Capacity Region of Power Controlled Fading CDMA: Transmit Strategies and Convexity Issues[1]

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We consider a symbol synchronous CDMA system with processing gain N, where all K users transmit to a single receiver. In the presence of fading, the received signal is given by,

$$\mathbf{r} = \sum_{i=1}^{K} \sqrt{p_i(\mathbf{h}) h_i}\, b_i \mathbf{s}_i + \mathbf{n} \tag{1}$$

where, for user i, b_i denotes the coded information symbol with $\mathrm{E}[b_i^2] = 1$, $\mathbf{s}_i = [s_{i1}, \cdots, s_{iN}]^\top$ denotes the signature sequence, $\sqrt{h_i}$ denotes the random and continuously distributed channel gain, and p_i denotes the transmit power; $\mathbf{n}$ is a zero-mean Gaussian random vector with covariance $\sigma^2 \mathbf{I}_N$. We assume that the receiver and all of the transmitters have perfect knowledge of the channel states of all users represented as a vector $\mathbf{h} = [h_1, \cdots, h_K]^\top$. The transmitters are then able to choose their powers as a function of the channel state information (CSI), subject to the average power constraints $E_{\mathbf{h}}[p_i(\mathbf{h})] \leq \bar{p}_i$. We first characterize the set of long term achievable rates, i.e., the capacity region, for fading CDMA. Hanly and Tse [1, Thm. 2.1] have characterized the capacity region for a power controlled scalar multi-access channel. We generalize both forward and converse parts of the proof of this theorem to the CDMA channel by incorporating the methods and results from [2, Prop. 1] and [3, Thm. 1]. We state the capacity region of the fading CDMA channel in the following theorem.

Theorem 1 *Let $\mathcal{P} = \{\mathbf{p}(\mathbf{h}) : E_{\mathbf{h}}[p_i(\mathbf{h})] \leq \bar{p}_i, \forall i\}$ denote the family of valid power allocation policies. The capacity region $\mathcal{C}$ of a fading CDMA channel under additive white Gaussian noise, where users have perfect CSI and allocate their powers as a function of the CSI is given by,*

$$\bigcup_{\mathbf{p}(\mathbf{h}) \in \mathcal{P}} \left\{ \mathbf{R} : \sum_{i \in \Gamma} R_i \leq E_{\mathbf{h}} \left[\frac{1}{2} \log \left| \mathbf{I}_N + \sigma^{-2} \sum_{i \in \Gamma} h_i p_i(\mathbf{h}) \mathbf{s}_i \mathbf{s}_i^\top \right| \right], \quad \forall \Gamma \subset \{1, \cdots, K\} \right\} \tag{2}$$

Fig. 1 illustrates a typical capacity region for some fixed signature sequences $\mathbf{s}_1$ and $\mathbf{s}_2$ in a two user setting. Each of the pentagons corresponds to a valid power allocation policy. Note the flat portion on the capacity region, which in fact is the dominant face of one of the pentagons. Unlike scalar multi-access channel capacity region [1], the capacity region for fading CDMA may contain such a flat region, and therefore, in general is not strictly convex. Consequently, the rate pairs on the line segment $|AB|$ in the figure are in general achieved by timesharing between the points A and B. This is stated more precisely in the following theorem, for K users.

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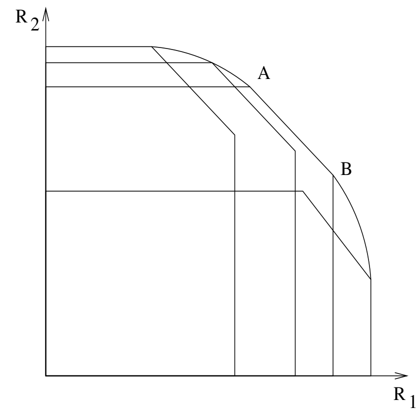

Fig. 1: Two user capacity region.

Theorem 2 *The capacity region of a power controlled fading CDMA channel is not strictly convex, provided $\exists\ i, j \in \{1, \cdots, K\}$ such that $i \neq j$ and $0 < \left|\mathbf{s}_i^\top \mathbf{s}_j\right| < 1$.*

Note that the pentagon containing $|AB|$ corresponds to the power control policy (or policies, if sum capacity function is not strictly concave, see [4]) that maximizes the sum capacity. Theorem 2 is proved by observing the following property of the sum capacity achieving power control policy [4, 5].

Theorem 3 *The sum capacity maximizing power control policy dictates that there exists a non-zero probability region of fading states $\mathbf{h}$ where a subset $E \subset \{1, \cdots, K\}$ of users transmit simultaneously, if and only if $\left\{\mathbf{s}_i \mathbf{s}_i^\top\right\}_{i \in E}$ are linearly independent.*

The existence of a simultaneous transmission region for at least two users with non-orthogonal signature sequences implies that the sum-capacity achieving pentagon (polymatroid) does not degenerate into a rectangle (orthotope), and therefore, a flat region on the capacity boundary exists. Then, Theorem 2 is proved by noting that for two users i and j, the linear independence of $s_i s_i^T$ and $s_j s_j^T$ is equivalent to $|s_i^T s_j| < 1$.

References

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