Last Updated: 23.02.2017 March 8, 2016

IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM I

Q1 |	Q2 |	Student ID:	Row No:
Q3 |			
Last Name:		First Name:	

I pledge my honour that I have not violated the honour code during this examination. Bu sınavda onur yasamızı ihlal etmediğime şerefim üzerine yemin ederim.	**Signature :**

1. (16 points) 12 females and 8 males are taking an exam in a classroom. After they finish with the exam they leave the room randomly one by one. What is the probability that the second students leaving the room is a female? (Write down all of the work. Use the methods of probability. Don't just write numbers!)

Solution :

Define the following events:

$$A = \text{the second student is a female,}$$
$$B = \text{the first student is a female.}$$

Need to compute

$$\mathbb{P}(A) = \mathbb{P}(A \cap B) + \mathbb{P}(A \cap B^c)$$

$$= \mathbb{P}(A|B)\mathbb{P}(B) + \mathbb{P}(A|B^c)\mathbb{P}(B^c)$$

$$= \frac{11}{19} \cdot \frac{12}{20} + \frac{12}{19} \cdot \frac{8}{20}$$

2. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let $\mathbb{P}(.)$ denote a probability measure on S.
(*Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

i.	$\mathbb{P}(A\|B) = 1 - \mathbb{P}(A^c\|B)$	Ⓣ	*F*
ii.	If $\mathbb{P}(A) = 0$ then $A = \emptyset$.	*T*	Ⓕ
iii.	If an experiment has only 2 outcomes, each outcome has probability 0.5.	*T*	Ⓕ
iv.	Mutually exclusive events are independent.	*T*	Ⓕ
v.	If $\mathbb{P}(A\|B) = \mathbb{P}(B\|A)$ then A and B are equally likely events	*T*	Ⓕ
vi.	If $\mathbb{P}(A \cap B \cap C) = \mathbb{P}(A)\mathbb{P}(B)\mathbb{P}(C)$ then A, B and C are independent.	*T*	Ⓕ
vii.	Probability mass function is always positive.	Ⓣ	*F*
viii.	The sum of all coefficients in the expansion of $(x + y)^{20}$ is 2^{20}.	Ⓣ	*F*
ix.	The coefficient of $x^3y^2z^3$ in the expansion of $(x + y + z)^8$ is 560.	Ⓣ	*F*
x.	Cumulative distribution function is always positive.	Ⓣ	*F*

Student's Name : ____________________

3. (16 points) In a game you roll a fair dice TWICE. After EACH roll if the dice shows an even number you receive an award of 100 TL, if the dice shows 1 or 3, you loose 150 TL and if you roll a 5 then you receive 200 TL. Let X be the random variable which records your TOTAL gain after you roll the dice twice.

   i. Write the probability mass function (PMF) of X.

   Solution :

   $S_X = \{-300, -50, 50, 200, 300, 400\}$ and

$$p(x) = \begin{cases} \frac{2}{6} \cdot \frac{2}{6} & , x = -300, \\ 2 \cdot \frac{3}{6} \cdot \frac{2}{6} & , x = -50, \\ 2 \cdot \frac{2}{6} \cdot \frac{1}{6} & , x = 50, \\ \frac{3}{6} \cdot \frac{3}{6} & , x = 200, \\ 2 \cdot \frac{3}{6} \cdot \frac{1}{6} & , x = 300, \\ \frac{1}{6} \cdot \frac{1}{6} & , x = 400, \\ 0 & , \text{else.} \end{cases}$$

   ii. What is the probability that you leave this game with a negative gain?

   Solution :

   A negative gain occurs when you end up loosing 300 TL or 50 TL . Hence the probability of a negative gain is

$$p(-300) + p(-50) = \frac{2}{6} \cdot \frac{2}{6} + 2 \cdot \frac{3}{6} \cdot \frac{2}{6} = \frac{4}{36} \cdot 4 = \frac{4}{9}.$$

Q4 |	Q5 |	Student ID:	Row No:
Q6 |	Q7 |		
Last Name:		First Name:	

4. (15 points)

i. A suitcase contains 6 distinct pairs of socks and 4 distinct pairs of pants. If a traveler randomly picks 2 pairs of socks and then 3 pairs of pants, how many ways can this be done?

Solution :

(Choose 2 pairs of socks out of 6) · (Choose 3 pairs of pants out of 4)

$$= \binom{6}{2} \cdot \binom{4}{3}$$

ii. Seven cards are drawn from an ordinary deck. In how many ways is it possible to draw 2 spades and 5 hearts?

Solution :

$$\binom{13}{2} \cdot \binom{13}{5}$$

iii. A multiple-choice test has 15 questions, each having four possible answers, of which only one is correct. If the questions are answered at random, what is the probability of getting all of them right?

Solution :

Probability that a randomly answered question being correct is $\frac{1}{4}$. So the probability that all questions being correct provided that all are answered randomly is

$$\left(\frac{1}{4}\right)^{15}$$

5. (14 points) From a faculty of six professors, six associate professors, ten assistant professors, and twelve instructors, a committee of size six is formed randomly. What is the probability that

i. there are exactly two professors on the committee;

Solution :

$$\frac{\binom{6}{2} \cdot \binom{28}{4}}{\binom{34}{6}}$$

ii. all committee members are of the same rank?

Solution :

We need to consider the case where all are prof. or all are assoc. prof. or all are assist. prof or all are instructors. So that probability is the sum

$$\frac{\binom{6}{6}}{\binom{34}{6}} + \frac{\binom{6}{6}}{\binom{34}{6}} + \frac{\binom{10}{6}}{\binom{34}{6}} + \frac{\binom{12}{6}}{\binom{34}{6}}$$

6. (14 points) Approximately 1% of women aged 40-50 have breast cancer. A woman with breast cancer has a 90% chance of a positive test from a mammogram, while a woman without has a 10% chance of a false positive result. What is the probability a woman has breast cancer given that she just had a positive test? Formulate and solve the problem in terms of conditional probabilities.

Solution :

Define the following events:

$$\begin{aligned} B &= \text{a woman has breast cancer} \\ T &= \text{she tests positive} \end{aligned}$$

We are given:

$$\begin{aligned} \mathbb{P}(B) &= 0.01 \\ \mathbb{P}(T|B) &= 0.9 \\ \mathbb{P}(T|B^c) &= 0.1 \end{aligned}$$

We need to find:

$$\begin{aligned} \mathbb{P}(B|T) &= \frac{\mathbb{P}(T|B) \cdot \mathbb{P}(B)}{\mathbb{P}(T)} \\ &= \frac{\mathbb{P}(T|B) \cdot \mathbb{P}(B)}{\mathbb{P}(T|B) \cdot \mathbb{P}(B) + \mathbb{P}(T|B^c) \cdot \mathbb{P}(B^c)} \\ &= \frac{(0.9) \cdot (0.01)}{(0.9) \cdot (0.01) + (0.1) \cdot (1 - 0.01)} \end{aligned}$$

by Bayes' Theorem.

Student's Name : ____________________

7. (15 points) An urn contains five red and seven blue balls. Suppose that two balls are selected at random and without replacement. Let A and B be the events that the first and the second balls are red, respectively. Are the events A and B independent ? Verify your answer by arguing on the values of $\mathbb{P}(B)$ and $\mathbb{P}(B|A)$.

Solution :

Given

$$A = \text{the first ball is red}$$
$$B = \text{the second ball is red.}$$

Then we have

$$\mathbb{P}(A) = \frac{5}{12}$$
$$\mathbb{P}(B|A) = \frac{4}{11}$$

$$\mathbb{P}(B) = \mathbb{P}(B|A) \cdot \mathbb{P}(A) + \mathbb{P}(B|A^c) \cdot \mathbb{P}(A^c)$$

$$= \frac{4}{11} \cdot \frac{5}{12} + \frac{5}{11} \cdot \frac{7}{12}$$

$$= \frac{5}{11}\left(\frac{4}{12} + \frac{7}{12}\right)$$

$$= \frac{5}{11} \cdot \frac{11}{12} = \frac{5}{12}$$

Since $\mathbb{P}(B|A) \neq \mathbb{P}(B)$, they are dependent.