Energy Harvesting Cooperative Multiple Access Channel with Data Arrivals

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***Abstract*—We consider an energy harvesting two user cooperative Gaussian multiple access channel (MAC), where both of the users harvest energy from nature. The data packets arrive intermittently over time. The users overhear each other's transmitted signals and can cooperate by forming common messages. We find the optimal offline transmit power and rate allocation policy that maximize the departure region. We first show that there exists an optimal policy, in which the single user rate constraints in each time slot are tight, yielding a one to one relation between the powers and rates. Then, we formulate the departure region maximization problem as a weighted sum rate maximization in terms of rates only. Next, we propose a sequential convex approximation method to approximate the problem at each step and show that it converges to the optimal solution. Finally, we solve the approximate problems using an inner outer decomposition method. Numerically, we observe that higher data rates can be supported with the same amount of energy.**

I. Introduction

We consider the cooperative energy harvesting MAC model illustrated in Fig. 1. The data packets and the harvested energies arrive at the transmitters intermittently over time. We determine the optimum power and rate allocation policies of the users which maximize the departure region of the system.

There has been a considerable amount of recent work in power control for energy harvesting communications [1]–[24]. In [1], the transmission completion time minimization problem is solved for an unlimited-sized battery. In [2], the throughput maximization problem is solved and its equivalence to the transmission completion time minimization problem is shown for an arbitrarily-sized battery. In [3], [5]–[10] the problem is extended to fading, broadcast, multiple access and interference channels. Throughput maximization problem with battery imperfections is considered in [11], [12] and processing costs are incorporated in [13]–[15]. Two-hop communication is considered with energy harvesting nodes for half- or full-duplex relay settings in [16]–[21]. Energy cooperation is introduced in [22]. Of particular relevance to us are references [8], [9], [23]–[25] where optimal scheduling problems on a MAC are investigated. In [25], minimum energy scheduling problem over a MAC where data packets arrive over time is solved. In [8], a MAC with energy arrivals is considered with infinitely backlogged users, i.e., the data packets do not arrive over time. In [9], an energy harvesting MAC with additional maximum power constraints on each user is considered. Recently, in [23], a MAC with both energy and data arrivals is considered and in [24], a cooperative MAC with only energy arrivals is considered. In this paper, we consider a cooperative MAC with both energy and data arrivals.

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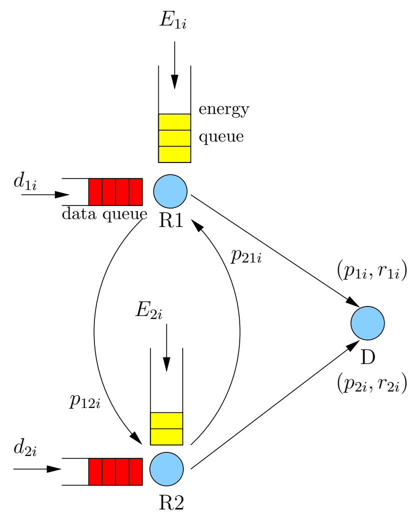

Fig. 1. Cooperative MAC with energy and data arrivals.

We first show that there exists an optimal rate and power allocation which is on the achievable rate region boundary of the cooperative MAC at every slot, instead of being strictly inside the achievable rate region. Then we formulate the problem in terms of data rates only, rather than both transmission powers and data rates. Although, this new problem is non-convex, we show that strong duality holds. As a result, we are able to employ a successive convex approximation technique in which non-convex constraints are approximated by suitable convex functions. Using this approximation, we solve the problem using an iterative algorithm which iterates between inner and outer maximization problems.

II. System Model and Problem Formulation

We consider an energy harvesting cooperative MAC with intermittent data and energy arrivals, as shown in Fig. 1. The harvested energies are saved in the corresponding batteries. There are N equal length slots. We use subscripts 1 and 2 to denote the parameters of users 1 and 2. In slot i, there are energy and data arrivals to both users with amounts E_{1i}, E_{2i}

and d_{1i}, d_{2i}, respectively. We denote the transmission powers and data rates of users 1 and 2 as p_{1i}, r_{1i} and p_{2i}, r_{2i}, respectively. The physical layer is a cooperative Gaussian MAC with unit-variance Gaussian noises at the users and σ^2 variance Gaussian noise at the receiver. We employ the delay constrained cooperation model proposed in [24]. The users cooperate in a slot by slot basis, by first exchanging information and then beamforming, to send the established common information, in each given slot. The specifics of the encoding and decoding policy can be found in [24, Section II]. The achievable rate region with transmitter sub-powers $p_{12i}, p_{21i}, p_{U1i}, p_{U2i}$ at each slot i is given as [24], [26]:

$$\mathcal{C}(p_{12i}, p_{21i}, p_{U1i}, p_{U2i}) = \left\{ \begin{aligned} & r_{1i} \leq f(1+p_{12i}), && (1) \\ & r_{2i} \leq f(1+p_{21i}), && (2) \\ & r_{1i} + r_{2i} \leq f(S_i/\sigma^2) && (3) \end{aligned} \right\}$$

where $f(x) = \frac{1}{2}\log(x)$ and $S_i = \sigma^2 + p_{1i} + p_{2i} + 2\sqrt{p_{U1i}p_{U2i}}$, $p_{1i} = p_{12i} + p_{U1i}$, $p_{2i} = p_{21i} + p_{U2i}$. For notational convenience, we denote the sub-power and rate sequences by the vectors $\mathbf{p}_{12}, \mathbf{p}_{21}, \mathbf{p}_{U1}, \mathbf{p}_{U2}, \mathbf{r}_1, \mathbf{r}_2$.

The energy that has not arrived yet cannot be used, leading to the following *energy causality constraints*:

$$\sum_{i=1}^{k} p_{1i} \leq \sum_{i=1}^{k} E_{1i}, \quad 1 \leq k \leq N, \tag{4}$$

$$\sum_{i=1}^{k} p_{2i} \leq \sum_{i=1}^{k} E_{2i}, \quad 1 \leq k \leq N. \tag{5}$$

The data that has not arrived yet cannot be transmitted, leading to the following *data causality constraints*:

$$\sum_{i=1}^{k} r_{1i} \leq \sum_{i=1}^{k} d_{1i}, \quad 1 \leq k \leq N, \tag{6}$$

$$\sum_{i=1}^{k} r_{2i} \leq \sum_{i=1}^{k} d_{2i}, \quad 1 \leq k \leq N. \tag{7}$$

The rate allocations must be achievable for the cooperative MAC in each slot:

$$(r_{1i}, r_{2i}) \in \mathcal{C}(p_{12i}, p_{21i}, p_{U1i}, p_{U2i}), \quad 1 \leq i \leq N. \tag{8}$$

The departure region maximization problem can be stated as a weighted sum rate maximization for given priorities $0 \leq \mu_1, \mu_2 \leq 1$, due to the convexity of the departure region:

$$\begin{aligned} \max_{\mathbf{p}_{12}, \mathbf{p}_{21}, \mathbf{p}_{U1}, \mathbf{p}_{U2}, \mathbf{r}_1, \mathbf{r}_2 \geq \mathbf{0}} \quad & \mu_1 \sum_{i=1}^{N} r_{1i} + \mu_2 \sum_{i=1}^{N} r_{2i} \\ \text{s.t.} \quad & \text{(4)-(8)} \end{aligned} \tag{9}$$

III. Necessary Conditions and Optimal Profile

In this section, we prove some properties of the optimal solution.

Lemma 1 *There exists an optimal profile that satisfies the following property,*

$$r_{1i} = f(1+p_{12i}), \quad r_{2i} = f(1+p_{21i}), \quad \forall i \tag{10}$$

Proof: We will prove this lemma by showing that for any policy that does not satisfy the above property, there exists another policy that satisfies it and achieves the same weighted sum rate. Assume there exists an optimal policy and slot i such that $r_{1i} < f(1+p_{12i})$. Now consider the modified policy, $q_{12i} = p_{12i} - \epsilon$, $q_{U1i} = p_{U1i} + \epsilon$ while keeping the remaining variables fixed. In this modified policy, $q_{1i} = q_{12i} + q_{U1i} = p_{12i} + p_{U1i} = p_{1i}$, therefore the new policy spends the same amount of energy as the previous one and is energy feasible. It is easy to check that this modification increases S_i and (r_{1i}, r_{2i}) still belongs to the set $\mathcal{C}(q_{12i}, p_{21i}, q_{U1i}, p_{U2i})$. Since we have not changed the rates, the data causality constraints are still feasible. By repeating this process we will reach a profile where $r_{1i} = f(1+p_{12i})$. By using similar arguments for r_{2i} and modifying p_{21i} and p_{U2i} we will reach a profile where $r_{2i} = f(1+p_{21i})$. Since we have not changed the rates, the weighted sum rate is the same and the policy is still optimal. This proves the lemma. ■

By using Lemma 1 and enforcing the constraints in (10) the sum rate constraints in (3) can be written as

$$f(1+p_{12i}) + f(1+p_{21i}) \leq f(S_i/\sigma^2), \quad \forall i \tag{11}$$

In addition to the rate-power relationships dictated by Lemma 1, we further introduce the auxiliary rate variables, r_{U1i}, r_{U2i}, and perform the variable changes, $r_{U1i} = f(1+p_{U1i})$, $r_{U2i} = f(1+p_{U2i})$. Then $S_i = \sigma^2 + 2^{2r_{1i}} + 2^{2r_{U1i}} + 2^{2r_{2i}} + 2^{2r_{U2i}} + 2\sqrt{(2^{2r_{U1i}}-1)(2^{2r_{U2i}}-1)} - 4$. We formulate the problem only in terms of rates as,

$$\begin{aligned} \max_{\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{U1}, \mathbf{r}_{U2}} \quad & \sum_{i=1}^{N} \mu_1 r_{1i} + \mu_2 r_{2i} \\ \text{s.t.} \quad & \sum_{i=1}^{k} 2^{2r_{1i}} + 2^{2r_{U1i}} \leq \sum_{i=1}^{k} (E_{1i}+2), \forall k, && (12) \\ & \sum_{i=1}^{k} 2^{2r_{2i}} + 2^{2r_{U2i}} \leq \sum_{i=1}^{k} (E_{2i}+2), \forall k, && (13) \\ & \sum_{i=1}^{k} r_{1i} \leq \sum_{i=1}^{k} d_{1i}, \quad \forall k, && (14) \\ & \sum_{i=1}^{k} r_{2i} \leq \sum_{i=1}^{k} d_{2i}, \quad \forall k, && (15) \\ & r_{1i} + r_{2i} \leq f(S_i/\sigma^2), \quad \forall i. && (16) \end{aligned}$$

The problem in (16) is a non-convex optimization problem due to the last set of constraints $r_{1i} + r_{2i} \leq f(S_i/\sigma^2)$, $\forall i$. We use the successive convex approximation technique to approximate the constraints in (16) as explained in [27]. We use the first order Taylor expansion to the function $f(S_i/\sigma^2)$ around the point $\mathbf{R}^n \triangleq (\mathbf{r}_1^n, \mathbf{r}_2^n, \mathbf{r}_{U1}^n, \mathbf{r}_{U2}^n)$ for iteration $n+1$, by

$$f(S_i/\sigma^2) \simeq C_i^n + \alpha_{1i}^n(r_{1i} - r_{1i}^n) + \alpha_{2i}^n(r_{2i} - r_{2i}^n)$$

$$+\beta_{1i}^n(r_{U1i}-r_{U1i}^n)+\beta_{2i}^n(r_{U2i}-r_{U2i}^n), \quad (17)$$

where the values of the coefficients are given in Appendix A and depend only on the solution of the previous iteration n. With this approximation the problem in (16) becomes

$$\begin{aligned} \max_{\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_{U1},\mathbf{r}_{U2}} \quad & \sum_{i=1}^N \mu_1 r_{1i}+\mu_2 r_{2i} \\ \text{s.t.} \quad & \text{(12)-(15)} \\ & (1-\alpha_{1i}^n)r_{1i}+(1-\alpha_{2i}^n)r_{2i}-\beta_{1i}^k r_{U1i} \\ & -\beta_{2i}^n r_{U2i} \le D_i^n, \quad \forall i, \end{aligned} \quad (18)$$

where $D_i^n \triangleq C_i^n-\alpha_{1i}^n r_{1i}^n-\alpha_{2i}^n r_{2i}^n-\beta_{1i}^n r_{U1i}^n-\beta_{2i}^n r_{U2i}^n$ and is a constant for this optimization problem. At iteration $n+1$, we evaluate the coefficients in (17) using the optimal rate allocations at iteration n, we solve the problem in (18) using these coefficients and we update the initial point as $\mathbf{R}^{n+1}=\mathbf{R}^*(n)$ where $\mathbf{R}^*(n)$ denotes the optimal values of the variables when (18) is solved. Now we show that this procedure stops at an optimal solution to the problem in (16). To achieve this, we first show that strong duality holds for (16). The proof is given in Appendix B.

Lemma 2 *Strong duality holds for the problem in (16).*

Now we show that the procedure converges to an optimal solution and the proof is given in Appendix C.

Lemma 3 $\mathbf{R}^n \to \mathbf{R}^*$ *where* $\mathbf{R}^*$ *solves (16).*

In the next section, we solve the problem in (18) for fixed n.

IV. Solution for Approximate Problems

In this section, we solve the approximate problems for iteration $n+1$. For notational convenience we drop the superscript n from the last constraints in (18) noting that they depend only on the solution of the problem at the previous iteration n. Therefore the coefficients $\alpha_{1i}^n,\alpha_{2i}^n,\beta_{1i}^n,\beta_{2i}^n$ are essentially constants for the problem at step $n+1$.

Lemma 4 *There exists an optimal solution in which* $(1-\alpha_{1i})r_{1i}+(1-\alpha_{2i})r_{2i}-\beta_{1i}r_{U1i}-\beta_{2i}r_{U2i}=D_i, \quad \forall i.$

Proof: Assume there exists a profile where $(1-\alpha_{1i})r_{1i}+(1-\alpha_{2i})r_{2i}-\beta_{1i}r_{U1i}-\beta_{2i}r_{U2i}<D_i$ for some slot i. Then we can decrease, r_{U1i} or r_{U2i} to achieve equality. ■

Invoking Lemma 4, the problem becomes,

$$\begin{aligned} \max_{\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_{U1},\mathbf{r}_{U2}} \quad & \sum_{i=1}^N \mu_1 r_{1i}+\mu_2 r_{2i} \\ \text{s.t.} \quad & \text{(12)-(15)} \\ & (1-\alpha_{1i})r_{1i}+(1-\alpha_{2i})r_{2i}-\beta_{1i} r_{U1i} \\ & -\beta_{2i} r_{U2i} = D_i, \quad \forall i. \end{aligned} \quad (19)$$

We solve the problem in (19) using a primal decomposition. We add a new optimization variable $\mathbf{t}\in\mathbb{R}^N$ and equivalently formulate (19) as follows:

$$\begin{aligned} \max_{\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_{U1},\mathbf{r}_{U2},\mathbf{t}} \quad & \sum_{i=1}^N \mu_1 r_{1i}+\mu_2 r_{2i} \\ \text{s.t.} \quad & \text{(12)-(15)} \\ & (1-\alpha_{1i})r_{1i}-\beta_{1i}r_{U1i}=D_i+t_i, \quad (20) \\ & (1-\alpha_{2i})r_{2i}-\beta_{2i}r_{U2i}=-t_i, \ \forall i. \quad (21) \end{aligned}$$

Let us define the function $z(\mathbf{t})$ which is a maximization over $(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_{U1},\mathbf{r}_{U2})$ for fixed $\mathbf{t}$:

$$\begin{aligned} z(\mathbf{t}) = \max_{\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_{U1},\mathbf{r}_{U2}} \quad & \sum_{i=1}^N \mu_1 r_{1i}+\mu_2 r_{2i} \\ \text{s.t.} \quad & \text{(12)-(15),(20),(21).} \end{aligned} \quad (22)$$

Then the original problem in (19) is equivalent to

$$\max_{\mathbf{t}} z(\mathbf{t}). \quad (23)$$

We solve (23) by separately solving the outer and inner maximization problems.

A. Inner Maximization

In this section, we focus on the inner problem in (22) for fixed $\mathbf{t}$. Note that when $\mathbf{t}$ is fixed, the variables $(\mathbf{r}_1,\mathbf{r}_{U1})$ and $(\mathbf{r}_2,\mathbf{r}_{U2})$ are decoupled and (22) can be separated into two sub-problems. We define $z_1(\mathbf{t})$ and $z_2(\mathbf{t})$ as

$$\begin{aligned} z_1(\mathbf{t}) = \max_{\mathbf{r}_1,\mathbf{r}_{U1}} \quad & \sum_{i=1}^N \mu_1 r_{1i} \\ \text{s.t.} \quad & \sum_{i=1}^k 2^{2r_{1i}}+2^{2r_{U1i}} \le \sum_{i=1}^k (E_{1i}+2), \quad (24) \\ & \sum_{i=1}^k r_{1i} \le \sum_{i=1}^k d_{1i}, \quad \forall k, \quad (25) \\ & (1-\alpha_{1i})r_{1i}-\beta_{1i}r_{U1i}=D_i+t_i, \ \forall i. \quad (26) \end{aligned}$$

$$\begin{aligned} z_2(\mathbf{t}) = \max_{\mathbf{r}_2,\mathbf{r}_{U2}} \quad & \sum_{i=1}^N \mu_2 r_{2i} \\ \text{s.t.} \quad & \sum_{i=1}^k 2^{2r_{2i}}+2^{2r_{U2i}} \le \sum_{i=1}^k (E_{2i}+2), \quad (27) \\ & \sum_{i=1}^k r_{2i} \le \sum_{i=1}^k d_{2i}, \quad \forall k, \quad (28) \\ & (1-\alpha_{2i})r_{2i}-\beta_{2i}r_{U2i}=-t_i, \ \forall i. \quad (29) \end{aligned}$$

and note that $z(\mathbf{t})=z_1(\mathbf{t})+z_2(\mathbf{t})$. First we concentrate on solving z_1. Let $w_{1i}=(1-\alpha_{1i})/\beta_{1i}, v_{1i}=2^{-2(D_i+t_i)/\beta_{1i}}$. Using the equality constraints in (26) we get,

$$\max_{\mathbf{r}_1} \quad \sum_{i=1}^N \mu_1 r_{1i}$$

$$\text{s.t.} \quad \sum_{i=1}^{k} 2^{2r_{1i}} + v_{1i} 2^{2w_{1i}r_{1i}} \leq \sum_{i=1}^{k} (E_{1i}+2), \qquad (30)$$

$$\sum_{i=1}^{k} r_{1i} \leq \sum_{i=1}^{k} d_{1i}, \quad \forall k. \qquad (31)$$

This is a single-user problem with data arrivals d_{1i}, energy arrivals E_{1i} and a modified energy consumption function $m(r_{1i}) = 2^{2r_{1i}} + v_{1i}2^{2w_{1i}r_{1i}}$. In order to solve it: first, we perform directional waterfilling on the data arrivals d_{1i}. Second, we perform directional waterfilling on the energy arrivals E_{1i} with the understanding that $m'(r_{1i})$ is a generalized water level and the quantity to be kept constant over the slots. Then, we take the minimum of the two solutions ensuring that any unused data or energy must be carried over to the future slots.

Now we solve z_2. Let $w_{2i} = (1-\alpha_{2i})/\beta_{2i}$ and $v_{2i} = 2^{t_i/\beta_{2i}}$. Using the equality constraints of (29) we get,

$$\max_{\mathbf{r}_2} \quad \sum_{i=1}^{N} \mu_2 r_{2i}$$

$$\text{s.t.} \quad \sum_{i=1}^{k} 2^{2r_{2i}} + v_{2i} 2^{2w_{2i}r_{1i}} \leq \sum_{i=1}^{k} (E_{2i}+2), \qquad (32)$$

$$\sum_{i=1}^{k} r_{2i} \leq \sum_{i=1}^{k} d_{2i}, \quad \forall k. \qquad (33)$$

This problem is solved similarly as in the case of z_1.

B. Outer Maximization

The outer maximization problem is that of finding optimal $\mathbf{t}$ in (23). The equality constraints in (20) and (21) impose some feasibility constraints on $\mathbf{t}$. Then the problem is equivalent to

$$\max_{\mathbf{t}} \quad z(\mathbf{t})$$

$$\text{s.t.} \quad z_1(\mathbf{t}), z_2(\mathbf{t}) \text{ are feasible.} \qquad (34)$$

It can be shown that $z(\mathbf{t})$ is concave in $\mathbf{t}$. Solving this problem can be performed efficiently by iterating over feasible $\mathbf{t}$ such that every iteration increases the objective function, for example, using the method described in [28, Section III.B]. Due to convexity, the convergence to an optimal solution is guaranteed. The overall solution algorithm is given in Algorithm 1. The solution to outer maximization problem is in lines 2 to 16.

Algorithm 1 Algorithm to solve (16)

Initialize

1: Find initial feasible $\mathbf{R}^0 \triangleq (\mathbf{r}_1^0, \mathbf{r}_2^0, \mathbf{r}_{U1}^0, \mathbf{r}_{U2}^0)$

Define function to find z(t)

2: **function** SOLVEZ($\alpha_{1i}^n, \alpha_{2i}^n, \beta_{1i}^n, \beta_{2i}^n, D_i^n$) ▷ Solves z
3: Set $\mathbf{u} \leftarrow \mathbf{0}, \mathbf{t}_1 \leftarrow \mathbf{u}, \mathbf{t}_2 \leftarrow \mathbf{u}$
4: Solve $z_1(\mathbf{u}), z_2(\mathbf{u})$ as explained after (31) and (33)
5: $z(\mathbf{u}) \leftarrow z_1(\mathbf{u}) + z_2(\mathbf{u})$
6: **for** $i = 1 : N$ **do**
7: $t_{1i} \leftarrow u_i + \epsilon, t_{2i} = u_i - \epsilon$
8: Solve $z_1(\mathbf{t}_1), z_2(\mathbf{t}_1), z_1(\mathbf{t}_2), z_2(\mathbf{t}_2)$
9: $z(\mathbf{t}_1) = z_1(\mathbf{t}_1) + z_2(\mathbf{t}_1), z(\mathbf{t}_2) = z_1(\mathbf{t}_2) + z_2(\mathbf{t}_2)$
10: **if** $[z(\mathbf{t}_1) > z(\mathbf{u})]$ **then** $\mathbf{u} = \mathbf{t}_1$
11: **else if** $[z(\mathbf{t}_2) > z(\mathbf{u})]$ **then** $\mathbf{u} = \mathbf{t}_2$
12: **end if**
13: **end for**
14: Go to (6) until convergence
15: **return** last found optimal $(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_{U1}, \mathbf{r}_{U2})$
16: **end function**

Main Algorithm

17: **repeat**
18: Find $A_i^n, \alpha_{1i}^n, \alpha_{2i}^n, \beta_{1i}^n, \beta_{2i}^n, C_i^n$ from (35) - (40)
19: $D_i^n \leftarrow C_i^n - \alpha_{1i}^n r_{1i}^n - \alpha_{2i}^n r_{2i}^n - \beta_{1i}^n r_{U1i}^n - \beta_{2i}^n r_{U2i}^n$
20: $\mathbf{R}^{n+1} \leftarrow$ SOLVEZ($\alpha_{1i}^n, \alpha_{2i}^n, \beta_{1i}^n, \beta_{2i}^n, D_i^n$)
21: $n \leftarrow n+1$
22: **until** convergence

V. NUMERICAL RESULTS

In this section, we demonstrate that user cooperation improves the achievable departure region of a MAC, under data and energy arrival constraints. In Fig. 3 we plot the achievable departure region of the proposed cooperative MAC model with energy and data arrival constraints. For comparison, we also plot the departure region of the MAC with energy and data arrivals which was studied in [23] and the departure region of the cooperative MAC with only energy arrivals which was studied in [24]. For direct comparison with [24], we use capacity and achievable rate formulas for bandlimited Gaussian channels, that yield the capacity and achievable rates in bits per second. We select the bandwidth and the equivalent noise variance (obtained by taking into account the bandwidth, noise spectral density and path loss) as in [24]. The Gaussian noise variances on the direct links are therefore selected as 10^{-2} W and the transmission bandwidth is selected as 1 MHz. For the cooperative MAC only, the inter-user channels are assumed to be AWGN channels with variance 5×10^{-3} W, which translates to inter-user links having a 3-dB SNR advantage over the direct links.

The energy and data arrivals are chosen as $\mathbf{E}_1 = [5, 0, 5, 0, 0, 0, 0, 10, 0, 0]$ mJ, $\mathbf{E}_2 = [5, 0, 0, 0, 0, 10, 0, 0, 5, 0]$ mJ, $\mathbf{d}_1 = [1.4, 1.4, 0, 1.4, 0, 7, 14, 0, 14, 0] \times 10^{-1}$ Mbits, $\mathbf{d}_2 = [7, 2.8, 0, 14, 0, 0, 1.4, 2.8, 0, 0] \times 10^{-1}$ Mbits. The transmission deadline is chosen as 10 seconds. The existence of data arrivals in the cooperative MAC has an impact on the departure region and this effect is more apparent in the single user rates. We also observe that cooperation has enhanced the departure region when we compare the ordinary MAC and cooperative MAC both with data and energy arrrivals.

Additionally, we plot the data departure curves for both users in Fig. 2 in the case of sum rate maximization, i.e., $\mu_1 = \mu_2 = 1$. We see that the possibility of user cooperation allows for higher data rates to be sustained using the same amount of energy.

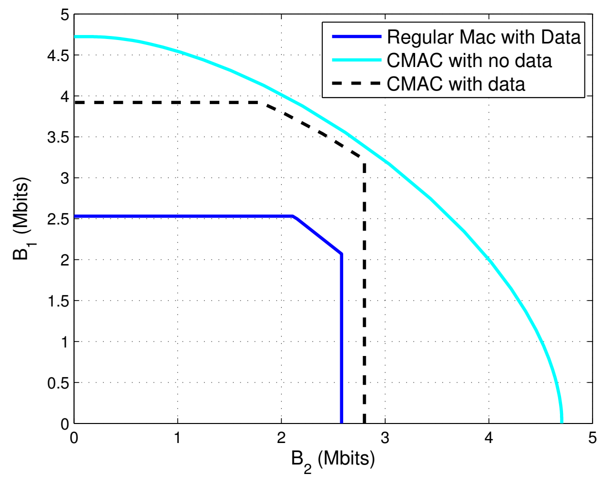

Fig. 2. Departure regions of cooperative MAC with and without data arrivals vs the capacity region of regular MAC with data arrivals.

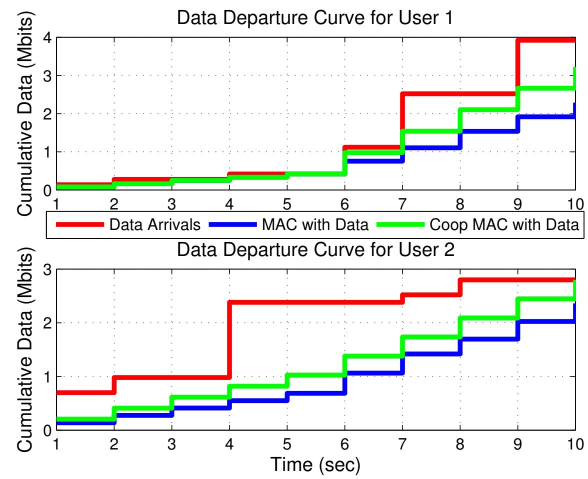

Fig. 3. Data departure curves for both users in the case of $\mu_1 = \mu_2 = 1$.

VI. Conclusion

We considered a cooperative MAC with intermittent data and energy arrivals. We found the optimal offline power and rate allocation policy that maximize the departure region. We first showed that there exists an optimal policy, in which the single user rate constraints in each time slot are tight. Then, we formulated the departure region maximization problem as a weighted sum rate maximization in terms of rates only. Next, we proposed a sequential convex approximation method and showed that it converges to the optimal solution. Finally, we solved the approximate problems with an inner outer decomposition method. Numerically, we observed that higher data rates can be sustained using the same amount of energy.

Appendix A
Coefficients of (17)

By differentiating $f(S_i/\sigma^2)$ the coefficients are,

$$A_i^n = 2^{2r_{1i}^n} + 2^{2r_{U1i}^n} + 2^{2r_{2i}^n} + 2^{2r_{U2i}^n} + 2\sqrt{(2^{2r_{U1i}^n}-1)(2^{2r_{U2i}^n}-1)} - 4, \quad (35)$$

$$\alpha_{1i}^n \triangleq \frac{\partial g}{\partial r_{1i}}\Big|_{r_{1i}^n} = \frac{0.5}{1+A_i^n/\sigma^2} 2^{2r_{1i}^n}, \quad (36)$$

$$\alpha_{2i}^n \triangleq \frac{\partial g}{\partial r_{2i}}\Big|_{r_{2i}^n} = \frac{0.5}{1+A_i^n/\sigma^2} 2^{2r_{2i}^n}, \quad (37)$$

$$\beta_{1i}^n \triangleq \frac{\partial g}{\partial r_{U1i}}\Big|_{r_{U1i}^n} = \frac{0.5}{1+A_i^n/\sigma^2} 2^{2r_{U1i}^n} \left(1 + \frac{\sqrt{2^{2r_{U2i}^n}-1}}{\sqrt{2^{2r_{U1i}^n}-1}}\right), \quad (38)$$

$$\beta_{2i}^n \triangleq \frac{\partial g}{\partial r_{U2i}}\Big|_{r_{U2i}^n} = \frac{0.5}{1+A_i^n/\sigma^2} 2^{2r_{U2i}^n} \left(1 + \frac{\sqrt{2^{2r_{U1i}^n}-1}}{\sqrt{2^{2r_{U2i}^n}-1}}\right), \quad (39)$$

$$C_i^n = \frac{1}{2}\log_2\left(1 + \frac{A_i^n}{\sigma^2}\right). \quad (40)$$

Appendix B
Proof of Lemma 2

We will prove a more general result. Assume we have two optimization problems (P1) and (P2) as given below.

$$\textbf{(P1):} \quad \min_{\mathbf{x}} \quad f_0(\mathbf{x}) \quad \text{s.t.} \quad f_i(\mathbf{x}) \le 0,\ i = 1, \dots, m. \quad (41)$$

$$\textbf{(P2):} \quad \min_{\mathbf{y}} \quad f_0(h(\mathbf{y})) \quad \text{s.t.} \quad f_i(h(\mathbf{y})) \le 0,\ i = 1, \dots, k, \quad f_i(h(\mathbf{y})) = 0,\ i = k+1, \dots, m. \quad (42)$$

Here $\{f_i\}_{i=1}^m$ are convex, differentiable functions and $h(\mathbf{y})$ is a collection of one-to-one, invertible functions. (P2) is obtained from (P1) by enforcing some inequality constraints with equality and by a change of variables, $\mathbf{x} = h(\mathbf{y})$. Since (P1) is a convex optimization problem, strong duality holds [29]. We denote the primal optimal values of problems (P1) and (P2) as p_1^*, p_2^* respectively. We show the following lemma.

Lemma 5 *If $p_1^* = p_2^*$, then strong duality also holds for (P2).*

Proof: The dual function and the Lagrange dual problem for (P1) are,

$$g_1(\boldsymbol{\lambda}) = \min_{\mathbf{x}} [f_0(\mathbf{x}) + \sum_{i=1}^m \lambda_i f_i(\mathbf{x})], \quad (43)$$

$$d_1^* = \max_{\boldsymbol{\lambda} \ge \mathbf{0}} g_1(\boldsymbol{\lambda}), \quad (44)$$

where $\boldsymbol{\lambda}$ are the Lagrange multipliers corresponding to the inequality constraints in (41) and d_1^* denotes the optimal dual value. Similarly for (P2),

$$g_2(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \min_{\mathbf{y}} [f_0(h(\mathbf{y})) + \sum_{i=1}^k \beta_i f_i(h(\mathbf{y})) + \sum_{i=k+1}^m \gamma_i f_i(h(\mathbf{y}))], \quad (45)$$

$$d_2^* = \max_{\boldsymbol{\beta} \geq \mathbf{0}, \boldsymbol{\gamma}} g_2(\boldsymbol{\beta}, \boldsymbol{\gamma}), \tag{46}$$

where β_i and γ_i correspond to the inequality and equality constraints in (42), respectively. We do not have the constraints $\boldsymbol{\gamma} \geq \mathbf{0}$ since $\boldsymbol{\gamma}$ corresponds to equality constraints. Since h is invertible, we let $\mathbf{x} = h^{-1}(\mathbf{y})$ and rewrite (45) as,

$$g_2(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \min_{\mathbf{x}} [f_0(\mathbf{x}) + \sum_{i=1}^{k} \beta_i f_i(\mathbf{x}) + \sum_{i=k+1}^{m} \gamma_i f_i(\mathbf{x})]. \tag{47}$$

Now we have,

$$d_2^* \geq \max_{(\boldsymbol{\beta}, \boldsymbol{\gamma}) \geq \mathbf{0}} g_2(\boldsymbol{\beta}, \boldsymbol{\gamma}) = \max_{\boldsymbol{\lambda} \geq \mathbf{0}} g_2(\boldsymbol{\lambda}) = \max_{\boldsymbol{\lambda} \geq \mathbf{0}} g_1(\boldsymbol{\lambda}) = d_1^*, \tag{48}$$

where the first inequality follows from the fact that $\boldsymbol{\gamma} \geq \mathbf{0}$ yields to a more restricted feasible set, the first equality is a rewriting of the problem in terms of variable $\boldsymbol{\lambda}$, the second equality follows from comparing (45) to (43). Furthermore,

$$d_2^* \geq d_1^* = p_1^* = p_2^*, \quad d_2^* \leq p_2^*, \tag{49}$$

where $d_1^* = p_1^*$ follows from strong duality of (P1) and $p_1^* = p_2^*$ from assumption and $d_2^* \leq p_2^*$ follows from weak duality of (P2) which always holds irrespective of convexity of the problem. Then we have $d_2^* = p_2^*$ and strong duality holds. ■

The problem in (16) is obtained from (9) similar to how (P2) is obtained from (P1) without changing the primal objective value and the problem in (9) is a convex problem. Therefore the problem in (16) has strong duality.

Appendix C
Proof of Lemma 3

In [27] a non-convex problem is solved by a convex approximation method, in which non-convex constraints $g(\mathbf{x})$ are approximated around point $\mathbf{x}^n$ by a differentiable convex function $\bar{g}(\mathbf{x}, \mathbf{x}^n)$. Each function $\bar{g}(\mathbf{x}, \mathbf{x}^n)$ must satisfy:

- $g(\mathbf{x}) \leq \bar{g}(\mathbf{x}, \mathbf{x}^n)$ for all feasible $\mathbf{x}$,
- $g(\mathbf{x}) = g(\mathbf{x}^n, \mathbf{x}^n)$,
- $\partial g(\mathbf{x}^n)/\partial \mathbf{x}^n = \partial \bar{g}(\mathbf{x}^n, \mathbf{x}^n)/\partial \mathbf{x}^n$.

In our problem, the non-convex constraint function g is given as $r_{1i} + r_{2i} - f(S_i/\sigma^2) \leq 0$. The last two properties are satisfied when $\bar{g}$ is taken as the Taylor expansion of the function g. The function $f(S_i/\sigma^2)$ is a convex function since it is of the form $\log(\sum 2^x)$. Then, g is concave. The first property is satisfied since linear approximations are over-estimators for concave functions. By [27, Theorem 1], $\mathbf{R}^n$ converges to $\mathbf{R}^*$ where $\mathbf{R}^*$ is a Kuhn-Tucker point of the problem in (16). From Lemma 2, strong duality holds and therefore Kuhn-Tucker conditions are both necessary and sufficient for global optimality. Therefore $\mathbf{R}^*$ is a global optimal solution to (16).

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