March 16, 2015

IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM I

Q1 |	**Q2** |	**Student ID:**	**Row No:**
Q3 |			
Last Name:		**First Name:**	

1. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let $\mathbb{P}(.)$ denote a probability measure on S.
(*Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

i.	$\mathbb{P}(A \cup B \mid F) = \mathbb{P}(A \mid F) + \mathbb{P}(B \mid F)$	*T*	F
ii.	If $\mathbb{P}(A \mid B) = \mathbb{P}(A)$ then $\mathbb{P}(B \mid A) = \mathbb{P}(B)$.	T	*F*
iii.	$\mathbb{P}(A \mid B^c) = 1 - \mathbb{P}(A \mid B)$.	*T*	F
iv.	$\mathbb{P}(A^c \mid B) = 1 - \mathbb{P}(A \mid B)$.	T	*F*
v.	If $\mathbb{P}(A \mid B) = \mathbb{P}(B \mid A)$ then A and B are equally likely events	*T*	F
vi.	If A and B are independent, then $\mathbb{P}(A \mid B) = \mathbb{P}(A \mid B^c)$.	T	*F*
vii.	If $\mathbb{P}(A) = (\mathbb{P}(A))^2$ then A is independent of A.	T	*F*
viii.	The sum of all coefficients in the expansion of $(x+y)^{10}$ is 2^{10}.	T	*F*
ix.	The coefficient of x^2y^3z in the expansion of $(x+y+z)^6$ is 60.	T	*F*
x.	If $\mathbb{P}(A) \le \mathbb{P}(B)$ then $A \subset B$.	*T*	F

2. (16 points) On rainy days, Joe is late to work with probability 0.3; on nonrainy days, he is late with probability 0.1. With probability 0.7, it will rain tomorrow.

   i. Find the probability that Joe is early tomorrow.

   Solution :
   Define events

$$R : \text{the event that the weather will rain tomorrow,}$$
$$J : \text{the event that Joe is late to work.}$$

   Then

$$\begin{aligned} P(J^c) &= P(R) \cdot P(J^c|R) + P(R^c) \cdot P(J^c|R^c) \\ &= (0.7) \cdot (0.7) + (0.3) \cdot (0.9) \\ &= 0.49 + 0.27 = 0.76. \end{aligned}$$

   ii. Given that Joe was early, what is the conditional probability that it rained?

   Solution :
   By using Bayes' Theorem,

$$P(R|J^c) = \frac{P(J^c|R) \cdot P(R)}{P(J^c)} = \frac{(0.7) \cdot (0.7)}{0.76} = \frac{0.49}{0.76}$$

3. (16 points) Suppose B and C are two independent events, and A is an event depending on both B and C. If $\mathbb{P}(B) = 0.4$ and $\mathbb{P}(A \cap B | C) = 0.1$ then find the probability $\mathbb{P}(A | B \cap C)$.

Solution :

If B and C are two independent events, then

$$P(B \cap C) = P(B) \cdot P(C).$$

We have

$$0.1 = P(A \cap B | C) = \frac{P(A \cap B \cap C)}{P(C)}.$$

And so

$$P(A \cap B \cap C) = (0.1) \cdot P(C).$$

Therefore

$$P(A | B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{(0.1) \cdot P(C)}{P(B) \cdot P(C)} = \frac{0.1}{0.4} = 0.25.$$

Q4	Q5	Student ID:	Row No:
Q6	Q7		
Last Name:		First Name:	

4. (16 points)

   i. If a person visits doctor, suppose that the probability that he will have blood test is 0.44, the probability that he will have an X-ray is 0.24, the probability that he will have an MRI scan is 0.21, the probability that he will have blood test and an X-ray is 0.08, the probability that he will have blood test and an MRI is 0.11, the probability that he will have an X-ray and an MRI is 0.07, and the probability that he will have blood test, an X-ray, and an MRI is 0.03. What is the probability that a person visiting his doctor will have at least one of these things done to him/her?

Solution :
Define events

$$\begin{aligned} B &= \text{person has blood test}, \\ X &= \text{person has X-ray}, \\ M &= \text{person has MRI scan}. \end{aligned}$$

We are given

$$\begin{aligned} P(B) &= 0.44, & P(X) &= 0.24, & P(M) &= 0.21, \\ P(B \cap X) &= 0.08, & P(B \cap M) &= 0.11, & P(X \cap M) &= 0.07, \\ & & P(B \cap X \cap M) &= 0.03. \end{aligned}$$

Hence we have

$$\begin{aligned} P(B \cup M \cup X) &= P(B) + P(X) + P(M) - P(B \cap X) - P(B \cap M) - P(X \cap M) \\ &\quad + P(B \cap X \cap M) \\ &= 0.89 - 0.26 + 0.03 \\ &= 0.66. \end{aligned}$$

ii. An airport security has two checkpoints. Let A be the event that the first checkpoint is busy, and let B be the event the second checkpoint is busy. Assume that $\mathbb{P}(A) = 0.2$, $\mathbb{P}(B) = 0.3$ and $\mathbb{P}(A \cap B) = 0.06$. Find the probability that **neither** of the two checkpoints is busy.

Solution :
We are asked for the probability of the event

$$A^c \cap B^c.$$

Then

$$P(A^c \cap B^c) = P((A \cup B)^c) = 1 - P(A \cup B) = 1 - 0.44 = 0.56$$

where

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.2 + 0.3 - 0.06 = 0.44.$$

5. (13 points) A machinist produces 22 items during a shift. Three of the 22 items are defective and the rest are not defective. In how many different orders can the 22 items be arranged if all the defective items are considered identical and all the nondefective items are identical of a different class?

Solution :

$$\frac{22!}{3! \cdot 19!}$$

6. (13 points) A small town contains 4 people that repair cars. If 4 cars break down, what is the probability that exactly 2 of the repairers are called?

Solution :

R : choose 2 repairer out of 4 possible people: $\binom{4}{2}$

C : distribute these 2 repairers to 4 cars: $\binom{4}{1}\binom{3}{3} + \binom{4}{2}\binom{2}{2} + \binom{4}{3}\binom{1}{1}$

A : total number of ways that 4 repairers can be distributed to 4 cars: 4^4

Then

$$P(\text{exactly 2 of the repairers are called}) = \frac{|R| \cdot |C|}{|A|} = \frac{\binom{4}{2} \cdot \left[\binom{4}{1}\binom{3}{3} + \binom{4}{2}\binom{2}{2} + \binom{4}{3}\binom{1}{1}\right]}{4^4}$$

7. (16 points)

i. If a die is rolled 4 times, what is the probability that 6 comes up at least once?

Solution :
Define events

$$
\begin{aligned}
D_1 &= \text{no 6 appears when the dice is rolled only 1 time,}\\
D_4 &= \text{no 6 appears when the dice is rolled 4 times,}\\
A &= \text{6 appears at least once when the dice is rolled 4 times.}
\end{aligned}
$$

Then we see that

$$
\begin{aligned}
P(D_1) &= 1 - \frac{1}{6} = \frac{5}{6}\\
P(D_4) &= \left(\frac{5}{6}\right)^4\\
P(A) &= 1 - P(D_4) = 1 - \left(\frac{5}{6}\right)^4.
\end{aligned}
$$

ii. A group of 6 physicist and 6 mathematicians is randomly divided into 2 groups of size 6 each. What is the probability that both groups will have the same number of mathematicians?

Solution :

$$
\frac{\binom{6}{3}\cdot\binom{3}{3}\cdot\binom{6}{3}\cdot\binom{3}{3}}{\binom{12}{6}\cdot\binom{6}{6}}.
$$