April 10, 2018

IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM

Q1 |	**Q2** |	**Student ID:**	**Row No:**
Q3 |	**Q4** |	**Q5** | | **Q6** |	**TOTAL** |
Last Name:		**First Name:**	

1. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let $\mathbb{P}(.)$ denote a probability measure on S.
(*Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

   i. Empty event is independent from any other event. *T* *F*

   ii. Probability can be a negative number. *T* *F*

   iii. For any 2 events A and B, it is always true that $\mathbb{P}(A|B) \leq \mathbb{P}(B)$ *T* *F*

   iv. If $B \subseteq A$ then $\mathbb{P}(A|B) = 1$. *T* *F*

   v. CDF is a decreasing function. *T* *F*

   vi. If $p(x)$ is a PMF, then $p(x) \leq 1$. *T* *F*

   vii. Expectation of a random variable can be negative. *T* *F*

   viii. For any 2 events A and B, $\mathbb{P}(A|B) = 1 - \mathbb{P}(A^c|B)$. *T* *F*

   ix. The coefficient of x^2y^4z in the expansion of $(x+y+z)^7$ is 210. *T* *F*

   x. In an equally likely sample space, each outcome has the same probability as another. *T* *F*

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2. (16 pts) Three men and three women sit in a row of six seats. Find the probability that

   (a) the three men sit together;

   (b) the men and women sit in alternate seats.

3. (20 points) A driver will be hired for a job. The applicants filled an application form and stated their experience in terms of years. Out of 10 applicants, 3 have only one year of experience, 5 have two years of experience and 2 have three years of experience. If a driver is selected randomly from these applicants and if X is the number of years of experience of the selected driver, answer the following questions accordingly.

   i. Write the set S_X.

   ii. What is the PMF (probability mass function) of X?

   iii. Find the driver's expected number of years of experience.

   iv. What is the driver's variance of number of years of experience?

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4. (20 points) When a cellphone dials a number, the signal goes through one of 3 towers, namely A, B or C. The tower A has a probability to transmit the signal successfully 0.8, whereas the tower B has the probability 0.7 and the tower C has the probability 0.5 to transmit the signal successfully. A cellphone decides which tower to connect randomly. According to the statistics, tower A is chosen 60% of the time by the cellphones, tower B is chosen 30% of the time and tower C is chosen 10% of the time.

   i. When you dial a number, what is the probability that your call will be successful?

   ii. When you dial a number and it goes through succesfully, what is the probability that the cellphone connected to the tower A?

5. (16 pts) Suppose X is a random variable with the CDF

$$F(x) = \begin{cases} 0 & , x < -2 \\ 0.2 & , -2 \leq x < -1 \\ 0.3 & , -1 \leq x < 0 \\ 0.6 & , 0 \leq x < 2 \\ 1 & , x \geq 2. \end{cases}$$

i. Write the set S_X.

ii. What is the probability $\mathbb{P}(X^2 - 1 \geq 0)$?

iii. What is the probability $\mathbb{P}(X > 1)$?

iv. Find the expected value $\mathbb{E}(2^X)$?

6. (18 pts) Assume A, B and C are three independent events with the given probabilities

$$\mathbb{P}(A^c) = 0.3 \quad , \quad \mathbb{P}(B|A) = 0.5 \quad , \quad \mathbb{P}(C^c|B) = 0.1.$$

   i. Find the probability that at least A or B or C occurs.

   ii. Are A and B mutually exclusive? State your reason.

   iii. What is the conditional probabilty $\mathbb{P}(A|B \cap C)$?