Power Control in the Cognitive Cooperative Multiple Access Channel

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***Abstract*—We extend several encoding and decoding techniques from cooperative communications framework, to a cognitive radio system consisting of a primary user (PU) and a secondary user (SU), sending their messages to a common receiver. Assuming that the transmitters and the receiver have full channel state information (CSI) collected and distributed by the common receiver, and that the SU knows the PU's codebook, the cooperation is obtained by block Markov superposition coding, and backwards decoding, which yield a causal overlay scenario. We formulate two rate optimization problems with the aim of, (i) maximizing the sum rate of the system, and (ii) maximizing the rate of the secondary user. We obtain the optimal power allocations for both cases, and the resulting rate regions. The power controlled cooperation turns out to be especially useful when maximizing the sum rate of the system, as it gives the PU significant rate rewards for allowing the cognitive transmitter to access its resources.**

I. Introduction

The widespread use of data intensive applications over wireless channels, makes it essential to utilize the available resources such as frequency, time, power and space more efficiently. Such efficient utilization calls for new techniques; involving more capable devices, with higher levels of awareness about, and higher abilities to adapt to their surroundings. Among such contending new techniques are cooperative communication and cognitive radio, both of which have been the focal point of most of the wireless communications research in the last decade. These two technologies essentially rely on very similar principles: in both approaches, the side information supplied by the wireless medium, and smart capable nodes which make use of this information, play key roles.

Cooperative communications root from the relay model introduced in [1], but the settings of more interest, where the relays have their own messages to be transmitted as well, are better captured using the multiple access channel with generalized feedback (MAC-GF) model. This model was treated in its abstract form in [2], applied to the wireless setup in [3], and power optimized in [4]. The MAC-GF treats the cooperating nodes as equals, which is its main difference from the traditional cognitive radio setup. Yet, it can also be viewed as a way of incorporating cognition in wireless networks, as will be demonstrated in this paper.

While the idea of cognitive radio has originally emerged from the goal of achieving a clever and efficient usage of the previously occupied spectrum [5], cognitive radios are now perceived as all around devices, which are aware of several properties of their medium including the messages, codewords and channel states of the other users sharing the medium; and which can use this awareness to increase their capacity/rates, without adversely affecting the communication quality of the existing primary users in the network [6]. When viewed from this perspective, these devices, whose originally envisioned operation relied on detecting the frequency voids in the spectrum and using them for their transmissions, called interweaving, or on transmitting at a conservative power level to satisfy a given interference requirement at the primary users, called underlay, may in fact be used more effectively in more aggressive modes, such as the overlay mode [6].

In overlay cognitive radio, which has attracted considerable interest rather recently, the cognitive users actively make use of the signals/codewords of the primary users: they may decode PU signals and use them while creating their codewords, or even relay the PU messages to communicate their own messages under better conditions. For instance, in [7], a system which consists of a primary, and a secondary transmitter receiver pair, where the secondary transmitter knows the primary transmitter's message non-causally, was studied. A protocol which makes sure that the PU's rate is not impaired by the SU was proposed, and the power level that needs to be allocated by the SU for cooperation was derived, in a non-fading scenario. In [8] a similar setup was treated, assuming that the message of one of the users in the interference channel model used, is the subset of the message of the other user. For the same model, the authors of [9] also incorporated rate splitting (RS) in the encoding strategy, and in [10], a strategy which allows RS at both users, and performs Gel'fand Pinsker (GP) binning differently, was developed. All of the aforementioned channel models have the downside that they assume non-causal knowledge of the PU message at the SU.

A more realistic model, where the common information generation is also taken into account, was first considered in [11]. There, two-phase protocols, based on time division were proposed to create primary user's information at the secondary user, and again RS and GP binning were utilized. Other recent works, such as [12]–[14], also focused on the causal cognitive overlay setup, but all of these works consider an underlying interference channel model for cognitive radio. While there is some work on resource allocation in underlay cognitive radio or in cases without cooperation [15], [16], the resource allocation problems in the overlay setups based on interference channel models are quite complicated, and are relatively untouched.

In this paper, we will focus on a somewhat simpler cognitive channel model instead: a cognitive cooperative fading multiple access channel (MAC). Our goal is to obtain the optimal power allocation policies that maximize either the sum rate, or the secondary user rate. Note that, cognitive transmissions and cooperation become significantly more feasible, if the primary and secondary transmitters share the same receiver; as the distribution of the side information can be managed by this common receiver. We modify the superposition block Markov encoding for a MAC-GF, to suit the causal cognitive radio setup, so that the only adaptation by the PU is power allocation as a function of the available channel states. We

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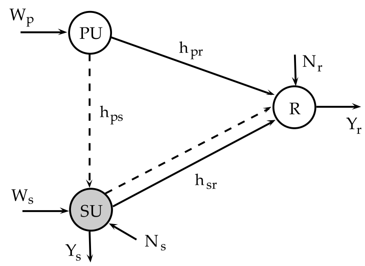

Fig. 1. Two-User Cooperative Cognitive Gaussian MAC

impose realistic constraints on the PU rates, taking into account the best possible rates achievable without cooperation, and we formulate the sum rate and SU rate optimization problems. We provide optimal power allocation strategies which solve these problems, in terms of the instantaneous channel coefficients, and obtain the resulting optimized average rates.

II. System Description

We consider a cognitive multiple access setting, where one primary and one secondary user share the same channel, while transmitting independent messages W_p and W_s respectively, to a common receiver. The SU listens to the channel, and is able to decode part of PU's message, and relays it to the receiver. The channel model, illustrated in Fig. 1, is described as

$$Y_r = h_{pr}X_p + h_{sr}X_s + N_r, \quad (1)$$
$$Y_s = h_{ps}X_p + N_s, \quad (2)$$

where h_{pr}, h_{sr} and h_{ps} are the amplitudes of the frequency flat fading over the links between the PU and the receiver, the SU and the receiver, and the PU and the SU respectively. N_r and N_s denote the independent additive white Gaussian noise variables at the receiver and the SU, both having zero-mean, and the respective variances σ_r^2 and σ_s^2. X_p and X_s are the codewords transmitted by the PU and and the SU, respectively.

This channel model can either be seen as a generalization of a relay channel, where the relay also has its own messages to transmit, or a special case of a MAC with generalized feedback [2], [3], where the cooperation signals from one of the users is disabled. We take the latter approach, and modify the superposition block Markov encoding strategy in [3]: we divide the PU's message into two submessages, i.e., $W_p = (W_{pr}, W_{ps})$. The submessage W_{pr} is the information sent directly to the receiver, and the submessage W_{ps} is the part that can be decoded by both the SU and the receiver. The SU message is not partitioned, as the PU should not aid the SU, due to the cognitive setup. Then, these messages are mapped to randomly generated codewords, whose entries are selected from unit Gaussian distributions, i.e.,

$$X_{sr}(W_s(b), W_{ps}(b-1)) \quad (3)$$
$$X_{pr}(W_{pr}(b), W_{ps}(b-1)) \quad (4)$$
$$X_{ps}(W_{ps}(b), W_{ps}(b-1)) \quad (5)$$
$$C(W_{ps}(b-1)) \quad (6)$$

where X_{sr} and X_{pr} are used to transmit fresh information $W_{sr}(b)$ and $W_{pr}(b)$ directly intended for the receiver in block b, X_{ps} is signal transmitted by the PU to allow potential cooperation from the SU in the next block, and C is the common signal which is used by both users to cooperatively transmit the PU's information $W_{ps}(b-1)$ from the previous block. The resulting codewords of the users are formed by superposition, where we also take into account the possibility of power control as in [4], as a function of the available channel state information, denoted by the the channel state vector $\mathbf{h} = [h_{pr}, h_{ps}, h_{sr}]$:

$$X_p = \sqrt{P_{pr}(\mathbf{h})}X_{pr} + \sqrt{P_{ps}(\mathbf{h})}X_{ps} + \sqrt{P_{pc}(\mathbf{h})}C, \quad (7)$$
$$X_s = \sqrt{P_{sr}(\mathbf{h})}X_{sr} + \sqrt{P_{sc}(\mathbf{h})}C. \quad (8)$$

The powers are required to satisfy the average power constraints,

$$P_p(\mathbf{h}) = P_{pr}(\mathbf{h}) + P_{ps}(\mathbf{h}) + P_{pc}(\mathbf{h}) \quad (9)$$
$$P_s(\mathbf{h}) = P_{sr}(\mathbf{h}) + P_{sc}(\mathbf{h}) \quad (10)$$
$$E\Big[P_i(\mathbf{h})\Big] \leq \bar{P}_i \ \text{ where } \ i \in \{p, s\} \quad (11)$$

Before characterizing the achievable rates, we would like to discuss the suitability of the model in (7)-(8), which is an application of the well known Block Markov encoding approach from cooperative communications, in the cognitive setup. In cognitive MACs, the PU's transmissions should not be altered substantially by the presence of the SU. However, in order to enable the cooperative overlay scenario, some level of collaboration from the PU is inevitable. Our proposed model keeps this level to a minimum, as follows: regardless of the presence of the SU, the PU always employs the coding strategy given in (7), using power levels given in (9), the values of which are potentially fed back to it by the receiver. Note that, if the PU was transmitting alone, it would still require the channel state information, or the transmit power value to be fed back from the receiver, hence the additional feedback required is limited to two additional power values, and the PU's modification of its transmit policy is limited to just changing these power levels upon the receiver's request. That means, the PU can in fact operate obliviously to the presence of the SU. When the SU is present, and willing to cooperate, it decodes the message W_{ps} using the codeword X_{ps}, treating X_{pr} as noise, and knowing C, which depends only on previously decoded information. Then, it can cooperatively send C, in addition to its own codeword X_{sr}, so that any potential rate penalty caused at the receiver by its codeword X_{sr}, is nullified, or even surpassed, by the gain from C. The receiver uses backwards decoding, and it is easy to check, by a direct extension from [4], that the achievable rate region is given by,

$$R_p < E\left\{\log\Big[1+s_{pr}P_{pr}(\mathbf{h})\Big]+\log\left[1+\frac{s_{ps}P_{ps}(\mathbf{h})}{s_{ps}P_{pr}(\mathbf{h})+1}\right]\right\} \quad (12)$$

$$R_s < E\left\{\log\Big[1+s_{sr}P_{sr}(\mathbf{h})\Big]\right\} \quad (13)$$

$$R_p+R_s \leq \min\left\{E\left\{\log\Big[1+s_{pr}P_{pr}(\mathbf{h})+s_{sr}P_{sr}(\mathbf{h})\Big]\right.\right.$$
$$\left.\left.+\log\left[1+\frac{s_{ps}P_{ps}(\mathbf{h})}{s_{ps}P_{pr}(\mathbf{h})+1}\right]\right\}, E\Big\{\log(A)\Big\}\right\} \quad (14)$$

where $A = 1 + s_{pr}P_p(\mathbf{h}) + s_{sr}P_s(\mathbf{h}) + 2\sqrt{s_{pr}s_{sr}P_{pc}P_{sc}}$. In (12)-(14), R_p and R_s denote the rates of primary and secondary users; and the channel fading coefficients, normalized by the noise powers are denoted as $s_{ij} = h_{ij}^2/\sigma_j^2$, where $i \in \{p, s\}$ and $j \in \{s, r\}$, $i \neq j$.

Now, we describe the crucial twist from the cooperative communication framework, due to the cognitive setup: not all rates satisfying the above constraints are necessarily achievable, as we should also guarantee that the PU's achievable rate is no worse than what it would be, had the PU been transmitting alone. Moreover, we have to assume that the PU would be able to use optimal power allocation [17], which is single user waterfilling, while computing the worst case rate requirement of the PU. Therefore, we need the constraint:

$$R_p \geq E\Big\{ \log\Big[1 + P_p^\star(h_{pr})s_{pr}\Big]\Big\} \triangleq B^* \tag{15}$$

where $P_p^\star(h_{pr})$ is the optimal power level for single user transmission, with $E[P_p^\star(h_{pr})] = \bar{P}_p$; and B^* is the resulting maximum data rate achievable by the PU, without cooperation.

In the next section, we solve the optimal power allocation problem for the cognitive cooperative scenario, with two separate objectives: sum rate maximization, which creates an extra incentive for the PU to allow cooperation, and SU rate maximization, which aims to accommodate as much rate for the cognitive user as possible, while still providing a maximum single-user rate guarantee for the PU.

III. Maximization of the Achievable Rates

We start by noting, also in light of the findings in [4] for the non-cognitive cooperative MAC, that for channel states which satisfy $s_{ps} > s_{pr}$, the optimal strategy is to set $P_{pr}(\mathbf{h}) = 0$, meaning no additional power should be allocated by the PU for direct transmission. Throughout this paper, we assume that we always operate in this regime; if $s_{ps} < s_{pr}$, the cooperation can simply be disabled. Note that, non-cooperative transmission is still possible even when $s_{ps} > s_{pr}$, as the SU may choose to ignore PU transmission, and the receiver still decodes X_{ps}. Setting $P_{pr}(\mathbf{h}) = 0$ in (12)-(14), and defining $\mathbf{P}(\mathbf{h}) = [P_{ps}(\mathbf{h}), P_{pc}(\mathbf{h}), P_{sr}(\mathbf{h}), P_{sc}(\mathbf{h})]$, we can state the power optimization problem in the following convex form:

$$\max_{\mathbf{P}(\mathbf{h})} \quad \alpha R_p + R_s \tag{16}$$

$$\text{s.t.} \quad R_p \leq E\Big[\log\Big(1 + s_{ps}P_{ps}(\mathbf{h})\Big)\Big] \tag{17}$$

$$R_s \leq E\Big[\log\Big(1 + s_{sr}P_{sr}(\mathbf{h})\Big)\Big] \tag{18}$$

$$R_p + R_s \leq E\Big[\log\Big(A\Big)\Big] \tag{19}$$

$$R_p \geq B^\star \tag{20}$$

$$E\Big[P_{ps}(\mathbf{h}) + P_{pc}(\mathbf{h})\Big] \leq \bar{P}_p \tag{21}$$

$$E\Big[P_{sr}(\mathbf{h}) + P_{sc}(\mathbf{h})\Big] \leq \bar{P}_s \tag{22}$$

$$P_{ps}(\mathbf{h}), P_{pc}(\mathbf{h}), P_{sr}(\mathbf{h}), P_{sc}(\mathbf{h}) \geq 0 \tag{23}$$

Note that, by setting $\alpha = 1$ in (16), we obtain the sum rate maximization for cognitive MAC, and by setting $\alpha = 0$, we obtain the SU rate maximization. We will treat both problems in parallel, and discuss their differences as they become apparent. First, by associating several Lagrange multipliers with the constraints in (17)-(23), we write the Lagrangian,

$$\begin{aligned}\mathcal{L} =& \alpha R_p + R_s + \gamma_1\Big\{E\Big[\log\Big(1 + s_{ps}P_{ps}(\mathbf{h})\Big)\Big] - R_p\Big\} \\ &+ \gamma_2\Big\{E\Big[\log\Big(1 + s_{sr}P_{sr}(\mathbf{h})\Big)\Big] - R_s\Big\} \\ &+ \gamma_3\Big\{E\Big[\log\Big(A\Big)\Big] - R_p - R_s\Big\} + \gamma_4\Big\{R_p - B^\star\Big\} \\ &+ \lambda_1\Big\{P_p - E\Big[P_{ps}(\mathbf{h}) + P_{pc}(\mathbf{h})\Big]\Big\} \\ &+ \lambda_2\Big\{P_s - E\Big[P_{sr}(\mathbf{h}) + P_{sc}(\mathbf{h})\Big]\Big\} \\ &+ \mu_1 P_{ps}(\mathbf{h}) + \mu_2 P_{pc}(\mathbf{h}) + \mu_3 P_{sr}(\mathbf{h}) + \mu_4 P_{sc}(\mathbf{h})\end{aligned} \tag{24}$$

Taking the partial derivatives with respect to the power components of primary and secondary users, as well as the rate variables, and employing complementary slackness constraints, it is easy to show that the following KKT conditions are necessary and sufficient for optimality:

$$\lambda_1 \geq \gamma_1\frac{s_{ps}}{1 + s_{ps}P_{ps}(\mathbf{h})} + \gamma_3\frac{s_{pr}}{A} \tag{25}$$

$$\lambda_2 \geq \gamma_2\frac{s_{sr}}{1 + s_{sr}P_{sr}(\mathbf{h})} + \gamma_3\frac{s_{sr}}{A} \tag{26}$$

$$\lambda_1 \geq \gamma_3\frac{s_{pr}\sqrt{P_{pc}(\mathbf{h})} + \sqrt{s_{pr}s_{sr}P_{sc}(\mathbf{h})}}{A\sqrt{P_{pc}(\mathbf{h})}} \tag{27}$$

$$\lambda_2 \geq \gamma_3\frac{s_{sr}\sqrt{P_{sc}(\mathbf{h})} + \sqrt{s_{pr}s_{sr}P_{pc}(\mathbf{h})}}{A\sqrt{P_{sc}(\mathbf{h})}} \tag{28}$$

$$1 = \gamma_2 + \gamma_3 \tag{29}$$

$$\alpha + \gamma_4 = \gamma_1 + \gamma_3 \tag{30}$$

The constraints (25), (26), (27), (28) are satisfied with equality, if the respective powers $P_{ps}(\mathbf{h}), P_{sr}(\mathbf{h}), P_{pc}(\mathbf{h}), P_{sc}(\mathbf{h})$ are positive. The Lagrange multipliers are selected to satisfy their respective constraints.

Let us first consider the sum rate maximization, i.e., $\alpha = 1$. From (29) and (30), we have $\gamma_1 = \gamma_2 + \gamma_4$, and the trick is to consider two cases separately: when $\gamma_4 = 0$, (20) is inactive, meaning the PU rate already satisfies the cognitive transmission constraint. Then, we are back to the non-cognitive scenario as in [4], and after some lengthy manipulations of (25)-(28), with $\gamma_1 = \gamma_2 = 1 - \gamma_3$, we get

$$P_{ps}(\mathbf{h}) = \left(\frac{\gamma_2(\lambda_2 s_{pr} + \lambda_1 s_{sr})}{\lambda_1^2 s_{sr}} - \frac{1}{s_{ps}}\right)^+, \tag{31}$$

$$P_{sr}(\mathbf{h}) = \left(\frac{\gamma_2(\lambda_2 s_{pr} + \lambda_1 s_{sr})}{\lambda_2^2 s_{sr}} - \frac{1}{s_{sr}}\right)^+, \tag{32}$$

$$P_{pc}(\mathbf{h}) = \frac{\frac{1-\gamma_2\left(s_{pr}+\lambda_1 s_{sr}/\lambda_2\right)}{\lambda_1} - D}{(s_{pr} + \lambda_1 s_{sr}/\lambda_2)^2}s_{pr}, \tag{33}$$

$$P_{sc}(\mathbf{h}) = \frac{\frac{1-\gamma_2\left(s_{sr}+\lambda_2 s_{pr}/\lambda_2\right)}{\lambda_2} - D}{(s_{sr} + \lambda_2 s_{pr}/\lambda_1)^2}s_{sr}, \tag{34}$$

where $D = 1 + s_{pr}P_{ps}(\mathbf{h}) + s_{sr}P_{sr}(\mathbf{h})$, provided $P_{pc}(\mathbf{h})$ and $P_{sc}(\mathbf{h})$ obtained from (33) and (34) are both positive. Otherwise, $P_{pc}(\mathbf{h})$ and $P_{sc}(\mathbf{h})$ both have to be set to zero in (25) and (26) through the variable A, (there is no other alternative, as having only one cooperative power non-zero would be strictly suboptimal), and one should re-solve (25) and (26) for $P_{ps}(\mathbf{h})$ and $P_{sr}(\mathbf{h})$, which are the positive roots of the following quadratic equation:

$$a_i P_{ij}(\mathbf{h})^2 + b_i P_{ij}(\mathbf{h}) + c_i = 0, \quad \{i,j\} \in \{\{p,s\},\{s,r\}\} \quad (35)$$

where the coefficients are given by

$$\begin{aligned}(a_p; b_p; c_p) &= (\lambda_1 s_{pr} s_{ps}; \lambda_1(s_{pr} + s_{ps} + s_{ps}s_{sr}P_{sr}(\mathbf{h})\\ &\quad - s_{pr}s_{ps}); \lambda_1(1 + s_{sr}P_{sr}(\mathbf{h}))\\ &\quad - \gamma_2(s_{ps} + s_{ps}s_{sr}P_{sr}(\mathbf{h}) - s_{pr} - s_{pr}))\\ (a_s; b_s; c_s) &= (\lambda_2 s_{sr}^2; \lambda_2(2s_{sr} + s_{pr}s_{sr}P_{ps}(\mathbf{h})) - s_{sr}^2;\\ &\quad \lambda_2(1 + s_{pr}P_{ps}(\mathbf{h})) - \gamma_2(s_{pr}s_{sr}P_{ps}(\mathbf{h})) - s_{sr})\end{aligned}$$

Now, we go back to the second possible case, $\gamma_4 > 0$, meaning that (20) is satisfied with equality, i.e., the PU rate is fixed to its minimum possible value. The key observation here is that the sum rate maximization problem then becomes equivalent to SU rate maximization, and solving the SU maximization problem will also complete the solution of the sum rate maximization. To do so, we may as well set $\alpha = 0$, and force equality in (20), by varying γ_4. Luckily, the KKT conditions, and the resulting optimal power expressions are almost identical to the previous case, except we now have to replace γ_2 in (31) by γ_1. Using (29)-(30), we now need to search for two Lagrange multipliers, γ_4 and γ_2 (or equivalently γ_1 and γ_2), rather than one. Once again, when the cooperative powers turn out to be negative, we set them to zero, and we instead solve $P_{ps}(\mathbf{h})$, $P_{sr}(\mathbf{h})$ that maximize the SU rate from (35), with

$$\begin{aligned}(a_p; b_p; c_p) &= (\lambda_1 s_{pr} s_{ps}; \lambda_1(s_{pr} + s_{ps} + s_{ps}s_{sr}P_{sr}(\mathbf{h}))\\ &\quad - (\gamma_1 - \gamma_2 + 1)s_{pr}s_{ps}; \lambda_1(1 + s_{sr}P_{sr}(\mathbf{h}))\\ &\quad - \gamma_1(s_{ps} + s_{ps}s_{sr}P_{sr}(\mathbf{h})) - (1 - \gamma_2)s_{pr})\\ (a_s; b_s; c_s) &= (\lambda_2 s_{sr}^2; \lambda_2(2s_{sr} + s_{pr}s_{sr}P_{ps}(\mathbf{h})) - s_{sr}^2;\\ &\quad \lambda_2(1 + s_{pr}P_{ps}(\mathbf{h})) - \gamma_2(s_{pr}s_{sr}P_{ps}(\mathbf{h})) - s_{sr})\end{aligned}$$

A powerful property of the optimal power allocation policy derived in this section is that, it lends itself to an iterative implementation over the users, which can be used to systematically search the Lagrange multipliers required in the solution. The detailed description of the iterative algorithm we have developed for this purpose, which is guaranteed to converge to the optimum, will be relegated to a journal version, while its numerical results will be demonstrated in detail, in the next section.

IV. Simulation Results

In this section, we provide some numerical examples to illustrate the performance of the described power allocation policies in cognitive cooperation. We assume an average total power of 1 for each user. We also set all noise variances to 1. In order to maintain that the links from the PU and the SU to the receiver are worse compared to PU-SU link, we assume uniformly distributed fading coefficients, taking values from the set $\{0.025, 0.050, ..., 0.25\}$ for the PU-receiver and SU-receiver links; and from the set $\{0.26, 0.27, ..., 0.35\}$ for

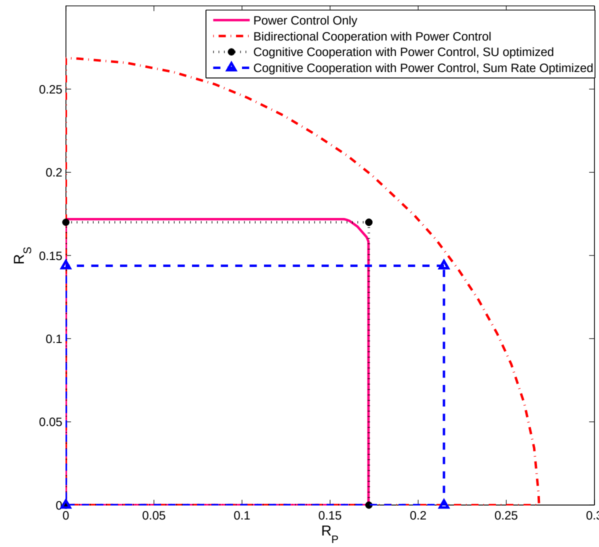

Fig. 2. Rates achievable with the power control and user cooperation for uniform fading.

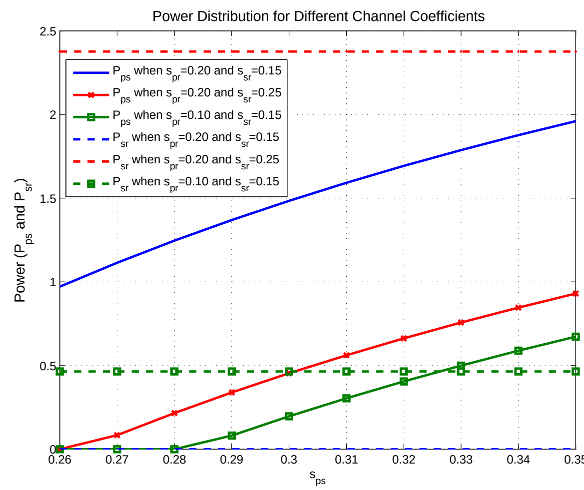

Fig. 3. Power distribution for different channel coefficients of s_{pr} and s_{sr}.

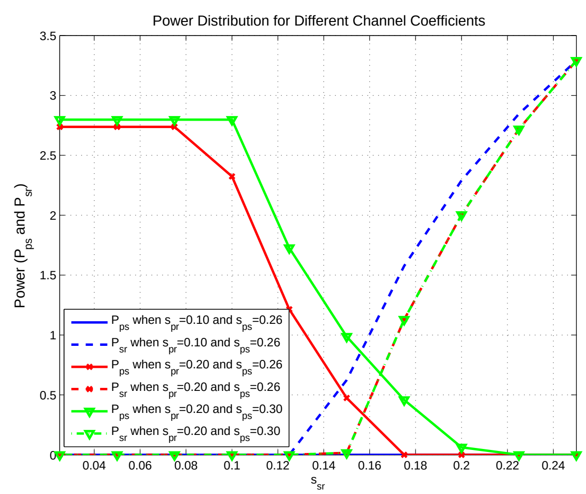

Fig. 4. Power distribution for different channel coefficients of s_{pr} and s_{ps}.

the PU-SU link. As discussed in the Section III, $P_{pr}(\mathbf{h})$ was set to zero.

Figure 2 illustrates the rate regions achievable by both sum rate maximizing and SU rate maximizing power control. For sum rate maximization, we see that the proposed power allocation policy remarkably improves the primary user's rate (see dashed curve) compared to a power controlled non-cooperative setting. In fact, we are able to achieve a rate pair, which is nearly on the bidirectional cooperation boundary (dash-dot), which is an upper bound. This result is particularly interesting, because it gives the PU a solid motivation to allow for cognitive cooperative transmissions from the SU. In fact, other variations, where one trades part of the additional rate achieved by the PU, for increased rate at the SU, may also be obtained by using our proposed policy (by selecting $0 < \alpha < 1$). In the present setup, the secondary user is able to reach the rate of 0.144 symbols/transmission while the primary user gets a much higher rate, so that the sum rate is maximized at 0.358 symbols/transmission.

For SU rate maximization, one has to take into account the fact that the PU's data rates should be neither below, nor above the single user optimal value obtained by power control. Hence, first the single user rate achievable by the PU using only power control was obtained from the non-cooperative rate region (solid); and this value, i.e., 0.172 symbols/transmission, was selected as B^*. The dotted region shows the rate pairs achievable by cognitive cooperation in this case, and at the optimal point, we observe that the cognitive user's rate is 0.17 symbols/transmission, which is above 0.158 symbols/transmission, it would have achieved by power control only, when PU rate was fixed to the single user optimum.

In Figure 3, we illustrate the sum-rate optimal power distributions as a function of s_{ps}, where we fix the direct link gains, s_{pr} and s_{sr}, to three different sets. Clearly, the power P_{ps} increases with increasing s_{ps}, as the primary user takes advantage of good channel conditions to create more common information. We observe a waterfilling type power distribution, as foreseen by (31). As the PU-receiver link s_{pr} gets better, the water level increases, and so does P_{ps}, as X_{ps} transmitted to the SU is also received at the receiver, and the PU looks to use this side benefit. As s_{sr} increases however, P_{ps}, decreases, which is also quite logical: good SU-receiver link presents a good opportunity to jointly transmit common information via C, hence it is natural that the PU cuts back P_{ps} in expense of P_{pc}, to boost the gain from coherent combining. The SU power clearly does not depend on s_{ps}, and decreases with increasing s_{pr} (to allow for cooperation) but it does increase with increasing s_{sr}, as expected.

In Figure 4, the illustrate the SU rate optimal power distribution as a function of s_{sr}, this time fixing the link gains of the primary user, s_{pr} and s_{ps}. As the quality of the SU-receiver link gets better, the the power allocated for SU transmission expectedly increases (but only after some point), and the power allocated for PU transmission towards SU decreases (again only after some point). The idea is, when s_{sr} is low, the channel is used to build up common information, especially if s_{ps} is strong. As s_{sr} increases, the SU uses some of its power for its own transmissions. We see that, above certain s_{sr}, PU ceases transmission of X_{ps}, and instead starts sending C, and also allows the SU to access the channel more aggressively to maximize its own rate. As expected, P_{sr} is unaffected by s_{ps}, when C is being transmitted, see (32).

V. Conclusion

In this paper, we developed a cooperative overlay model for cognitive transmissions in a fading MAC. We formulated two power optimization problems, one with the goal of maximizing the sum rate of the system, and the other with the goal of maximizing the secondary user's rate. We obtained the analytical expressions of the optimal power allocation policies for both settings, and computed the resulting powers, and the achievable rates using an iterative algorithm based on the KKT conditions. We analyzed the variation of the powers dedicated to different codewords, and showed their coupled relationships, as a function of the channel states. We demonstrated that, power control plays a very important role in rate optimization for our cognitive setup, and especially in sum rate maximization: the PU obtains quite remarkable gains, thereby justifying its willingness to participate in the cognitive setup. Hence, we conclude that, in networks with users having advanced cognitive capabilities, user cooperation is a very natural, and promising technique to be employed in conjunction with traditional cognitive transmission approaches.

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