May, 2018

IŞIK UNIVERSITY, MATH 230 FINAL EXAM

Q1 |	**Q2** |	**Student ID:**	**Row No:**
Q3 |	**Q4** |	**Q5** | | **Q6** |	**TOTAL** |
Last Name:		**First Name:**	

I pledge my honour that I have not violated the honour code during this examination. Bu sınavda onur yasamızı ihlal etmediğime şerefim üzerine yemin ederim.	**Signature :**

1. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let $\mathbb{P}(.)$ denote a probability measure on S.
(*Note: A statement is assumed to be true if it is true* **in any possible case***, and it is assumed to be false if it fails in at least one case.*):

i. An exponential random variable has no memory. *T F*

ii. For a random variable X, we have $\mathbb{E}(X^2) < (E(X))^2$. *T F*

iii. Variance can be negative for some random variables. *T F*

iv. Continuous random variables have non-zero PMFs. *T F*

v. CDF is a decreasing function. *T F*

vi. If $f(x)$ is a PDF, then the values of $f(x)$ can be greater than 1. *T F*

vii. For a discrete random variable X and $a \in S_X$, always $\mathbb{P}(X < a) = \mathbb{P}(X \leq a)$. *T F*

viii. For a continuous random variable X and $a \in S_X$, always $\mathbb{P}(X < a) = \mathbb{P}(X \leq a)$. *T F*

ix. If Z is a standard Gaussian then $\mathbb{P}(Z < 0) = \mathbb{P}(Z > 0)$. *T F*

x. If X, Y are discrete and independent random variables ,
then we have $\mathbb{P}(X = 2, Y = 3) = \mathbb{P}(X = 2)\mathbb{P}(Y = 3)$. *T F*

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2. (16 pts) Data shows that 20% of the e-mail a person receives is spam. We also know that 70% of spam e-mails contain words such as lottery, win, notification etc while only 5% of non-spam e-mails contain such words. Given that a message includes one of these words, what is the probability that it is spam? **Formulate and solve the problem using conditional probabilities.**

3. (16 points) Suppose X and Y are jointly continuous random variables with the joint bf <u>CDF</u>

$$F(x) = \begin{cases} \frac{1}{2}x^2y + \frac{1}{2}xy^3 & , 0 \leq x \leq 1, \quad 0 \leq y \leq 1, \\ \frac{1}{2}x^2 + \frac{1}{2}x & , 0 \leq x \leq 1, \quad y \geq 1, \\ \frac{1}{2}y + \frac{1}{2}y^3 & , 0 \leq y \leq 1, \quad x \geq 1, \\ 0 & , \text{otherwise.} \end{cases}$$

Find the probability $\mathbb{P}(X < Y)$.

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4. (20 points)

   i. Suppose that the number of rain drops in a certain area in a mild rain is a Poisson random variable with an average of 120 drops in 2 minutes. What is the probability of at most 3 drops in 3 minutes on the same area?

   ii. An operator at a call center is asked to dial random phone numbers to sell a product until someone answers. Statistics shows that 1 out of every 5 calls answer these type of promotional calls. What is the probability that the operator will have to dial at least 8 times?

5. (20 pts)

i. Suppose X is a random variable with the distribution $X \sim N(2, 9)$ and $Y = 2X - 1$. Find the mean and variance of Y. What is the probability $\mathbb{P}(Y < 2)$?

ii. Suppose that waiting time in a que of a call center is an exponential random variable with an average waiting time of 5 minutes. What is the probability that you have to wait at least 10 minutes when you call this call center?

6. (18 pts) Suppose X and Y are jointly continuous random variables with the joint PDF

$$f(x) = \begin{cases} c\,x^2 y & , 0 \le y \le x, \quad 0 \le y \le 1, \\ 0 & , \text{otherwise}, \end{cases}$$

where c is a constant.

i. What is the value of c?

ii. Are X and Y independent?

iii. What is the conditional density $f_{X\,|\,Y}(x\,|\,1/2)$?