

4.9 Expected Value of Sums of Random Variables

Question 12. If X and Y are two random variables, then what is

$$\mathbb{E}(X + Y) = ?$$

Theorem 4.9.1 i. $\mathbb{E}(X + Y) = \mathbb{E}(X) + \mathbb{E}(Y)$.
 ii. If X_1, X_2, \dots, X_n are random variables then $\mathbb{E}(X_1 + X_2 + \dots + X_n) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \dots + \mathbb{E}(X_n)$.

Proof. We postpone this proof to chapter 6. ■

■ **Example 4.33** Roll 5 dice. What is the expected value of the sum of the number showing on these dice?

X_1 = the number that the 1st dice show.

X_2 = the number that the 2nd dice show.

X_3 = the number that the 3rd dice show.

X_4 = the number that the 4th dice show.

X_5 = the number that the 5th dice show.

Then we are looking for

$$\mathbb{E}(X_1 + X_2 + X_3 + X_4 + X_5) = \mathbb{E}(X_1) + \mathbb{E}(X_2) + \mathbb{E}(X_3) + \mathbb{E}(X_4) + \mathbb{E}(X_5) = \frac{7}{2} + \frac{7}{2} + \frac{7}{2} + \frac{7}{2} + \frac{7}{2} = \frac{35}{2}.$$

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■ **Example 4.34** In a chocolate factory, each box of chocolate is filled with both white and dark chocolate balls. The number of chocolate balls are random for each box. It is known that the number of white balls are *Poisson*(5) and the number of dark balls are *Poisson*(7). Moreover each white ball weighs 2 gr, whereas each dark ball weighs 3 gr.

What is the expected weight of a full box if an empty box weighs 10 gr?

If we assume

$$X = \{\text{the number of white chocolate balls in a box}\}$$

and

$$Y = \{\text{the number of dark chocolate balls in a box}\}$$

then

$$X \sim \text{Poisson}(5) \quad , \quad Y \sim \text{Poisson}(7).$$

Let

$$Z = \{\text{total weight of a box}\}$$

Then

$$Z = 2X + 3Y + 10$$

and

$$\mathbb{E}(Z) = \mathbb{E}(2X + 3Y + 10) = 2\mathbb{E}(X) + 3\mathbb{E}(Y) + 10 = 2 \cdot 5 + 3 \cdot 7 + 10 = 41 \text{ gr.}$$

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