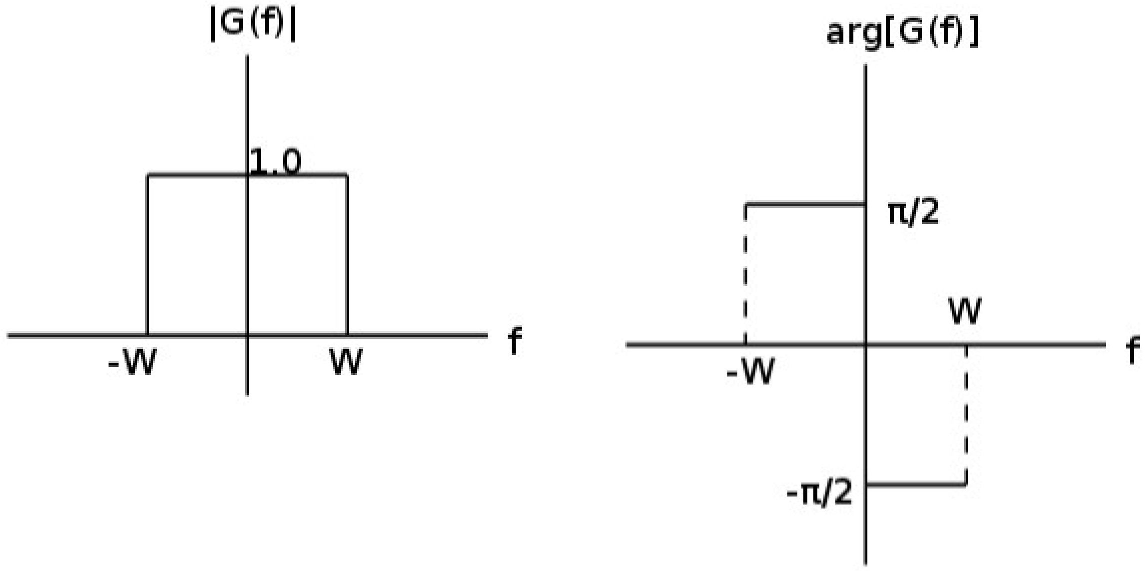
**Problem 1)** Determine the inverse Fourier transform of the frequency function *G(f)* defined by the amplitude and phase spectra shown in the figure below.




**Problem 2)** The Fourier transform of a signal *g(t)* is denoted by *G(f)*. Prove the following properties of the Fourier Transform:

**a)** If a real signal *g(t)* is an even function of time *t*, the Fourier transform *G(f)* is purely real. If a real signal *g(t)* is an odd function of time *t*, the Fourier transform *G(f)* is purely imaginary.

**b)** $t^n g(t) \Leftrightarrow \left(\frac{j}{2\pi}\right)^n G^{(n)}(f)$

where $G^{(n)}(f)$ is the *n*th derivative of *G(f)* with respect to *f*.

**c)** $\int_{-\infty}^{\infty} t^n g(t)\,dt = \left(\frac{j}{2\pi}\right)^n G^{(n)}(0)$

**d)** $g_1(t) g_2^*(t) \Leftrightarrow \int_{-\infty}^{\infty} G_1(\lambda) G_2^*(\lambda - f)\, d\lambda$

**e)** $\int_{-\infty}^{\infty} g_1(t) g_2^*(t) = \int_{-\infty}^{\infty} G_1(f) G_2^*(f)\, df$

**Problem 3)** Let $\hat{g}(t)$ denote the Hilbert transform of g(t). Derive the following set of Hilbert-transform pairs:

| $g(t)$ | $\hat{g}(t)$ |
|---|---|
| $\frac{\sin(t)}{t}$ | $\frac{1-\cos(t)}{t}$ |
| $rect(t)$ | $-\frac{1}{\pi}\ln\left(\left\|(t-\frac{1}{2})/(t+\frac{1}{2})\right\|\right)$ |
| $\delta(t)$ | $\frac{1}{\pi t}$ |
| $\frac{1}{1+t^2}$ | $\frac{1}{1+t^2}$ |

**Problem 4)** Consider the signal

$$s(t)=c(t)m(t)$$

whose *m(t)* is a low-pass signal whose Fourier transform *M(f)* vanishes for $|f|>W$, and *c(t)* is a high-pass signal whose Fourier transform *C(f)* vanishes for $|f|<W$. Show that the Hilbert transform of s(t) is

$$\hat{s}(t)=\hat{c}(t)m(t)$$

where $\hat{c}(t)$ is Hilbert transform of *c(t)*.