

## IŞIK UNIVERSITY, MATH 230 FINAL EXAM

Q1	Q2	Student ID:		Row No:		
Q3	Q4	$\mathbf{Q5}$	Bonus			
Last Name:		First Name:				
I pledge my honour that I have not violated the honour code during this examination.			Signature :			
Bu sınavda onur yasamızı ihlal etmediğime						
şerefim uzerine yemin ederim.						

- 1. (20 points)
  - i. (10 pts) There are some black and some white balls in an urn. Balls are selected with replacement one by one for a certain number of trials. It is given that the expected number of white balls selected is 2 whereas the variance of number of white balls selected is 1.6 in this experiment. What is the probability that at most 1 white ball is selected?

ii. (10 pts) The number of engine failures a company may have is a Poisson distribution with an average of 6 per year. What is the probability that this company will have at most 2 failures this month?

2.

(10 points) Determine whether the following statements are True or False. Circle <b>T</b> or <b>F</b> . No explanation is required. Let $A$ , $B$ , $C$ and $A_i$ denote events in a sample space $S$ , let $\mathbb{P}(.)$ denote a probability measure on $S$ nad $X$ denote a random variable.		
(Note: A statement is assumed to be true if it is true in any possible case, and it is assumed to be false if it fails in at least one case.):		
<i>i</i> . A uniform random variable can be defined on the interval $(0, \infty)$ .	Т	F
<i>ii</i> . Exponential random variable has no memory.	Т	F
<i>iii</i> . Joint distribution can always be obtained from marginal distributions.	Т	F
iv. The PDF of a continuous random variable must be always less than 1.	Т	F
v. The PDF of a continuous random variable must be always greater than 0.	Т	F
vi. The PMF of a continuous random variable is always zero.	Т	F
vii. If X and Y are independent random variables, then $\mathbb{P}(X < a, Y > b) = \mathbb{P}(X < a)\mathbb{P}(Y > b)$	Т	F
<i>viii</i> . Cumulative distribution function is decreasing.	Т	F
<i>ix.</i> If X is a discrete random variable, then $S_X$ has to be a finite set.	Т	F
x. If A, B and C are 3 events such that $C \subseteq B$ , then $\mathbb{P}(A \cap B C) = \mathbb{P}(A B \cap C)$	Т	F

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Student's Name :\_\_\_\_

- 3. (20 points)
  - i. (10 pts) The water level of a lake changes randomly based on the amount of rain. It is measured that the level at any time is normally distributed with a variance of  $100 \text{ } \text{cm}^2$ . It is also known that the probability that the level is above 400 cm is the same as the probability that the level is below 400 cm. What is the probability that the water level is below 416 cm?

ii. (10 pts) The lifetime of a battery is exponentially distributed. It is measured that this battery survives over 100 hours with probability 0.2. What is the probability that such a battery dies within the first 20 hours of purchase?

4. (20 points) Let X be a continuous random variable with the density function

$$f(x) = \begin{cases} \frac{c}{x} & , 1 < x < 5, \\ 0 & , else. \end{cases}$$

i. (6 pts) What is the value of c?

ii. (7 pts) Find the expectation of X?

iii. (7 pts) Find the expectation of  $X^3$ ?

Student's Name :\_\_\_\_\_

5. (30 points) Assume that X and Y are two jointly continuous random variables with the joint density

$$f_{X,Y}(x,y) = \begin{cases} c \cdot xy^2 &, 0 < y < x < 1\\ 0 &, else, \end{cases}$$

where c is a constant.

i. (6 pts) What is the value of c?

ii. (8 pts) What is the marginal density of X?

iii. (8 pts) What is the probability  $\mathbb{P}(Y \ge X^2)$ ?

iv. (8 pts) What is the conditional probability  $\mathbb{P}(Y < 0.2 | X = 0.5)$ ?

Bonus Question (20 points) Consider the real number interval [0, 1]. Let us fix a point inside this interval. Say, this fixed point is  $\frac{1}{3}$ .



Now choose a uniform random number u from the interval [0, 1] and cut it into two pieces at this selected point u. Let us denote the length of the piece including the point  $\frac{1}{3}$  by L.



What is the expected length L?

Entry is area A under the standard normal curve from  $-\infty$  to z(A)

z .00 .01 .02 .01   .0 .5000 .5040 .5080 .512   .1 .5398 .5438 .5478 .551   .2 .5793 .5832 .5871 .591   .3 .6179 .6217 .6255 .622   .4 .6554 .6591 .6628 .6664   .5 .6915 .6950 .6985 .70   .6 .7257 .7291 .7324 .73   .7 .7580 .7611 .7642 .76   .8 .7881 .7910 .7939 .792   .9 .8159 .8186 .8212 .82	A												
z .00 .01 .02 .01   .0 .5000 .5040 .5080 .512   .1 .5398 .5438 .5478 .551   .2 .5793 .5832 .5871 .591   .3 .6179 .6217 .6255 .623   .4 .6554 .6591 .6628 .666   .5 .6915 .6950 .6985 .70   .6 .7257 .7291 .7324 .73   .7 .7580 .7611 .7642 .76   .8 .7881 .7910 .7939 .794   .9 .8159 .8156 .8129 .8159													
.0 .5000 .5040 .5080 .511   .1 .5398 .5438 .5478 .551   .2 .5793 .5832 .5871 .593   .3 .6179 .6217 .6255 .623   .4 .6554 .6591 .6628 .666   .5 .6915 .6950 .6985 .70   .6 .7257 .7291 .7324 .73   .7 .7580 .7611 .7642 .76   .8 .7881 .7910 .7939 .794	.04	.05	.06	.07	.08	.09							
.1 .5398 .5438 .5478 .551   .2 .5793 .5832 .5871 .591   .3 .6179 .6217 .6255 .624   .4 .6554 .6591 .6628 .6664   .5 .6915 .6950 .6985 .700   .6 .7257 .7291 .7324 .733   .7 .7580 .7611 .7642 .766   .8 .7881 .7910 .7939 .792   .9 .8156 .8156 .8126 .812	.5160	.5199	.5239	.5279	.5319	.5359							
.2 .5793 .5832 .5871 .59   .3 .6179 .6217 .6255 .629   .4 .6554 .6591 .6628 .660   .5 .6915 .6950 .6985 .70   .6 .7257 .7291 .7324 .73   .7 .7580 .7611 .7642 .76   .8 .7881 .7910 .7939 .792	7 .5557	.5596	.5636	.5675	.5714	.5753							
.3 .6179 .6217 .6255 .624   .4 .6554 .6591 .6628 .660   .5 .6915 .6950 .6985 .70   .6 .7257 .7291 .7324 .73   .7 .7580 .7611 .7642 .76   .8 .7881 .7910 .7939 .794	0 .5948	.5987	.6026	.6064	.6103	.6141							
.4 .6554 .6591 .6628 .660   .5 .6915 .6950 .6985 .70   .6 .7257 .7291 .7324 .73   .7 .7580 .7611 .7642 .76   .8 .7881 .7910 .7939 .794	.6331	.6368	.6406	.6443	.6480	.6517							
.5 .6915 .6950 .6985 .70 .6 .7257 .7291 .7324 .73 .7 .7580 .7611 .7642 .76 .8 .7881 .7910 .7939 .794 .8 .159 .8186 .8212 .82	.6700	.6736	.6772	.6808	.6844	.6879							
.6 .7257 .7291 .7324 .733 .7 .7580 .7611 .7642 .76 .8 .7881 .7910 .7939 .794 9 .8159 .8186 .8212 .82	9 .7054	.7088	.7123	.7157	.7190	.7224							
.7 .7580 .7611 .7642 .76 .8 .7881 .7910 .7939 .79 9 8159 8186 8212 82	57 .7389	.7422	.7454	.7486	.7517	.7549							
.8 .7881 .7910 .7939 .794	3.7704	.7734	.7764	.7794	.7823	.7852							
0 8150 8186 8717 87	.7995	.8023	.8051	.8078	.8106	.8133							
.9 .0139 .0100 .0212 .02	38 .8264	.8289	.8315	.8340	.8365	.8389							
1.0 .8413 .8438 .8461 .84	35 .8508	.8531	.8554	.8577	.8599	.8621							
1.1 .8643 .8665 .8686 .87	8729	.8749	.8770	.8790	.8810	.8830							
1.2 .8849 .8869 .8888 .89	.8925	.8944	.8962	.8980	.8997	.9015							
1.3 .9032 .9049 .9066 .90	32 .9099	.9115	.9131	.9147	.9162	.9177							
1.4 .9192 .9207 .9222 .92	.9251	.9265	.9279	.9292	,9306	.9319							
1.5 .9332 .9345 .9357 .93	.9382	.9394	.9406	.9418	.9429	.9441							
1.6 .9452 .9463 .9474 .94	34 .9495	.9505	.9515	.9525	.9535	.9545							
1.7 .9554 .9564 .9573 .95	.9591	.9599	.9608	.9616	.9625	.9633							
1.8 .9641 .9649 .9656 .96	54 .9671	.9678	.9686	.9693	.9699	.9706							
1.9 .9713 .9719 .9726 .97	32 .9738	.9744	.9750	.9756	.9761	.9767							
2.0 .9772 .9778 .9783 .97	38 .9793	.9798	.9803	.9808	.9812	.9817							
2.1 .9821 .9826 .9830 .98	34 .9838	.9842	.9846	.9850	.9854	.9857							
2.2 .9861 .9864 .9868 .98	71 .9875	.9878	.9881	.9884	.9887	.9890							
2.3 .9893 .9896 .9898 .99	01 .9904	.9906	.9909	.9911	.9913	.9916							
2.4 .9918 .9920 .9922 .99	.9927	.9929	.9931	.9932	.9934	.9936							
2.5 .9938 .9940 .9941 .99	43 .9945	.9946	.9948	.9949	.9951	.9952							
2.6 .9953 .9955 .9956 .99	57 .9959	.9960	.9961	.9962	.9963	.9964							
2.7 .9965 .9966 .9967 .99	58 .9969	.9970	.9971	.9972	.9973	.9974							
2.8 .9974 .9975 .9976 .99	77 .9977	.9978	.9979	.9979	.9980	.9981							
2.9 .9981 .9982 .9982 .99	.9984	.9984	.9985	.9985	.9986	.9986							
3.0 .9987 .9987 .9987 .99	.9988	.9989	.9989	.9989	.9990	.9990							
3.1 .9990 .9991 .9991 .99	91 .9992	.9992	.9992	.9992	.9993	.9993							
3.2 .9993 .9993 .9994 .99	94 .9994	.9994	.9994	.9995	.9995	.9995							
3.3 .9995 .9995 .9995 .99	96 .9996	.9996	.9996	.9996	.9996	.9997							
3.4 .9997 .9997 .9997 .99	97 .9997	.9997	.9997	.9997	.9997	.9998							