① For coin A, the probability of landing on a head is equal to $\frac{1}{4}$ and the probability of landing on a tail is equal to $\frac{3}{4}$; coin B is a fair coin. Each coin is flipped four times. Let the random variable X denote the number of heads resulting from coin A, and Y denote the number of heads from coin B.

a) What is the probability that $X=Y=2$?

b) What is the probability that $X=Y$?

c) What is the probability that $X>Y$?

d) What is the probability that $X+5\leq 5$?

② X is a Gaussian random variable with mean 4 and variance 9, i.e., $X\sim N(4,9)$. Determine

a) $P(X>7)$

b) $P(0<X<9)$

③ Two random variables X and Y are distributed according to

$$f_{X,Y}(x,y)=\begin{cases}K(x+y) & 0\leq x,y\leq 1\\ 0 & \text{otherwise}\end{cases}$$

a-) Find K

b-) What is the probability that $X+Y>1$?

c-) Find $P(X>Y)$

d-) What is $P(X>Y\mid X+2Y>1)$?

e-) Find $P(X=Y)$

f-) What is $P(X>0.5\mid X=Y)$

g-) Find $f_X(x)$ and $f_Y(y)$

h-) Find $f_X(x\mid X+2Y>1)$ and $E[X\mid X+2Y>1]$

④ Let Θ be uniformly distributed on $[0,\pi]$, and let the random variables X and Y be defined by $X=\cos\Theta$ and $Y=\sin\Theta$. Show that X and Y are uncorrelated, but not independent.

⑤ Let X and Y be independent Gaussian random variables, each distributed according to $N(0,\sigma^2)$. Define $Z=X+Y$ and $W=2X-Y$, what is the joint PDF of Z and W? What is the covariance of Z and W?

⑥ Let the random process $X(t)$ be defined by $X(t)=A+Bt$ where A and B are independent random variables, each uniformly distributed on $[-1,1]$. Find $m_X(t)$ and $R_X(t_1,t_2)$. Is $X(t)$ WSS?