2.45 The *single sideband version of angle modulation* is defined by

$$s(t) = \exp[-\hat{\phi}(t)] \cos[2\pi f_c t + \phi(t)]$$

where $\hat{\phi}(t)$ is the Hilbert transform of the phase function $\phi(t)$, and f_c is the carrier frequency.

(a) Show that the spectrum of the modulated signal $s(t)$ contains no frequency components in the interval $-f_c < f < f_c$, and is of infinite extent.

(b) Given that the phase function

$$\phi(t) = \beta \sin(2\pi f_m t)$$

where β is the modulation index and f_m is the modulation frequency, derive the corresponding expression for the modulated wave $s(t)$.

Note: For Problems 2.44 and 2.45, you need to refer to Appendix 2 for a treatment of the Hilbert transform.

Noise in CW Modulation Systems

2.46 A DSB-SC modulated signal is transmitted over a noisy channel, with the power spectral density of the noise being as shown in Figure P2.46. The message bandwidth is 4 kHz and the carrier frequency is 200 kHz. Assuming that the average power of the modulated wave is 10 watts, determine the output signal-to-noise ratio of the receiver.

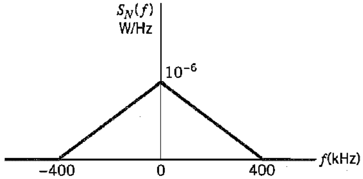

FIGURE P2.46

2.47 Evaluate the autocorrelation functions and cross-correlation functions of the in-phase and quadrature components of the narrowband noise at the coherent detector input for (a) the DSB-SC system, (b) an SSB system using the lower sideband, and (c) an SSB system using the upper sideband.

2.48 In a receiver using coherent detection, the sinusoidal wave generated by the local oscillator suffers from a phase error $\theta(t)$ with respect to the carrier wave $\cos(2\pi f_c t)$. Assuming that $\theta(t)$ is a sample function of a zero-mean Gaussian process of variance σ_Θ^2, and that most of the time the maximum value of $\theta(t)$ is small compared with unity, find the mean-square error of the receiver output for DSB-SC modulation. The mean-square error is defined as the expected value of the squared difference between the receiver output and the message signal component of the receiver output.

2.49 Following a procedure similar to that described in Section 2.11 for the DSB-SC receiver, extend this noise analysis to a SSB receiver. Specifically, evaluate the following:

(a) The output signal-to-noise ratio.

(b) The channel signal-to-noise ratio.

Hence, show that the figure of merit for the SSB receiver is exactly the same as that for the DSB-SC receiver. Note that unlike the DSB-SC receiver, the midband frequency of the spectral density function of the narrowband-filtered noise at the front end of the SSB receiver is offset from the carrier frequency f_c by an amount equal to $W/2$, where W is the message bandwidth.

2.50 Let a message signal $m(t)$ be transmitted using single-sideband modulation. The power spectral density of $m(t)$ is

$$S_M(f) = \begin{cases} a\dfrac{|f|}{W}, & |f| \le W \\ 0, & \text{otherwise} \end{cases}$$

where a and W are constants. White Gaussian noise of zero mean and power spectral density $N_0/2$ is added to the SSB modulated wave at the receiver input. Find an expression for the output signal-to-noise ratio of the receiver.

2.51 Consider the output of an envelope detector defined by Equation (2.92), which is reproduced here for convenience

$$y(t) = \{[A_c + A_c k_a m(t) + n_I(t)]^2 + n_Q^2(t)\}^{1/2}$$

(a) Assume that the probability of the event

$$|n_Q(t)| > \varepsilon A_c |1 + k_a m(t)|$$

is equal to or less than δ_1, where $\varepsilon \ll 1$. What is the probability that the effect of the quadrature component $n_Q(t)$ is negligible?

(b) Suppose that k_a is adjusted relative to the message signal $m(t)$ such that the probability of the event

$$A_c[1 + k_a m(t)] + n_I(t) < 0$$

is equal to δ_2. What is the probability that the approximation

$$y(t) \simeq A_c[1 + k_a m(t)] + n_I(t)$$

is valid?

(c) Comment on the significance of the result in part (b) for the case when δ_1 and δ_2 are both small compared with unity.

2.52 An unmodulated carrier of amplitude A_c and frequency f_c and band-limited white noise are summed and then passed through an ideal envelope detector. Assume the noise spectral density to be of height $N_0/2$ and bandwidth $2W$, centered about the carrier frequency f_c. Determine the output signal-to-noise ratio for the case when the carrier-to-noise ratio is high.

2.53 Let R denote the random variable obtained by observing the output of an envelope detector at some fixed time. Intuitively, the envelope detector is expected to be operating well into the threshold region if the probability that the random variable R exceeds the carrier amplitude A_c is 0.5. On the other hand, if this same probability is only 0.01, the envelope detector is expected to be relatively free of loss of message and the threshold effect.

(a) Assuming that the narrowband noise at the detector input is white, zero-mean, Gaussian with spectral density $N_0/2$ and the message bandwidth is W, show that the probability of the event $R \ge A_c$ is

$$P(R \ge A_c) = \exp(-\rho)$$

where ρ is the carrier-to-noise ratio:

$$\rho = \frac{A_c^2}{4WN_0}$$

(b) Using the formula for this probability, calculate the carrier-to-noise ratio when (1) the envelope detector is expected to be well into the threshold region, and (2) it is expected to be operating satisfactorily.

2.54 Consider a phase modulation (PM) system, with the modulated wave defined by

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

where k_p is a constant and $m(t)$ is the message signal. The additive noise $n(t)$ at the phase detector input is

$$n(t) = n_I(t) \cos(2\pi f_c t) - n_Q(t) \sin(2\pi f_c t)$$

Assuming that the carrier-to-noise ratio at the detector input is high compared with unity, determine (a) the output signal-to-noise ratio and (b) the figure of merit of the system. Compare your results with the FM system for the case of sinusoidal modulation.

2.55 An FDM system uses single-sideband modulation to combine 12 independent voice signals and then uses frequency modulation to transmit the composite baseband signal. Each voice signal has an average power P and occupies the frequency band 0.3 to 3.4 kHz; the system allocates it a bandwidth of 4 kHz. For each voice signal, only the lower sideband is transmitted. The subcarrier waves used for the first stage of modulation are defined by

$$c_k(t) = A_k \cos(2\pi k f_0 t), \qquad 0 \leq k \leq 11$$

The received signal consists of the transmitted FM signal plus white Gaussian noise of zero mean and power spectral density $N_0/2$.

(a) Sketch the power spectral density of the signal produced at the frequency discriminator output, showing both the signal and noise components.

(b) Find the relationship between the subcarrier amplitudes A_k so that the modulated voice signals have equal signal-to-noise ratios.

2.56 In the discussion on FM threshold effect presented in Section 2.13, we described the conditions for positive-going and negative-going clicks in terms of the envelope $r(t)$ and phase $\psi(t)$ of the narrowband noise $n(t)$. Reformulate these conditions in terms of the in-phase component $n_I(t)$ and quadrature component $n_Q(t)$ of $n(t)$.

2.57 By using the pre-emphasis filter shown in Figure 2.50a and with a voice signal as the modulating wave, an FM transmitter produces a signal that is essentially frequency-modulated by the lower audio frequencies and phase-modulated by the higher audio frequencies. Explain the reasons for this phenomenon.

2.58 Suppose that the transfer functions of the pre-emphasis and de-emphasis filters of an FM system are scaled as follows:

$$H_{pe}(f) = k\left(1 + \frac{jf}{f_0}\right)$$

and

$$H_{de}(f) = \frac{1}{k}\left(\frac{1}{1 + jf/f_0}\right)$$

The scaling factor k is to be chosen so that the average power of the emphasized message signal is the same as that of the original message signal $m(t)$.

(a) Find the value of k that satisfies this requirement for the case when the power spectral density of the message signal $m(t)$ is

$$S_M(f) = \begin{cases} \dfrac{S_0}{1 + (f/f_0)^2}, & -W \leq f \leq W \\ 0, & \text{elsewhere} \end{cases}$$

(b) What is the corresponding value of the improvement factor I produced by using this pair of pre-emphasis and de-emphasis filters? Compare this ratio with that obtained in Example 2.6. The improvement factor I is defined by Equation (2.160).