

UNDERGRADUATE
PROBABILITY:
LECTURE NOTES

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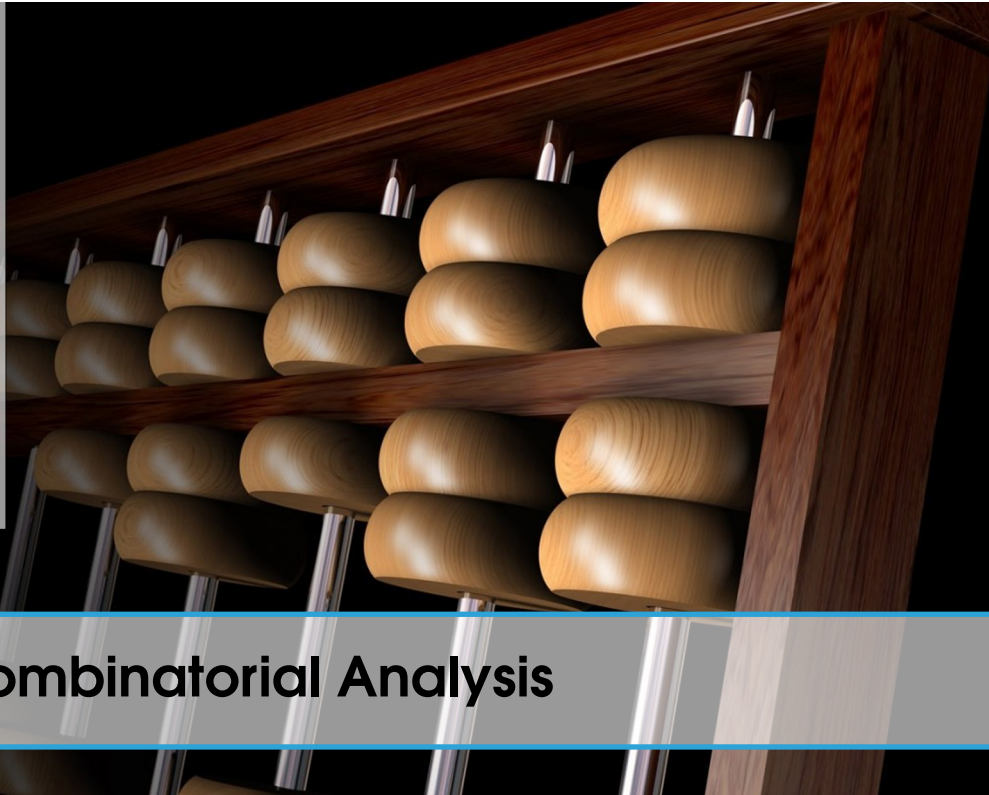
These notes are based on "A First Course in Probability Theory", 8th edition, by S. Ross.

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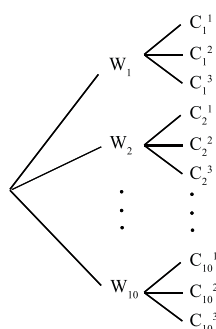


1 — Combinatorial Analysis

Before introduction of any notion about probability, it is important to go over some basics about counting principles.

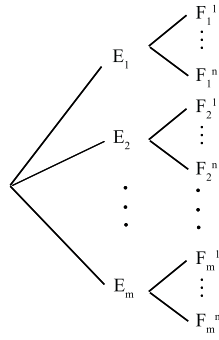
1.2 The Basic Principle of Counting

Question 1. A community consists of 10 women, each of whom has 3 children. How many children are present in total?



This simple diagram shows that there are $10 \cdot 3 = 30$ children. This is based on the following fact.

The Basic Principle of Counting: Suppose that two experiments are to be performed. Then if experiment 1 can result in any one of m possible outcomes and if, for each outcome of experiment 1, there are n possible outcomes of experiment 2, then together there are $m \cdot n$ possible outcomes.



The Generalized Basic Principle of Counting: Suppose that r experiments are to be performed. If

- 1st experiment may result in n_1 possible outcomes,
 2nd experiment may result in n_2 possible outcomes,
 ...
 r^{th} experiment may result in n_r possible outcomes,

then there is a total of

$$n_1 \cdot n_2 \cdot \dots \cdot n_r$$

possible outcomes of r experiments.

- **Example 1.1** i. How many different 2 digit-3 letter- 2 digit licence plates are possible? (No restriction) (e.g. 34 ABC 01)

$$\begin{array}{ccccccc} 10 & 10 & 29 & 29 & 29 & 10 & 10 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \text{Exp. 1} & \text{Exp. 2} & \text{E. 3} & \text{E. 4} & \text{E. 5} & \text{E. 6} & \text{E. 7} \end{array}$$

So there are $10^4 \cdot 29^3$ such plates.

- ii. How many different 2 digit-3 letter- 2 digit licence plates are possible if **no** repetition of letters is allowed?

$$\begin{array}{ccccccc} 10 & 10 & 29 & 28 & 27 & 10 & 10 \\ \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow & \uparrow \\ \text{Exp. 1} & \text{Exp. 2} & \text{E. 3} & \text{E. 4} & \text{E. 5} & \text{E. 6} & \text{E. 7} \end{array}$$

So there are $10^4 \cdot 29 \cdot 28 \cdot 27$ such plates. ■