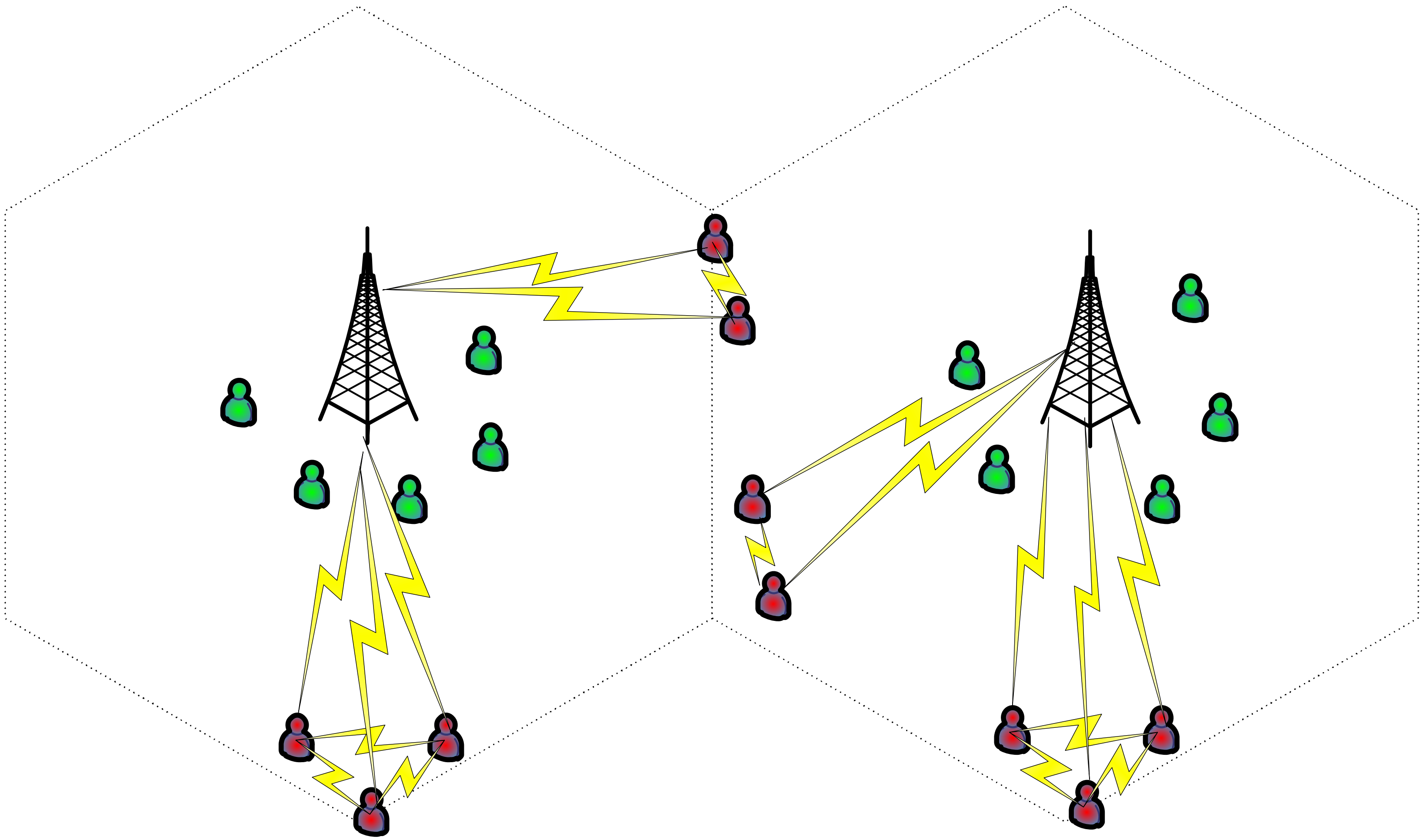
Motivation



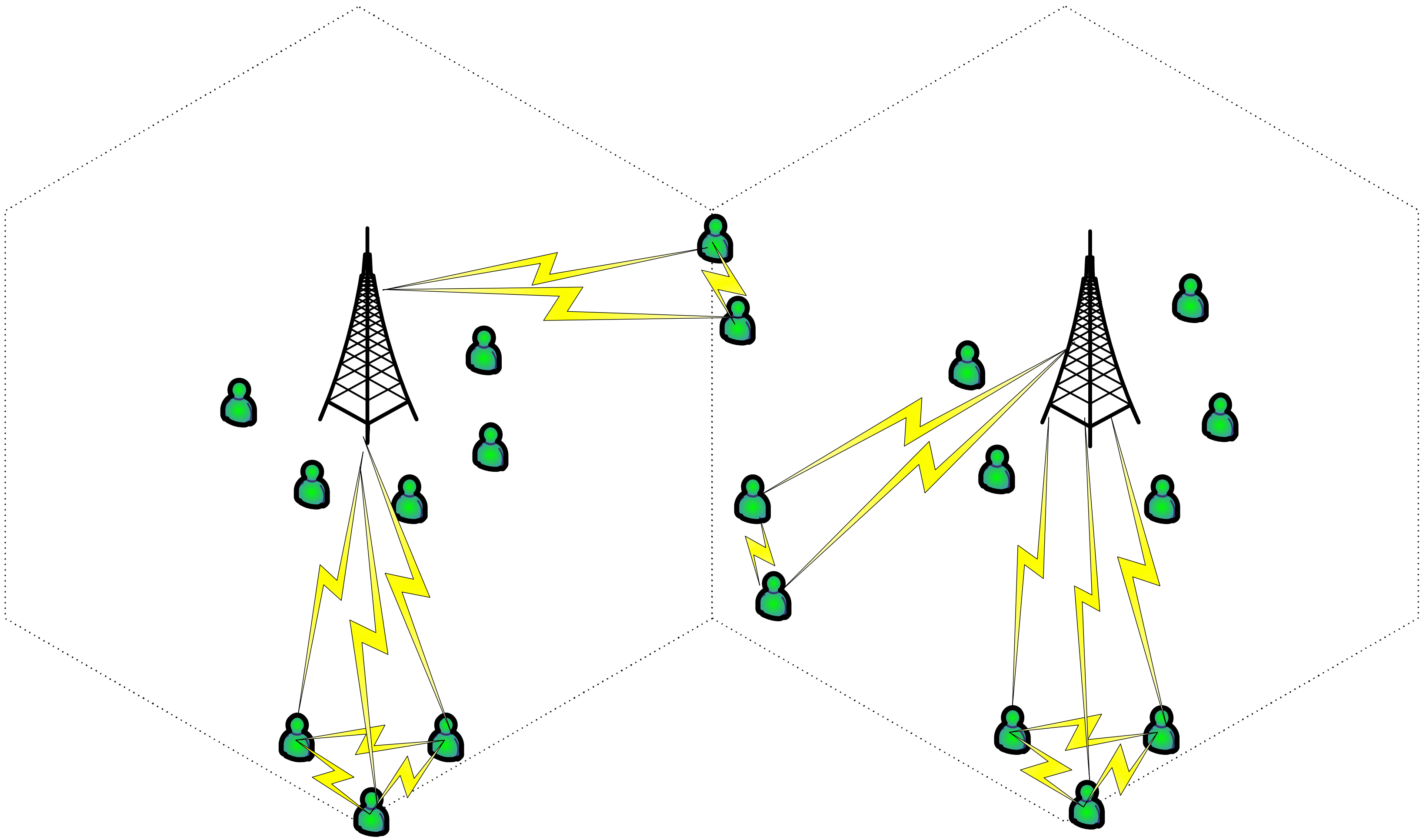
Motivation



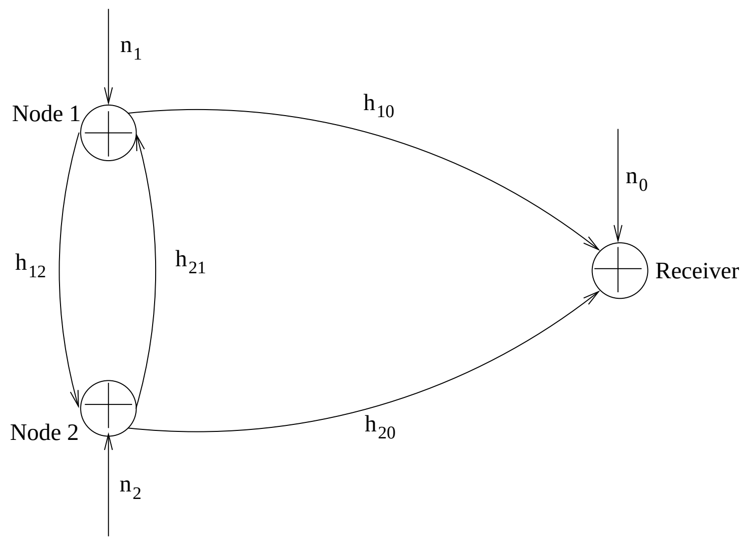
Motivation



Motivation



Optimum Power Allocation for the Two User Cooperative MAC



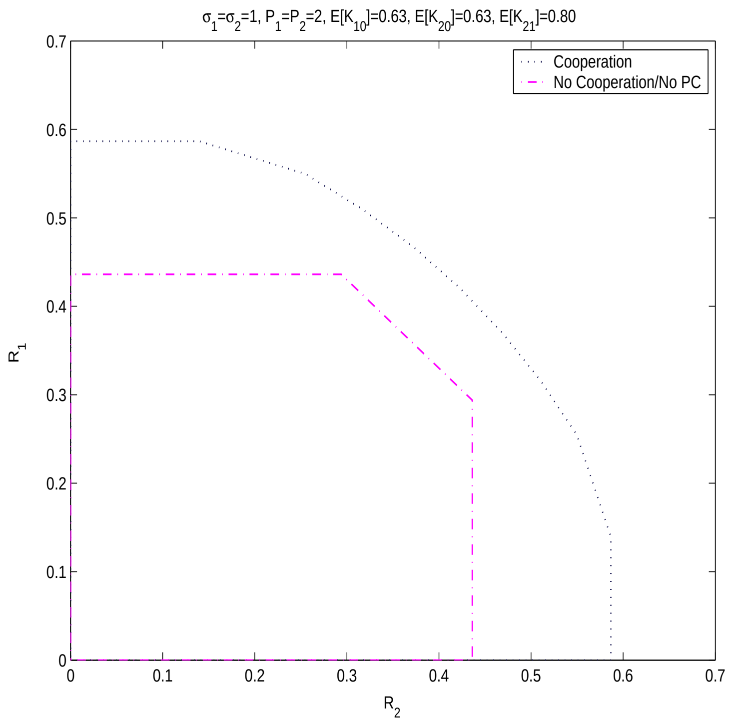
$$Y_0 = h_{10}X_1 + h_{20}X_2 + n_0$$
$$Y_1 = h_{21}X_2 + n_1$$
$$Y_2 = h_{12}X_1 + n_2$$

- Joint work with Sennur Ulukus

MAC with Generalized Feedback

- Gaussian MAC with cooperating encoders [Sendonaris, Erkip, Aazhang]
  - Special case of MAC with generalized feedback [Willems, van der Meulen, Schalkwijk]
- An achievable rate region is obtained by employing
  - Block Markov superposition encoding
    - Inject high rate fresh information to be resolved with the help of upcoming blocks.
    - Send resolution information for previous blocks.
  - Backward decoding
    - After receiving all blocks, decode the resolution information in the last block.
    - Using previously decoded resolution information, sequentially decode earlier blocks.

Gaussian MAC with User Cooperation – No Resource Allocation



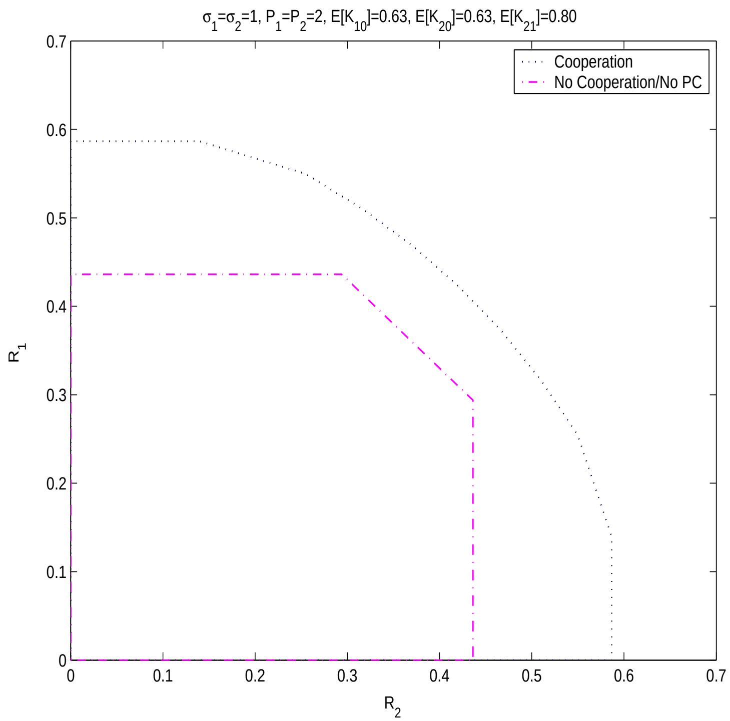
Block Markov superposition coding

- Build common information (X_{12}, X_{21})
- Cooperatively send (U)
- Inject new information (X_{10}, X_{20})

$$X_1 = \sqrt{p_{10}} X_{10} + \sqrt{p_{12}} X_{12} + \sqrt{p_{u1}} U$$
$$X_2 = \sqrt{p_{20}} X_{20} + \sqrt{p_{21}} X_{21} + \sqrt{p_{u2}} U$$

- Amplitude of the each channel's gain is assumed to be known at the corresponding receiver.
- Phases of all channel gains are assumed known at the receiver and the transmitters
  - Coherent combining.

Gaussian MAC with User Cooperation – Resource Allocation



Block Markov superposition coding

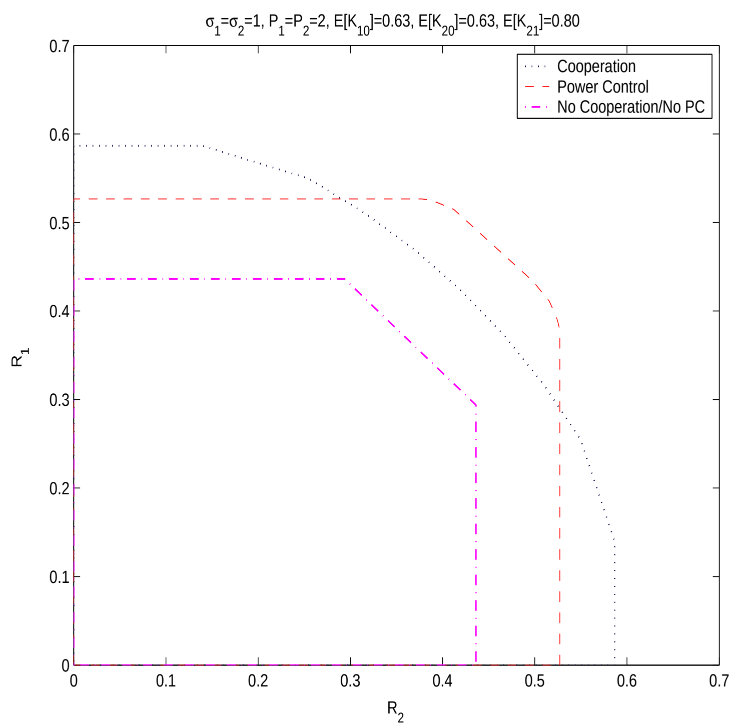
- Build common information (X_{12}, X_{21})
- Cooperatively send (U)
- Inject new information (X_{10}, X_{20})

$$X_1 = \sqrt{p_{10}(\mathbf{h})}X_{10} + \sqrt{p_{12}(\mathbf{h})}X_{12} + \sqrt{p_{u1}(\mathbf{h})}U$$

$$X_2 = \sqrt{p_{20}(\mathbf{h})}X_{20} + \sqrt{p_{21}(\mathbf{h})}X_{21} + \sqrt{p_{u2}(\mathbf{h})}U$$

- Complete channel state information at the transmitters and the receiver.
- Transmitted codewords can be modulated by channel adaptive power levels
  - Opportunistic cooperation and transmission – use available average power efficiently.

Gaussian MAC with User Cooperation – Resource Allocation



Block Markov superposition coding

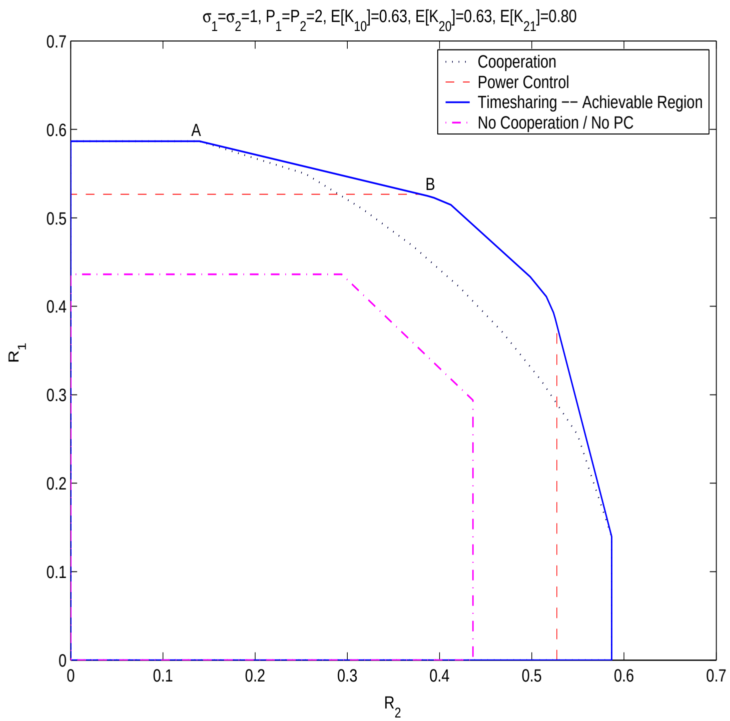
- Build common information (X_{12}, X_{21})
- Cooperatively send (U)
- Inject new information (X_{10}, X_{20})

$$X_1 = \sqrt{p_{10}(\mathbf{h})}X_{10}$$

$$X_2 = \sqrt{p_{20}(\mathbf{h})}X_{20}$$

- Complete channel state information at the transmitters and the receiver.
- Transmitted codewords can be modulated by channel adaptive power levels
  - Opportunistic cooperation and transmission – use available average power efficiently.

Gaussian MAC with User Cooperation – Resource Allocation



Block Markov superposition coding

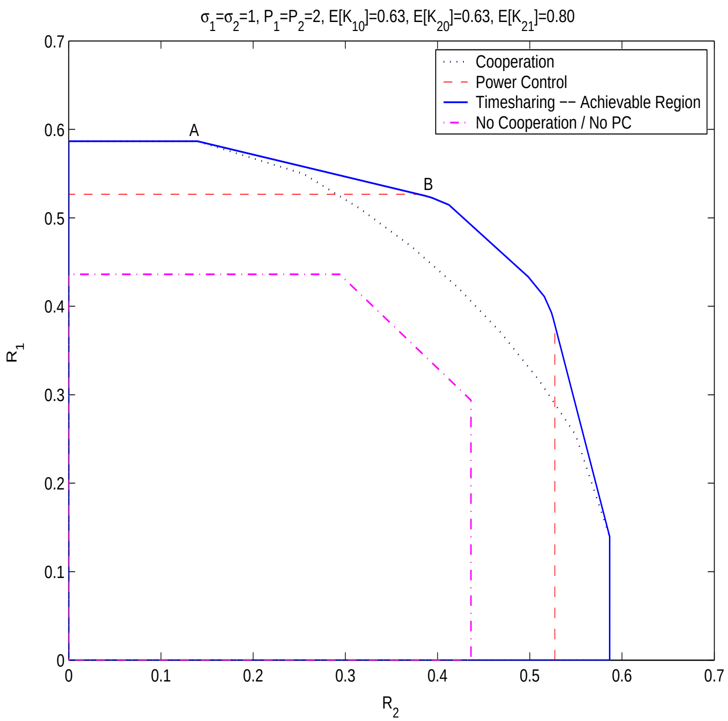
- Build common information (X_{12}, X_{21})
- Cooperatively send (U)
- Inject new information (X_{10}, X_{20})

$$X_1 = \sqrt{p_{10}(\mathbf{h})}X_{10}$$

$$X_2 = \sqrt{p_{20}(\mathbf{h})}X_{20}$$

- Complete channel state information at the transmitters and the receiver.
- Transmitted codewords can be modulated by channel adaptive power levels
  - Opportunistic cooperation and transmission – use available average power efficiently.

Gaussian MAC with User Cooperation – Resource Allocation



Block Markov superposition coding

- Build common information (X_{12}, X_{21})
- Cooperatively send (U)
- Inject new information (X_{10}, X_{20})

$$X_1 = \sqrt{p_{10}(\mathbf{h})}X_{10} + \sqrt{p_{12}(\mathbf{h})}X_{12} + \sqrt{p_{u1}(\mathbf{h})}U$$
$$X_2 = \sqrt{p_{20}(\mathbf{h})}X_{20} + \sqrt{p_{21}(\mathbf{h})}X_{21} + \sqrt{p_{u2}(\mathbf{h})}U$$

- Complete channel state information at the transmitters and the receiver.
- Transmitted codewords can be modulated by channel adaptive power levels
  - Opportunistic cooperation and transmission – use available average power efficiently.

Achievable Region of Rates with Power Control

- Union over all valid policies $E[p_{i0}(\mathbf{h}) + p_{ij}(\mathbf{h}) + p_{U_i}(\mathbf{h})] \leq \overline{p_i}$ of pairs $\{R_1, R_2\}$ that satisfy

$$R_1 < E\left[\log\left(1+\frac{h_{12}p_{12}(\mathbf{h})}{h_{12}p_{10}(\mathbf{h})+\sigma_2^2}\right)+\log\left(1+\frac{h_{10}p_{10}(\mathbf{h})}{\sigma_0^2}\right)\right]$$

$$R_2 < E\left[\log\left(1+\frac{h_{21}p_{21}(\mathbf{h})}{h_{21}p_{20}(\mathbf{h})+\sigma_1^2}\right)+\log\left(1+\frac{h_{20}p_{20}(\mathbf{h})}{\sigma_0^2}\right)\right]$$

$$R_1+R_2 < \min\left\{E\left[\log\left(1+\frac{h_{10}p_1(\mathbf{h})+h_{20}p_2(\mathbf{h})+2\sqrt{h_{10}h_{20}p_{U_1}(\mathbf{h})p_{U_2}(\mathbf{h})}}{\sigma_0^2}\right)\right],\right.$$
$$\left.E\left[\log\left(1+\frac{h_{12}p_{12}(\mathbf{h})}{h_{12}p_{10}(\mathbf{h})+\sigma_2^2}\right)+\log\left(1+\frac{h_{21}p_{21}(\mathbf{h})}{h_{21}p_{20}(\mathbf{h})+\sigma_1^2}\right)+\log\left(1+\frac{h_{10}p_{10}(\mathbf{h})+h_{20}p_{20}(\mathbf{h})}{\sigma_0^2}\right)\right]\right\}$$

- Bounds not concave in power vector $\mathbf{p}(\mathbf{h}) = [p_{10}(\mathbf{h})\ p_{12}(\mathbf{h})\ p_{U_1}(\mathbf{h})\ p_{20}(\mathbf{h})\ p_{21}(\mathbf{h})\ p_{U_2}(\mathbf{h})]$

Properties of Sum-Rate-Optimal Power Allocation

Proposition 1 *Let the effective channel gains normalized by the noise powers be defined as $s_{ij} = h_{ij}/\sigma_j^2$. Then, for the power control policy $\mathbf{p}^*(\mathbf{h})$ that maximizes the sum rate, we need*

- $p_{10}^*(\mathbf{h}) = p_{20}^*(\mathbf{h}) = 0$, if $s_{12} > s_{10}$ and $s_{21} > s_{20}$
- $p_{10}^*(\mathbf{h}) = p_{21}^*(\mathbf{h}) = 0$, if $s_{12} > s_{10}$ and $s_{21} \leq s_{20}$
- $p_{12}^*(\mathbf{h}) = p_{20}^*(\mathbf{h}) = 0$, if $s_{12} \leq s_{10}$ and $s_{21} > s_{20}$
- $$\left.\begin{array}{c} p_{12}^*(\mathbf{h}) = p_{21}^*(\mathbf{h}) = 0 \\ \text{OR} \\ p_{10}^*(\mathbf{h}) = p_{21}^*(\mathbf{h}) = 0 \\ \text{OR} \\ p_{12}^*(\mathbf{h}) = p_{20}^*(\mathbf{h}) = 0 \end{array}\right\} \text{if } s_{12} \leq s_{10} \text{ and } s_{21} \leq s_{20}$$

Implications of the Optimal Power Allocation

- Block Markov superposition coding is simpler than originally thought.
  - Each transmitter either sends a cooperation signal or fresh information, but not both!
- The choice at each channel state "only" depends on the channel state.
  - Channel statistics, power constraints play no role in deciding which signals to transmit.
  - Except for the tiny little last case... which usually has very insignificant probability.
- The achivable rate expressions are greatly simplified, and are now concave.
- This simplified coding policy not only maximizes the sum rate, but also the individual rate constrains on R_1 and R_2, and is optimal in terms of the entire rate region.
- Concave optimization problem over a convex constraint set, but non-differentiable.

Simplified Rate Region – Example

- Assume $s_{12} > s_{10}$, $s_{21} > s_{20}$ to illustrate the simplified rate region.

$$R_1 < E\left[\log\left(1 + s_{12}p_{12}(\mathbf{h})\right)\right]$$

$$R_2 < E\left[\log\left(1 + s_{21}p_{21}(\mathbf{h})\right)\right]$$

$$R_1 + R_2 < \min\left\{E\left[\log\left(1 + s_{10}p_1(\mathbf{h}) + s_{20}p_2(\mathbf{h}) + 2\sqrt{s_{10}s_{20}p_{U_1}(\mathbf{h})p_{U_2}(\mathbf{h})}\right)\right],\right.$$
$$\left. E\left[\log\left(1 + s_{12}p_{12}(\mathbf{h})\right) + \log\left(1 + s_{21}p_{21}(\mathbf{h})\right)\right]\right\}$$

- Inequalities define either a pentagon like in the traditional MAC, or a triangle.
- All bounds concave in powers, and so is any weighted sum $\mu_1 R_1 + \mu_2 R_2$ at the corners.
- Sum rate not differentiable where the arguments of the min are equal.

Rate Maximization Using Subgradient Method

- Points on the rate region boundary can be obtained by maximizing $C_{\boldsymbol{\mu}} = \mu_1 R_1 + \mu_2 R_2$.
- The optimization problem for arbitrary priorities μ_1 and μ_2 is given by

$$
\begin{aligned}
\max_{\mathbf{p}(\mathbf{h})} \ & \mu_1 R_1 + \mu_2 R_2 \\
\text{s.t. } & E_{3,4}\left[p_{10}(\mathbf{h})\right] + E_{1,2}\left[p_{12}(\mathbf{h})\right] + E\left[p_{U_1}(\mathbf{h})\right] \le \bar{p}_1 \\
& E_{2,4}\left[p_{20}(\mathbf{h})\right] + E_{1,3}\left[p_{21}(\mathbf{h})\right] + E\left[p_{U_2}(\mathbf{h})\right] \le \bar{p}_2
\end{aligned}
$$

- $\{R_1, R_2\}$ is the corner of the pentagon obtained for a given power allocation policy.
- Gradient of the objective function does not exist everywhere, find subgradient $\mathbf{g}$ instead

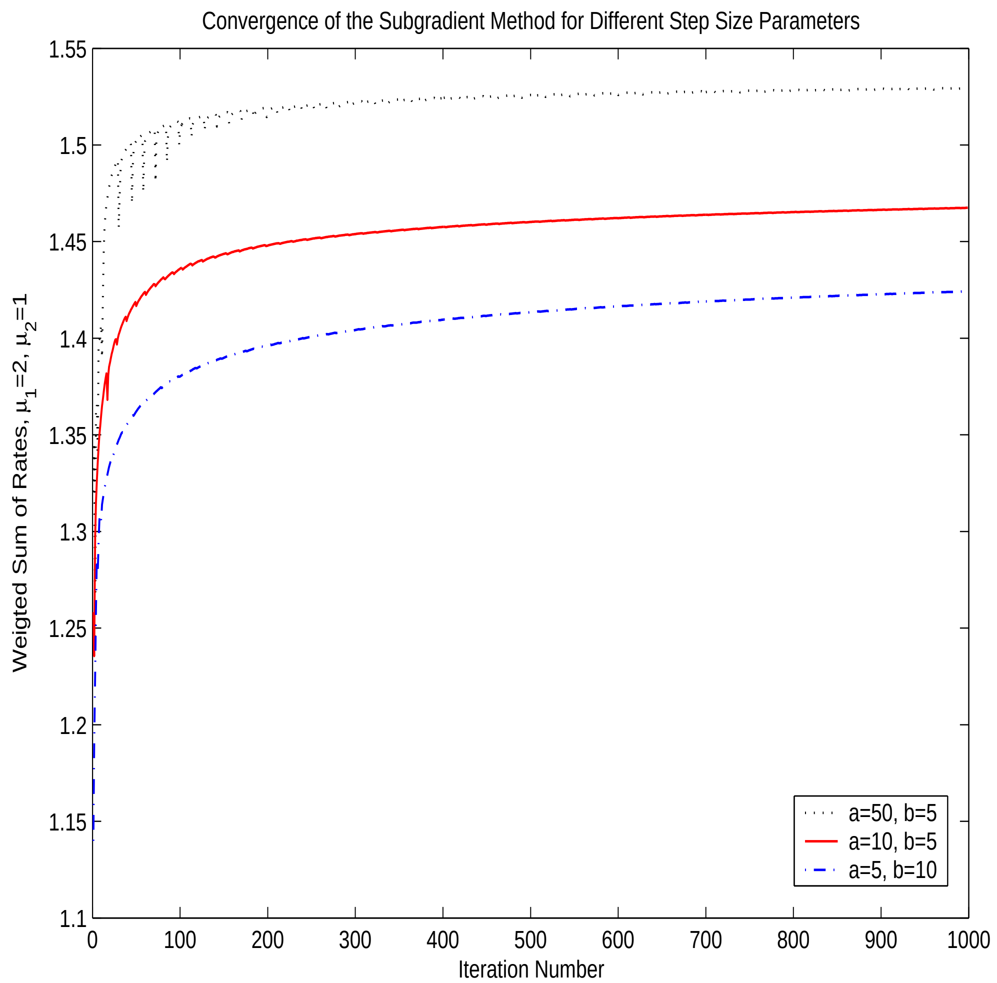
$$C_{\boldsymbol{\mu}}(\mathbf{p}') \le C_{\boldsymbol{\mu}}(\mathbf{p}) + (\mathbf{p}' - \mathbf{p})\mathbf{g}$$

- Use projected subgradient method to maximize $C_{\boldsymbol{\mu}}$

$$\mathbf{p}(k+1) = \left[\mathbf{p}(k) + \alpha_k \mathbf{g}_k\right]^+$$

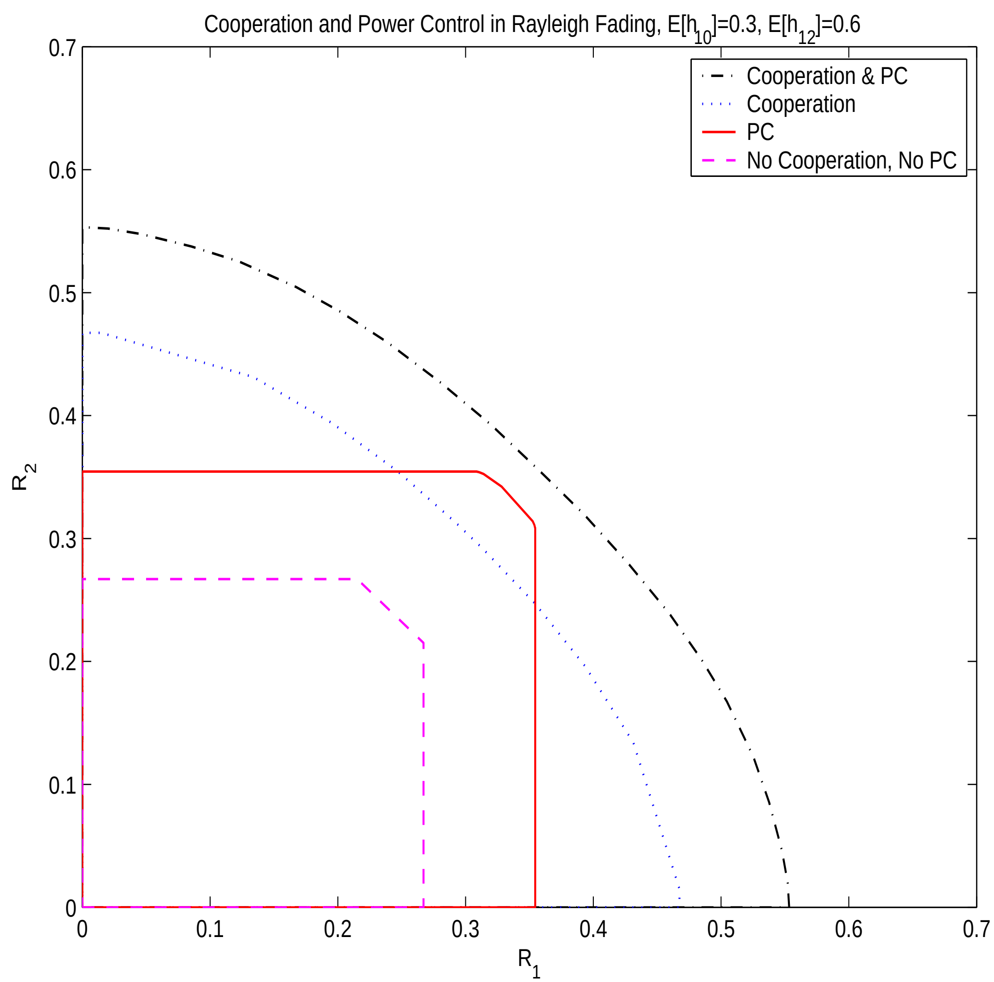
- Provably converges for a diminishing stepsize α_k [Shor].

Convergence of the Projected Subgradient Algorithm



- Rate of convergence depends on the stepsize parameter.
- Subgradient method need not give a monotonically increasing function value.

Achievable Rate Region for Joint Power Control and User Cooperation



- Optimized power levels enlarge the achievable rate region significantly.

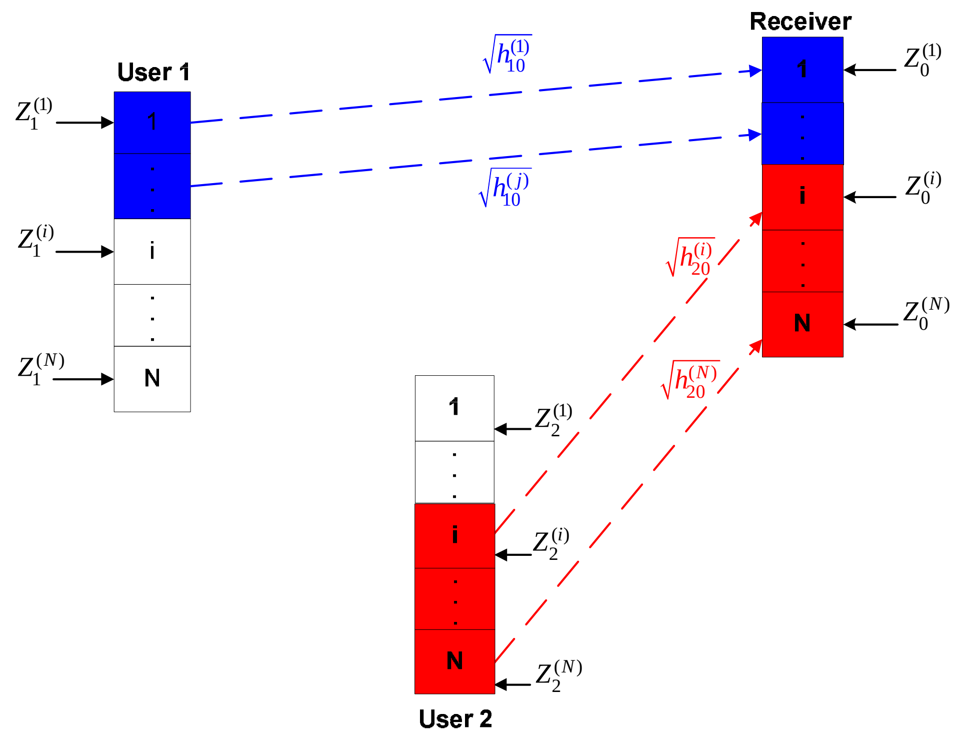
Summary and Conclusions

- Characterized the power control policies that are jointly optimal with Block Markov superposition coding.
- Using sub-gradient methods, obtained optimal power levels and corresponding rate region.
- Joint usage of cooperative diversity and time diversity: major improvements in capacity.
- Encoding and decoding is significantly simplified.
  - Transmitters send either cooperation or fresh information signals, but noth both.
- Optimal power policies also dictate MAC and routing policies
  - Cross layer design.

Two User Cooperative OFDMA

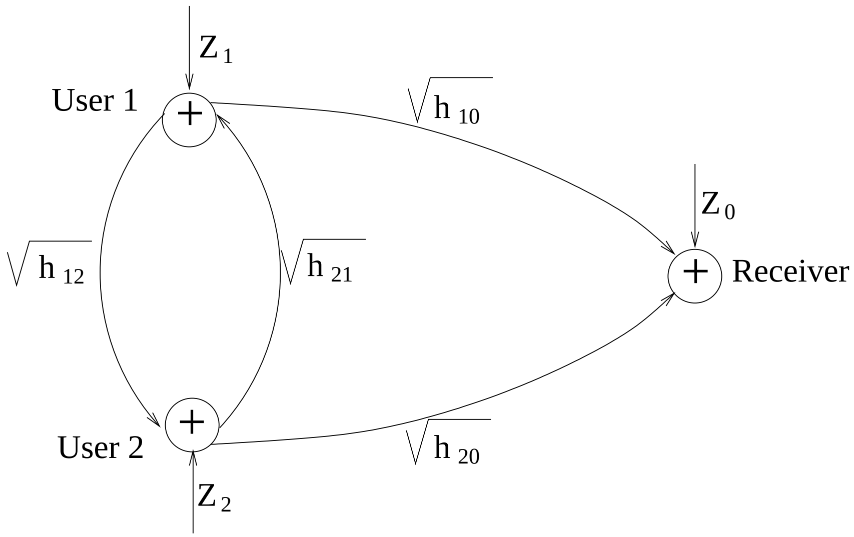
- *Joint work with Sezi Bakım

OFDMA



- Divides the entire transmission bandwidth into N orthogonal subchannels.
- Converts a frequency selective fading channel into parallel flat fading subchannels.
- Creates diversity across subchannels.
- Avoids interference, but incurs rate penalty due to orthogonalization of transmissions.

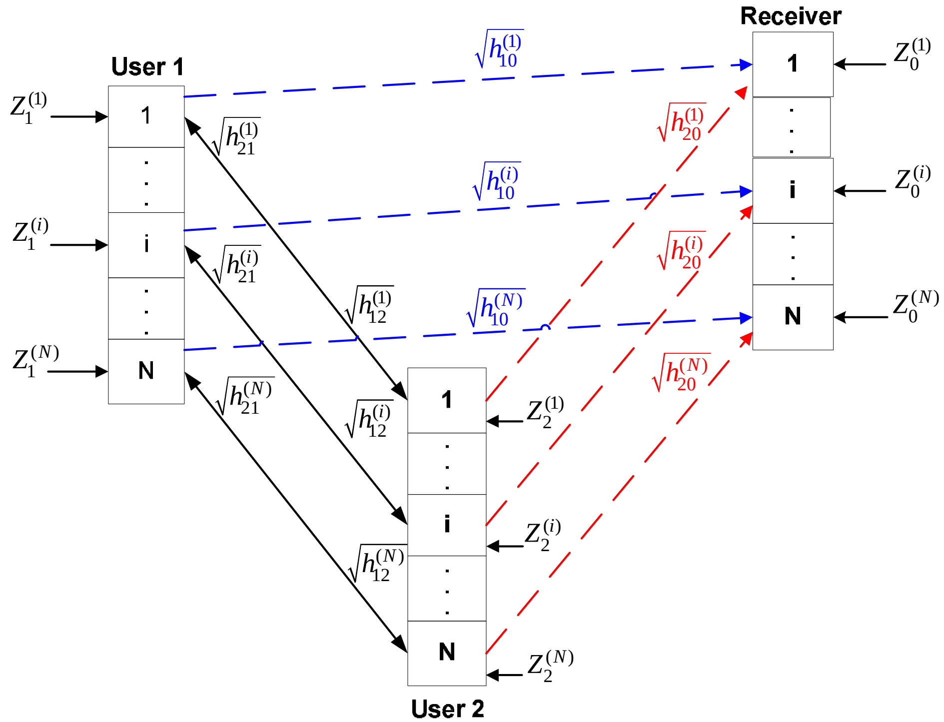
User Cooperation



$$Y_0 = \sqrt{h_{10}} X_1 + \sqrt{h_{20}} X_2 + Z_0$$
$$Y_1 = \sqrt{h_{21}} X_2 + Z_1$$
$$Y_2 = \sqrt{h_{12}} X_1 + Z_2$$

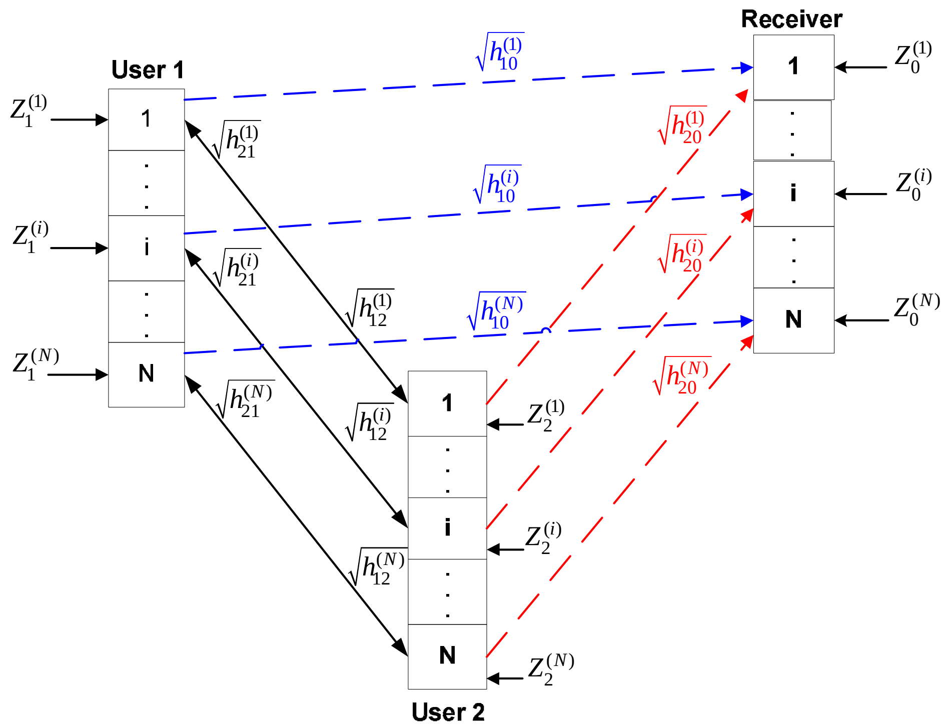
- Interference is information.
- Why not take advantage of overheard information in OFDMA?

Two User Cooperative OFDMA Channel Model



- Equivalent to N orthogonal cooperative MACs.
- Both users may TX & RX on the same subchannel: makes use of overheard information.
- May cooperate independently over each subchannel (intra-subchannel cooperation),
- May cooperate across subchannels (inter-subchannel cooperation).

Two User Cooperative OFDMA Channel Model



$$Y_0^{(i)} = \sqrt{h_{10}^{(i)}} X_1^{(i)} + \sqrt{h_{20}^{(i)}} X_2^{(i)} + Z_0^{(i)}$$

$$Y_1^{(i)} = \sqrt{h_{21}^{(i)}} X_2^{(i)} + Z_1^{(i)}$$

$$Y_2^{(i)} = \sqrt{h_{12}^{(i)}} X_1^{(i)} + Z_2^{(i)}$$

Scalar MAC – Block Markov Superposition Encoding

- Two user cooperation: each user's message is divided into two sub-messages
  - $w_1 = (w_{10}, w_{12}), \quad w_2 = (w_{20}, w_{21})$

- Block Markov superposition coding

Purpose	Codeword
Build common information	$X_{kj}\left(w_{kj}[b], w_{kj}[b-1], \hat{w}_{jk}[b-1]\right)$
Cooperatively send	$U_k\left(w_{kj}[b-1], \hat{w}_{jk}[b-1]\right)$
Inject new information	$X_{k0}\left(w_{k0}[b], w_{kj}[b-1], \hat{w}_{jk}[b-1]\right)$

$$X_1 = \sqrt{p_{10}}X_{10} + \sqrt{p_{12}}X_{12} + \sqrt{p_{U_1}}U_1$$

$$X_2 = \sqrt{p_{20}}X_{20} + \sqrt{p_{21}}X_{21} + \sqrt{p_{U_2}}U_2$$

$$p_{k0} + p_{kj} + p_{Uk} = P_k$$

OFDMA: Intra Subchannel Cooperative Encoding

- Two user cooperation: each user's message is divided into two sub-messages
  - $w_1 = (w_{10}, w_{12}), \quad w_2 = (w_{20}, w_{21})$
- These two sub-messages are further divided into N submessages each
  - $w_{k0} = \left\{ w_{k0}^{(1)}, \ldots, w_{k0}^{(N)} \right\}, \quad w_{kj} = \left\{ w_{kj}^{(1)}, \ldots, w_{kj}^{(N)} \right\}$
- Block Markov superposition coding

Purpose	Codeword
Build common information	$X_{kj}^{(i)}\left(w_{kj}^{(i)}[b], w_{kj}^{(i)}[b-1], \hat{w}_{jk}^{(i)}[b-1]\right)$
Cooperatively send	$U_k^{(i)}\left(w_{kj}^{(i)}[b-1], \hat{w}_{jk}^{(i)}[b-1]\right)$
Inject new information	$X_{k0}^{(i)}\left(w_{k0}^{(i)}[b], w_{kj}^{(i)}[b-1], \hat{w}_{jk}^{(i)}[b-1]\right)$

$$X_1^{(i)} = \sqrt{p_{10}^{(i)}} X_{10}^{(i)} + \sqrt{p_{12}^{(i)}} X_{12}^{(i)} + \sqrt{p_{U_1}^{(i)}} U_1^{(i)}$$

$$X_2^{(i)} = \sqrt{p_{20}^{(i)}} X_{20}^{(i)} + \sqrt{p_{21}^{(i)}} X_{21}^{(i)} + \sqrt{p_{U_2}^{(i)}} U_2^{(i)}$$

$$\sum_{i=1}^{N} p_{k0}^{(i)} + p_{kj}^{(i)} + p_{Uk}^{(i)} = P_k$$

Intra-Subchannel Cooperative Encoding – Rate Constraints

- Rate constraints for reliable decoding at users:

$$R_{12}^{(i)} < E\left[\log\left(1+\frac{s_{12}^{(i)}p_{12}^{(i)}}{s_{12}^{(i)}p_{10}^{(i)}+1}\right)\right]$$

$$R_{21}^{(i)} < E\left[\log\left(1+\frac{s_{21}^{(i)}p_{21}^{(i)}}{s_{21}^{(i)}p_{20}^{(i)}+1}\right)\right]$$

- Rate constraints for reliable decoding at receiver:

$$R_{10}^{(i)} < E\left[\log\left(1+s_{10}^{(i)}p_{10}^{(i)}\right)\right]$$

$$R_{20}^{(i)} < E\left[\log\left(1+s_{20}^{(i)}p_{20}^{(i)}\right)\right]$$

$$R_{10}^{(i)}+R_{20}^{(i)} < E\left[\log\left(1+s_{10}^{(i)}p_{10}^{(i)}+s_{20}^{(i)}p_{20}^{(i)}\right)\right]$$

$$R_{12}^{(i)}+R_{21}^{(i)}+R_{10}^{(i)}+R_{20}^{(i)} < C_s^{(i)} \triangleq E\left[\log\left(1+s_{10}^{(i)}p_1^{(i)}+s_{20}^{(i)}p_2^{(i)}+2\sqrt{s_{10}^{(i)}s_{20}^{(i)}p_{u_1}^{(i)}p_{u_2}^{(i)}}\right)\right]$$

Intra-Subchannel Cooperative Encoding: Issues and Limitations

$$
\begin{aligned}
R_1 &< \sum_i \min\left\{E\left[\log\left(1+\frac{s_{12}^{(i)}p_{12}^{(i)}}{s_{12}^{(i)}p_{10}^{(i)}+1}\right)\right]+E\left[\log\left(1+s_{10}^{(i)}p_{10}^{(i)}\right)\right], C_s^{(i)}\right\} \\
R_2 &< \sum_i \min\left\{E\left[\log\left(1+\frac{s_{21}^{(i)}p_{21}^{(i)}}{s_{21}^{(i)}p_{20}^{(i)}+1}\right)\right]+E\left[\log\left(1+s_{20}^{(i)}p_{20}^{(i)}\right)\right], C_s^{(i)}\right\} \\
R_1+R_2 &< \sum_i \min\left\{C_s^{(i)}, E\left[\log\left(1+\frac{s_{12}^{(i)}p_{12}^{(i)}}{s_{12}^{(i)}p_{10}^{(i)}+1}\right)\right]+E\left[\log\left(1+\frac{s_{21}^{(i)}p_{21}^{(i)}}{s_{21}^{(i)}p_{20}^{(i)}+1}\right)\right]\right. \\
&\quad \left. + E\left[\log\left(1+s_{10}^{(i)}p_{10}^{(i)}+s_{20}^{(i)}p_{20}^{(i)}\right)\right]\right\}
\end{aligned}
$$

- Each submessage is retransmitted over the same subchannel it was received on.
- Does not take advantage of diversity created by OFDMA.
- Rate over each subchannel is limited by the worst link.

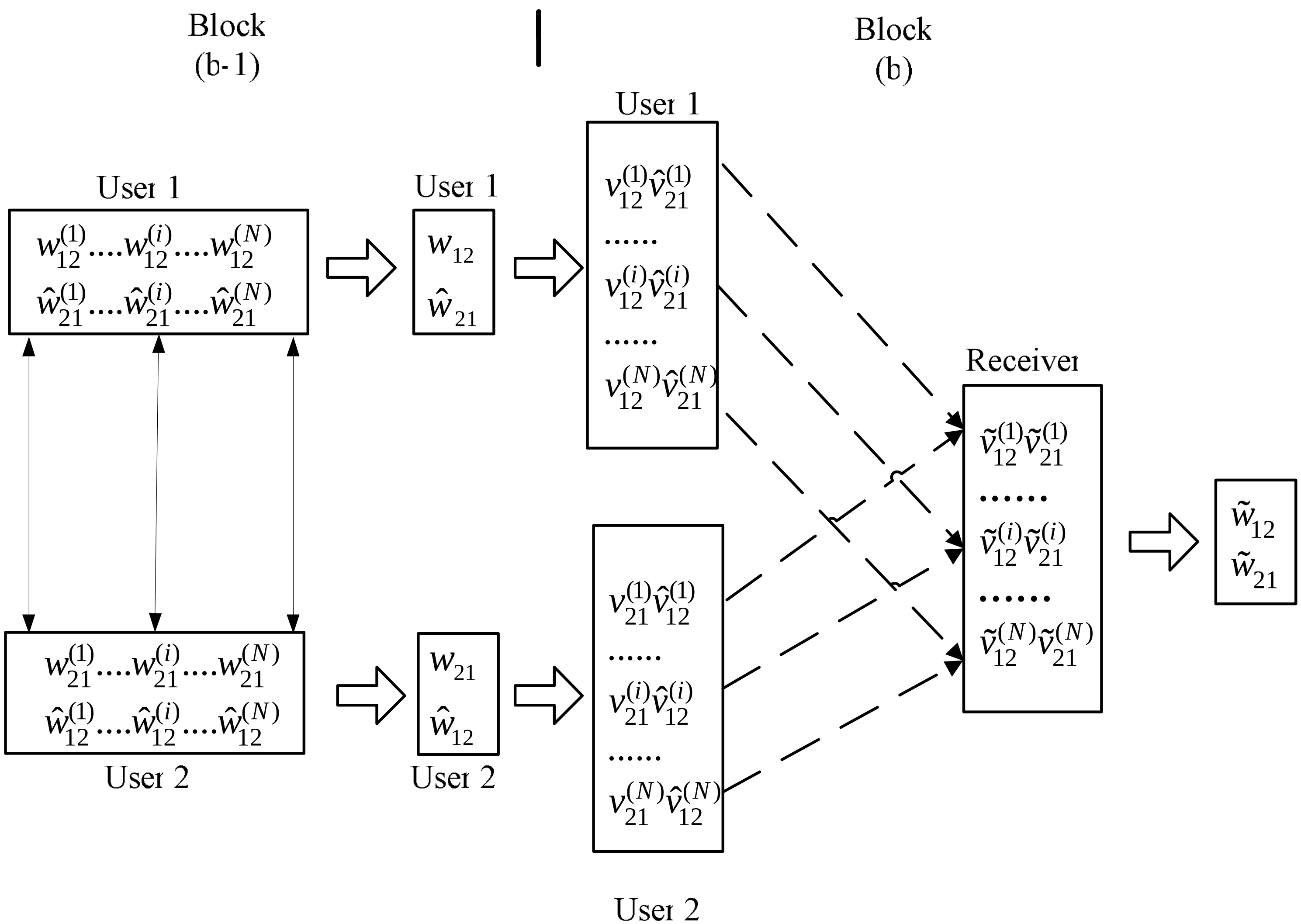
Inter-Subchannel Cooperative Encoding

- Can re-partition and re-encode the overall message received over the subchannels:
  - w_{kj} can be divided into new submessages with rates $R_{kj}^{\prime(i)}$
    - $w_{12} = \left\{v_{12}^{(1)}, \dots, v_{12}^{(N)}\right\}, \quad w_{21} = \left\{v_{21}^{(1)}, \dots, v_{21}^{(N)}\right\}$
- $\left\{w_{kj}^{(i)}\right\}_{i=1}^{N}$ and $\left\{v_{kj}^{(i)}\right\}_{i=1}^{N}$, are just different partitionings of the same message w_{kj}, so their total rates have to be the same:

$$2^{nR_{12}} = 2^{nR_{12}^{(1)}+\dots+nR_{12}^{(N)}} = 2^{nR_{12}^{\prime(1)}+\dots+nR_{12}^{\prime(N)}},$$
$$2^{nR_{21}} = 2^{nR_{21}^{(1)}+\dots+nR_{21}^{(N)}} = 2^{nR_{21}^{\prime(1)}+\dots+nR_{21}^{\prime(N)}}.$$

- Re-encode the new partition onto the subchannels
  - The information received over a subchannel no longer required to be sent over the same subchannel.

Inter-Subchannel Cooperative Encoding – Message Repartitioning



Inter-Subchannel Cooperation: Encoding and Decoding

- Encoding

Purpose	Codeword
Build common information	$X_{kj}^{(i)}\left(w_{kj}^{(i)}[b], v_{kj}^{(i)}[b-1], \hat{v}_{jk}^{(i)}[b-1]\right)$
Cooperatively send	$U_{k}^{(i)}\left(v_{kj}^{(i)}[b-1], \hat{v}_{jk}^{(i)}[b-1]\right)$
Inject new information	$X_{k0}^{(i)}\left(w_{k0}^{(i)}[b], v_{kj}^{(i)}[b-1], \hat{v}_{jk}^{(i)}[b-1]\right)$

- Decoding
  - Each user uses joint typicality check at the end of each block.
  - Receiver uses backwards decoding to determine the transmitted messages
    * For each subchannel determine $\tilde{v}_{21}^{(i)}[b-1], \tilde{v}_{12}^{(i)}[b-1], \tilde{w}_{10}^{(i)}[b]$ and $\tilde{w}_{20}^{(i)}[b]$
    * Estimates of the re-partitioned cooperative messages $\tilde{v}_{kj}^{(i)}[b-1]$ are converted to estimates of the cooperative messages $\tilde{w}_{kj}^{(i)}[b-1]$ via match-up table available at the users and the receiver.

Inter-Subchannel Cooperative Encoding – Rate Constraints

- Rate constraints for reliable decoding at users:

$$R_{12}^{(i)} < E\left[\log\left(1+\frac{s_{12}^{(i)}p_{12}^{(i)}}{s_{12}^{(i)}p_{10}^{(i)}+1}\right)\right]$$

$$R_{21}^{(i)} < E\left[\log\left(1+\frac{s_{21}^{(i)}p_{21}^{(i)}}{s_{21}^{(i)}p_{20}^{(i)}+1}\right)\right]$$

- Rate constraints for reliable decoding at receiver:

$$R_{10}^{(i)} < E\left[\log\left(1+s_{10}^{(i)}p_{10}^{(i)}\right)\right]$$

$$R_{20}^{(i)} < E\left[\log\left(1+s_{20}^{(i)}p_{20}^{(i)}\right)\right]$$

$$R_{10}^{(i)}+R_{20}^{(i)} < E\left[\log\left(1+s_{10}^{(i)}p_{10}^{(i)}+s_{20}^{(i)}p_{20}^{(i)}\right)\right]$$

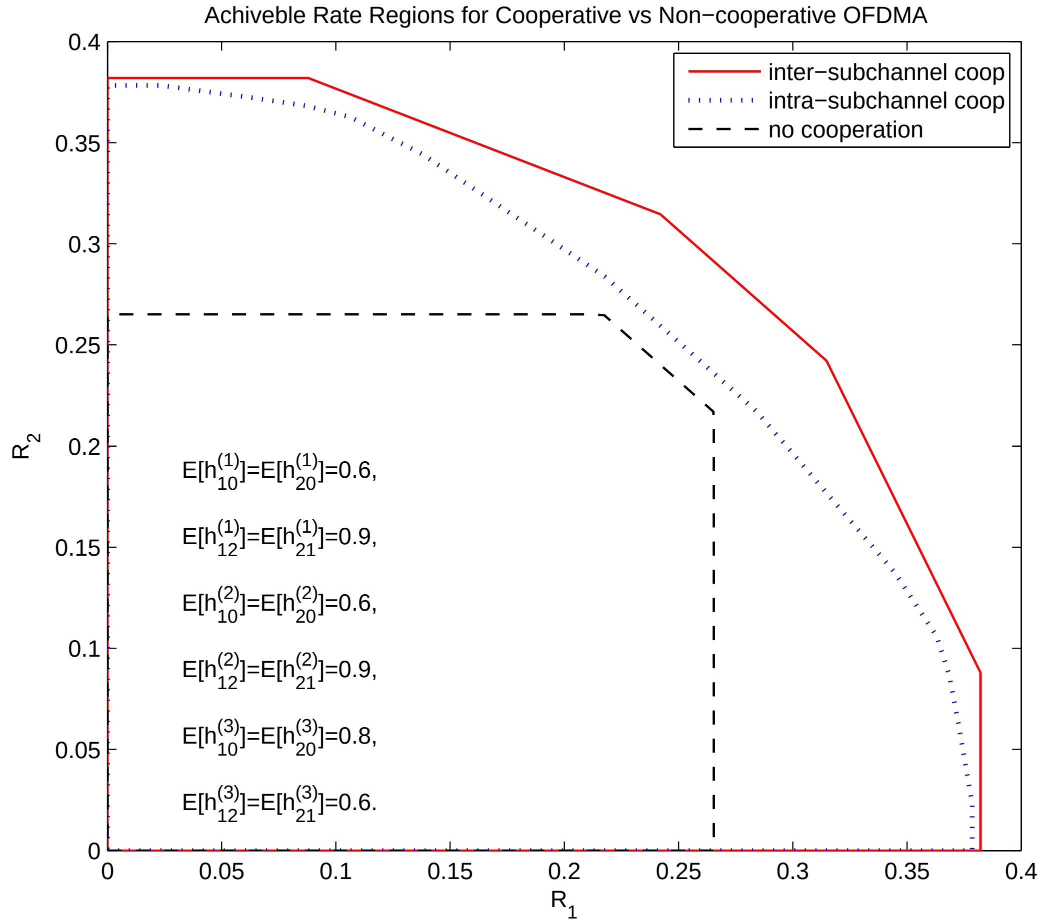
$$R_{12}^{\prime(i)}+R_{21}^{\prime(i)}+R_{10}^{(i)}+R_{20}^{(i)} < C_s^{(i)} \triangleq E\left[\log\left(1+s_{10}^{(i)}p_1^{(i)}+s_{20}^{(i)}p_2^{(i)}+2\sqrt{s_{10}^{(i)}s_{20}^{(i)}p_{u_1}^{(i)}p_{u_2}^{(i)}}\right)\right]$$

Inter-Subchannel Cooperative Encoding - Achievable Rate Region

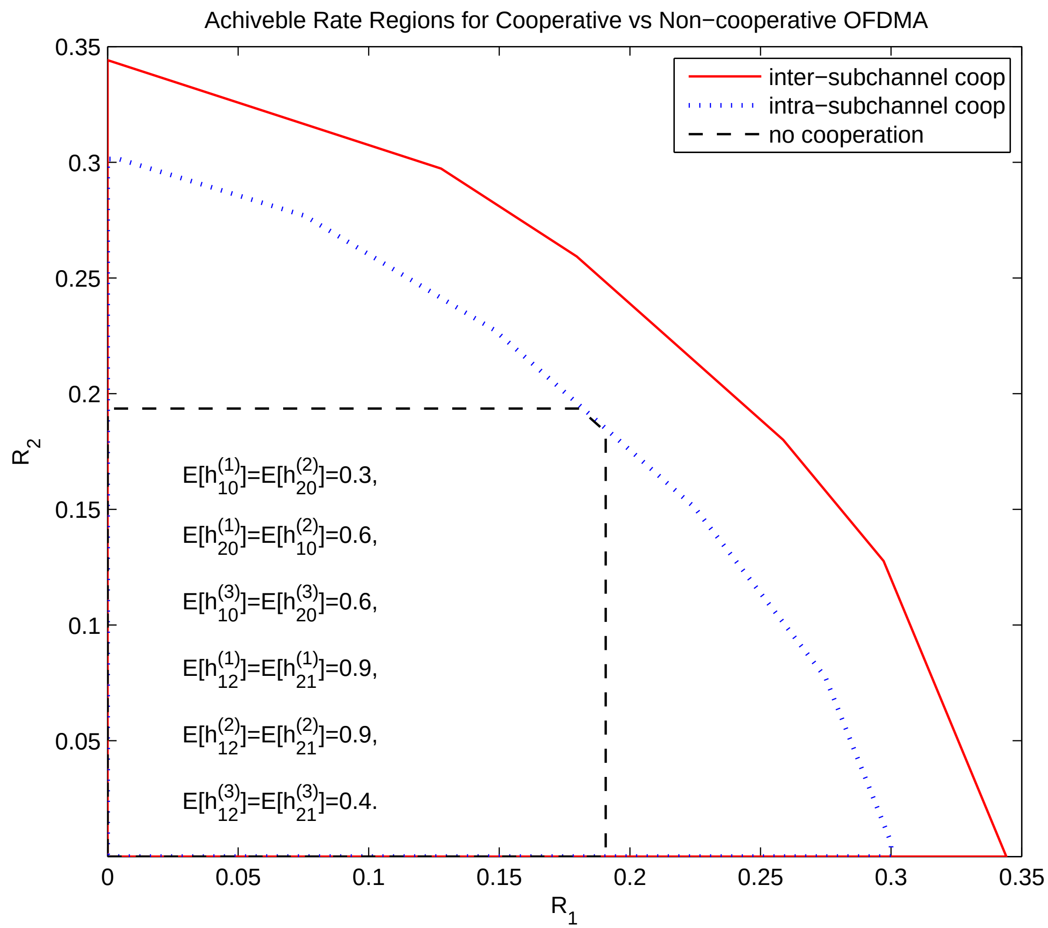
- Achievable rate region is equivalent to the closure of the convex hull of all rate pairs:

$$
\begin{aligned}
R_1 &< \sum_i E\left[\log\left(1+\frac{s_{12}^{(i)}p_{12}^{(i)}}{s_{12}^{(i)}p_{10}^{(i)}+1}\right)\right] + E\left[\log\left(1+s_{10}^{(i)}p_{10}^{(i)}\right)\right] \\
R_2 &< \sum_i E\left[\log\left(1+\frac{s_{21}^{(i)}p_{21}^{(i)}}{s_{21}^{(i)}p_{20}^{(i)}+1}\right)\right] + E\left[\log\left(1+s_{20}^{(i)}p_{20}^{(i)}\right)\right] \\
R_1+R_2 &< \min\left\{\sum_i C_s^{(i)}, \sum_i E\left[\log\left(1+\frac{s_{12}^{(i)}p_{12}^{(i)}}{s_{12}^{(i)}p_{10}^{(i)}+1}\right)\right] + E\left[\log\left(1+\frac{s_{21}^{(i)}p_{21}^{(i)}}{s_{21}^{(i)}p_{20}^{(i)}+1}\right)\right]\right. \\
&\qquad \left. + E\left[\log\left(1+s_{10}^{(i)}p_{10}^{(i)}+s_{20}^{(i)}p_{20}^{(i)}\right)\right]\right\}
\end{aligned}
$$

Achievable Rates for Two User Cooperative OFDMA



Achievable Rates for Two User Cooperative OFDMA



Summary and Conclusions

- Introduced a two user cooperative OFDMA system, and proposed two encoding strategies based on block Markov superposition encoding:
  - Intra-subchannel cooperative encoding
  - Inter-subchannel cooperative encoding
- Derived rate region expressions and obtained the achievable rate regions for both encoding strategies
- Showed that re-partitioning and re-encoding of the cooperative messages across subchannels;
  - Always superior to intra-subchannel cooperative encoding.
  - Significant improvement with respect to non-cooperative OFDMA.
- Can we do any better? Yes! Power control.

Power control for cooperative OFDMA

- The structure of the problem is very silmilar to the scalar case.
  - Now, we have an additional sum constraint for powers, over the sub-channels.
- The dimensionality of the problem is N times the scalar case.
- Can still use subgradients. A little slow, but it works.
- Can also exploit the convex nature of the problem, if we formulate it correctly.

Differentiable Reformulation of Sum-Rate Maximization Problem

- Idea: get rid of the minimum operation:

$$
\begin{aligned}
\max_{p^{(i)}(\mathbf{s})} \quad & r \\
\text{s.t.} \quad & r \le \sum_i E\left[\log\left(1 + s_{10}^{(i)}(p_{12}^{(i)}(\mathbf{s}) + p_{U_1}^{(i)}(\mathbf{s})) + s_{20}^{(i)}(p_{21}^{(i)}(\mathbf{s}) + p_{U_2}^{(i)}(\mathbf{s}))\right.\right. \\
& \left.\left. +2\sqrt{s_{10}^{(i)} s_{20}^{(i)} p_{U_1}^{(i)}(\mathbf{s}) p_{U_2}^{(i)}(\mathbf{s})}\right)\right] \\
& r \le \sum_i E\left[\log(1 + p_{12}^{(i)}(\mathbf{s}) s_{12}^{(i)}) + \log(1 + p_{21}^{(i)}(\mathbf{s}) s_{21}^{(i)})\right] \\
& \sum_i \left(E\left[p_{12}^{(i)}(\mathbf{s})\right] + E\left[p_{U_1}^{(i)}(\mathbf{s})\right]\right) \le \bar{p}_1 \\
& \sum_i \left(E\left[p_{21}^{(i)}(\mathbf{s})\right] + E\left[p_{U_2}^{(i)}(\mathbf{s})\right]\right) \le \bar{p}_2 \\
& p_{12}^{(i)}(\mathbf{s}), p_{U_1}^{(i)}(\mathbf{s}), p_{21}^{(i)}(\mathbf{s}), p_{U_2}^{(i)}(\mathbf{s}) \ge 0, \quad \forall \mathbf{s}
\end{aligned}
$$

Lagrangian Approach

$$
\begin{aligned}
L = r + \gamma_1 &\left(\sum_i \left(E\left[\log(1 + s_{12}^{(i)} p_{12}^{(i)}(\mathbf{s})) + \log(1 + s_{21}^{(i)} p_{21}^{(i)}(\mathbf{s})) \right] \right) - r \right) \\
&+ \gamma_2 \left(\sum_i E\left[\log\left(1 + s_{10}^{(i)}(p_{12}^{(i)}(\mathbf{s}) + p_{U_1}^{(i)}(\mathbf{s})) + s_{20}^{(i)}(p_{21}^{(i)}(\mathbf{s}) + p_{U_2}^{(i)}(\mathbf{s})) \right. \right. \right. \\
&\quad \left. \left. \left. + 2\sqrt{s_{10}^{(i)} s_{20}^{(i)} p_{U_1}^{(i)}(\mathbf{s}) p_{U_2}^{(i)}(\mathbf{s})} \right) \right] - r \right) + \lambda_1 \left(\bar{p}_1 - \sum_i \left(E\left[p_{12}^{(i)}(\mathbf{s}) + p_{U_1}^{(i)}(\mathbf{s}) \right] \right) \right) \\
&+ \lambda_2 \left(\bar{p}_2 - \sum_i \left(E\left[p_{21}^{(i)}(\mathbf{s}) + p_{U_2}^{(i)}(\mathbf{s}) \right] \right) \right) + \varepsilon_1^{(i)}(\mathbf{s}) p_{12}^{(i)}(\mathbf{s}) + \varepsilon_2^{(i)}(\mathbf{s}) p_{U_1}^{(i)}(\mathbf{s}) \\
&+ \varepsilon_3^{(i)}(\mathbf{s}) p_{21}^{(i)}(\mathbf{s}) + \varepsilon_4^{(i)}(\mathbf{s}) p_{U_2}^{(i)}(\mathbf{s}).
\end{aligned}
$$

Karush-Kuhn-Tucker Conditions

$$\gamma_1 \frac{s_{12}^{(i)}}{1 + s_{12}^{(i)} p_{12}^{(i)}(\mathbf{s})} + \gamma_2 \frac{s_{10}^{(i)}}{D^{(i)}} \leq \lambda_1$$

$$\gamma_1 \frac{s_{21}^{(i)}}{1 + s_{21}^{(i)} p_{21}^{(i)}(\mathbf{s})} + \gamma_2 \frac{s_{20}^{(i)}}{D^{(i)}} \leq \lambda_2$$

$$\gamma_2 \frac{\sqrt{s_{10}^{(i)} s_{20}^{(i)} p_{U_2}^{(i)}(\mathbf{s})} + s_{10}^{(i)} \sqrt{p_{U_1}^{(i)}(\mathbf{s})}}{D^{(i)} \sqrt{p_{U_1}^{(i)}(\mathbf{s})}} \leq \lambda_1$$

$$\gamma_2 \frac{\sqrt{s_{10}^{(i)} s_{20}^{(i)} p_{U_1}^{(i)}(\mathbf{s})} + s_{20}^{(i)} \sqrt{p_{U_2}^{(i)}(\mathbf{s})}}{D^{(i)} \sqrt{p_{U_2}^{(i)}(\mathbf{s})}} \leq \lambda_2$$

- $\gamma_1 + \gamma_2 = 1$
- Each condition satisfied with strict equality, if the corresponding power is positive.
- All we need to do is find λ_i and γ_1

Structure of Optimal Power Allocation

- When p_{U_1} and p_{U_2} are both positive,

$$p_{12}^{(i)}(\mathbf{s}) = \left(\frac{\gamma_1 \left(\lambda_2 s_{10}^{(i)} + \lambda_1 s_{20}^{(i)} \right)}{\lambda_1^2 s_{20}^{(i)}} - \frac{1}{s_{12}^{(i)}} \right)^+$$

$$p_{21}^{(i)}(\mathbf{s}) = \left(\frac{\gamma_1 \left(\lambda_2 s_{10}^{(i)} + \lambda_1 s_{20}^{(i)} \right)}{\lambda_2^2 s_{10}^{(i)}} - \frac{1}{s_{21}^{(i)}} \right)^+$$

- When both are zero, p_{12} and p_{21} solved from,

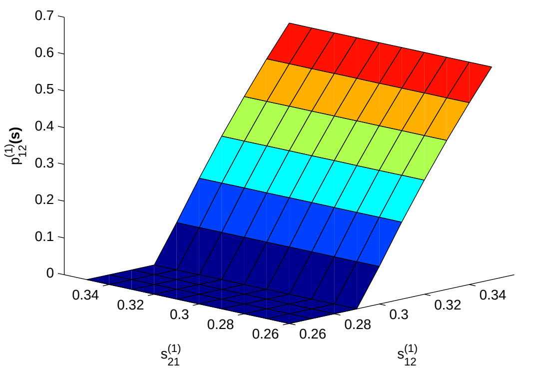
$$\gamma_1 \frac{s_{12}^{(i)}}{1 + s_{12}^{(i)} p_{12}^{(i)}(\mathbf{s})} + \gamma_2 \frac{s_{10}^{(i)}}{1 + s_{10}^{(i)} p_{12}^{(i)}(\mathbf{s}) + s_{20}^{(i)} p_{21}^{(i)}(\mathbf{s})} \leq \lambda_1$$

$$\gamma_1 \frac{s_{21}^{(i)}}{1 + s_{21}^{(i)} p_{21}^{(i)}(\mathbf{s})} + \gamma_2 \frac{s_{20}^{(i)}}{1 + s_{10}^{(i)} p_{12}^{(i)}(\mathbf{s}) + s_{20}^{(i)} p_{21}^{(i)}(\mathbf{s})} \leq \lambda_2$$

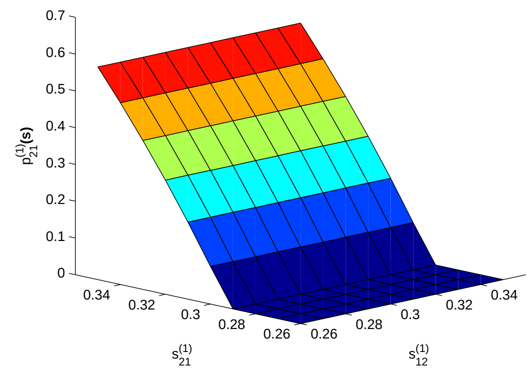
Iterative Power Allocation Algorithm

- All powers can be computed using KKT conditions, by iteratively searching for Lagrange multipliers.
- Not exactly closed form: p_{U_1} and p_{U_2}'s depend on p_{12} and p_{21}, and vice versa.
- Objective function concave, constraints strictly convex, Cartesian nature across users:
  - Can solve the users' powers iteratively – one user at a time.
  - Start by assuming p_U's positive, and iterate. Converges to optimum.

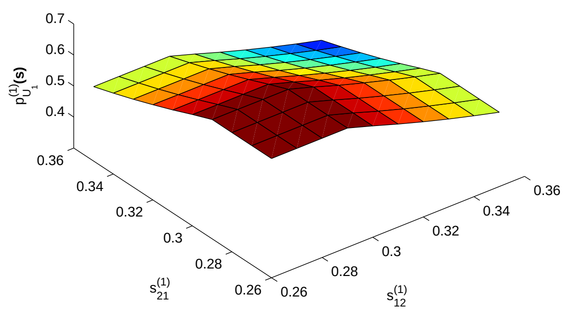
Optimal Power Allocation over Fading States– U-D links high



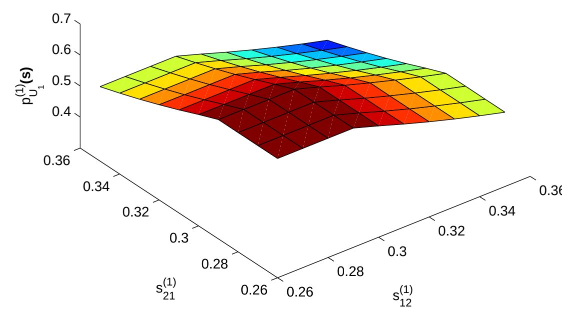
(a) Power levels, $p^{(1)}_{12}$



(b) Power level, $p^{(1)}_{21}$

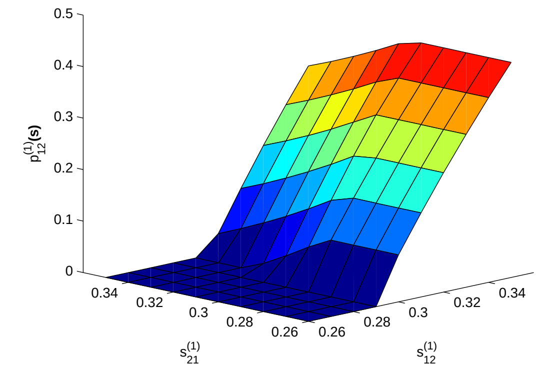


(c) Power level, $p^{(1)}_{U1}$

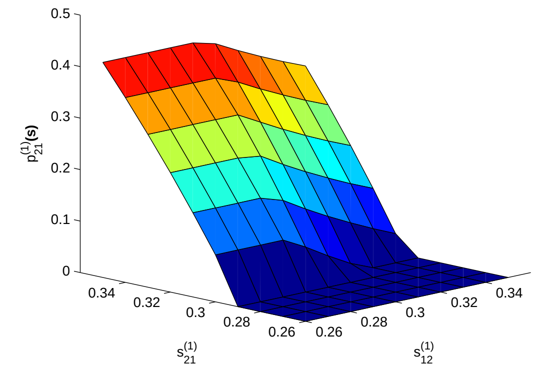


(d) Power level, $p^{(1)}_{U2}$

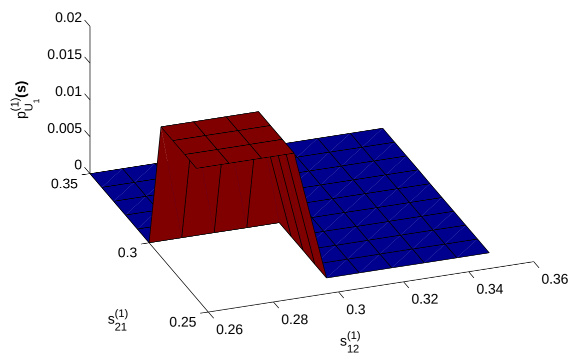
Optimal Power Allocation over Fading States– U-D links moderate



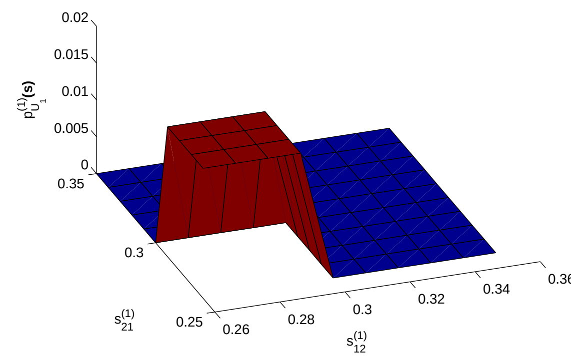
(e) Power level, $p^{(1)}_{12}$



(f) Power level, $p^{(1)}_{21}$



(g) Power level, $p^{(1)}_{U1}$



(h) Power level, $p^{(1)}_{U2}$

Cooperative OFDMA – Achievable Rates with Power Control