

IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM

Q1	Q2	Student ID:	Row No:
Q3	Q4		
Last Name:		First Name:	

1. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A , B , and A_i denote events in a sample space S and let $\mathbb{P}(\cdot)$ denote a probability measure on S .
(Note: A statement is assumed to be true if it is true in any possible case, and it is assumed to be false if it fails in at least one case.);

- i.* For any random variable X , we have $\mathbb{E}(X^2) \geq (\mathbb{E}(X))^2$. *T F*
- ii.* Probability of the empty set must be always zero. *T F*
- iii.* If A and B are any two events then $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$. *T F*
- iv.* If A and B are any two events then $\mathbb{P}(A|B)\mathbb{P}(B) = \mathbb{P}(B|A)\mathbb{P}(A)$. *T F*
- v.* If $\mathbb{P}(A|B) = \mathbb{P}(B|A)$ and $\mathbb{P}(A \cap B) \neq 0$ then A and B are equally likely events. *T F*
- vi.* If $p(x)$ is a PMF, then $p(x) \geq 0$. *T F*
- vii.* If $\mathbb{P}(A) = (\mathbb{P}(A))^2$ then A is independent of A . *T F*
- viii.* The sum of all coefficients in the expansion of $(x + y)^{10}$ is 2^{10} . *T F*
- ix.* The coefficient of $x^3y^2z^2$ in the expansion of $(x + y + z)^7$ is 210. *T F*
- x.* If A and B are mutually exclusive events then they are independent. *T F*

2. (15 points) In a study group of 20 males, men are tested for their systolic blood pressures. According to this study 4 of the males have blood pressure about 110, 8 have their blood pressure about 120, 3 have 130 and the rest have 140. Let X be the random variable denoting the systolic blood pressure of a randomly chosen male from this study group.

i. What is the PMF (probability mass function) of X ?

ii. Find the expected systolic blood pressure of this study group.

iii. What is the variance of systolic blood pressure of this study group.

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3. (13 points) Approximately 1% of women aged 40-50 have breast cancer. A woman with breast cancer has a 90% chance of a positive test from a mammogram, while a woman without has a 10% chance of a false positive result. What is the probability a woman has breast cancer given that she just had a positive test?

4. (10 points) For 3 events A , B and C , we have $\mathbb{P}(A \cup B|C) = 0.5$ and $\mathbb{P}(A \cap B|C) = 0.1$. Moreover, it is known that probability of A given C and probability of B given C are equal. What is the probability $\mathbb{P}(A \setminus B|C)$? (Do not use venn diagrams. Use proper mathematical equalities to receive credit.)

Q5	Q6	Student ID:	Row No:
Q7	Q8		
Last Name:		First Name:	

5. (12pts)

- (a) We randomly draw 13 cards from a shuffled standard deck of 52 playing cards. This is called a “hand”. **How many hands** exist where there is at least one spade ? (In a deck, there are 13 spades, 13 diamonds, 13 hearts and 13 clubs)
- (b) We randomly draw 13 cards from a shuffled standard deck of 52 playing cards. This is called a “hand”. **How many hands** exist where there is at most one spade ?
- (c) How many numbers can we construct using any of the numerals 1,2,3,4,5 and 6 at most once ?

6. (15pts)

- (a) An urn contains 5 white and 10 black balls. We draw two balls one after the other randomly from the urn by replacement (that is, after we withdraw the ball, we check its colour and then put the ball back into the urn before we do the next draw). What is the probability that the first ball we draw is white and the second is black ?
- (b) Same question, what is the probability that one of the balls that we draw is white and the other is black (in any order)
- (c) We simultaneously draw 5 balls from this urn. What is the probability that two of them are white and three of them are black ?

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7. (15pts) (This question should be formulated and solved using conditional probability approach.) In an urn there are 100 dice. 25 of these 100 dice are biased (not fair) in a way that they come up “6” with probability $1/2$. The rest are fair dice. We randomly draw a die from the urn and roll it. What is the probability that it is 6 ?

8. (10pts) The Parker family has 2 children. Define events:

Event A: *The parker family has two children having different sexes (one is a girl, one is a boy)*

Event B: *The parker family has at least one daughter*

Find whether the events A and B are independent MATHEMATICALLY. If you just say “independent” or “dependent” without the mathematical analysis necessary (or by doing the wrong math), you will not get any points