April 10, 2019

IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM

Q1 |	**Q2** |	**Student ID:**	**Row No:**
Q3 |	**Q4** |	**Q5** | | **Q6** |	
Last Name:		**First Name:**	

I pledge my honour that I have not violated the honour code during this examination. Bu sınavda onur yasamızı ihlal etmediğime şerefim üzerine yemin ederim.	**Signature :**

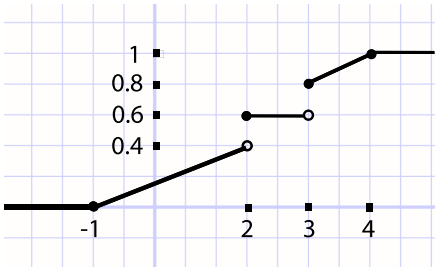
1. (10 points) Consider a Pois(λ) random variable X.

   i. (2 pts) What is the expectation of X?

   ii. (8 pts) Show/prove that the expectation of X is indeed what you claimed in part (i.).

2. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let $\mathbb{P}(.)$ denote a probability measure on S.

   The following graph is the graph of the cumulative distribution function of a random variable X. Answer questions regarding X with respect to this graph.



   i. $P(X \leq 2) = 0.6$. **T** **F**

   ii. $P(X < 2) = 0.6$. **T** **F**

   iii. $P(3 \leq X < 5) = 0.4$ **T** **F**

   iv. $P(2 < X \leq 3) = 0.4$ **T** **F**

   v. $P(X > 3) = 0.2$ **T** **F**

   vi. $\lim_{x \to \infty} \mathbb{P}(X \leq x) = 1$ **T** **F**

   vii. $\mathbb{P}(A^c|B) = 1 - \mathbb{P}(A|B)$. **T** **F**

   viii. If A and B are independent events then $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$ **T** **F**

   ix. The coefficient of x^3yz^5 in the expansion of $(x + y + z)^9$ is 72. **T** **F**

   x. If $p(\cdot)$ is a PMF, then $p(x) < 0$ is possibe for some x. **T** **F**

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3. (20 points) In a city of 100 people, some carry a gene which displays a certain property. People will be tested one by one whether they carry this gene. It is expected to observe the first positive test (first person with this gene) will show on the 20th person tested.

   i. (10 pts) What is the probability that the first person with this gene shows on the 5th test?

   ii. (10 pts) What is the probability that at most 1 of the people in this city has this gene?

4. (20 points)

   i. (10 pts) 40 students will be distributed to 4 rooms for an orientation program. They are allowed to choose any room they like, and they make the selection randomly. (So all students may choose the same room if they prefer.) What is the probability that each room contains the same number of students?

   ii. (10 pts) A committee of 3 students will be formed to join meetings of the University Senate. There are 10 candidates out of whom 6 are women and 4 are men. The selection will be made randomly. What is the probability that there are more women then men in the committee?

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5. (20 points) An international company hires people from Asia, Europe and Africa. When looked at the past hiring data, it is observed that 40% of the applicants from Asia, 50% of the applicants from Europe and 60% of the applicants from Africa has been hired in the past. For this year, there are 20 people from Asia, 30 people from Europe and 25 people from Africa applied for a single job opening. If one person is hired out of these applicants, what is the probability that the hired person is from Africa?

6. (20 points) Aslı plays a game by picking 2 random balls from an urn containing 2 red and 3 black balls. Aslı earns 10 TL for each red ball and loses 5 TL for each black ball in her selection. She plays this game only once. Let X denote her total earning after this game.

   i. (2 pts) What is S_X?

   ii. (8 pts) Write the PMF of X.

   iii. (6 pts) Find the expectation of X.

   iv. (4 pts) Find the variance of X.