IŞIK UNIVERSITY, MATH 230 FINAL EXAM

Q1	Q2	Student ID:	Row No:	
Q3	Q4			
Last Name:		First Name:		

1.	 (10 points) Determine whether the following statements are True or False. Circle T or F. No explanation is required. Let A, B, and A_i denote events in a sample space S and let P(.) denote a probability measure on S. (Note: A statement is assumed to be true if it is true in any possible case, and it is assumed to be false if it fails in at least one case.): 					
	i.	Variance of a random variable may be a negative number.	T	F		
	ii.	Expectation of a random variable may be a negative number.	T	F		
	iii.	If X and Y are independent then $\mathbb{P}(X < 2, Y < 3) = \mathbb{P}(X < 2)\mathbb{P}(Y < 3)$.	T	F		
	iv.	If X is a constant random variable then its variance is zero.	Т	F		
	v.	If the joint density, $f_{X,Y}$, of X and Y is given, then one can find $\mathbb{E}(X)$.	T	F		
	$v\iota.$	If X is a Gaussian with variance σ^2 and $\mathbb{P}(X > a) = \mathbb{P}(X < a)$ then $Y = (X - a)/\sigma^2$ is a standard Gaussian.	Т	F		
	vii.	If $A \subset B$ and $\mathbb{P}(A) = 1$ then $\mathbb{P}(B^c) = 0$.	Т	F		
	viii.	If $\mathbb{P}(A \cup B) + \mathbb{P}(A \cap B) > 1$ then A and B cannot be disjoint.	Т	F		
	ix.	If marginal densities of X and Y are known then their joint density can be written.	Т	F		
	x.	For any random variable X, we have $\mathbb{E}(X^2) - \mathbb{E}(2X) + 1 \ge 0$.	T	F		

2. (35 points) Consider the joint pdf $f_{X,Y}$ of random variables X and Y which is given by

$$f_{X,Y}(x,y) = \begin{cases} c \cdot y & , y < x < 2-y, \quad y > 0, \\ 0 & , \text{ otherwise,} \end{cases}$$

where c is a constant.

i. Find the value of the constant c.

ii. Find the marginal pdf, f_Y , of Y.

iii. What is the probability $\mathbb{P}(X - Y > 1)$?

iv. What is the probability $\mathbb{P}(Y > 1/2)$?

v. What is the expectation $\mathbb{E}(Y)$?

vi. Find the conditional pdf $f_{X|Y}(x|1/2)$.

vii. Find the conditional probability $\mathbb{P}(X < 1|Y = 1/2)$.

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3. (7 points) Suppose X is a Gaussian random variable with mean 30 and variance 400. What is the probability $\mathbb{P}(X > -5)$?

4. (7 points) It is assumed that a particular ram module has the exponential lifetime with a mean of 1000 hours. What is the probability that a ram module of this type dies within the first 200 hours of its production?

$\mathbf{Q5}$	$\mathbf{Q6}$	Student ID:	Row No:
Q7	Q8	. S	
Last Name:		First Name:	

5. (10pts) We have two events A and B. We know that $\mathbb{P}(A) = 1/2$ and $\mathbb{P}(A|B) = \mathbb{P}(B|A) = 1/4$. Find $\mathbb{P}(A^c|B^c)$ and $\mathbb{P}(A|B^c)$.

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6. (11pts) Data shows that 20% of the e-mail a person receives is spam. We also know that 70% of spam e-mails contain words such as "lottery", "win", "notification" etc while only 5% of non-spam e-mails contain such words. Given that a message includes one of these words, what is the probability that it is spam? Formulate and solve the problem using conditional probabilities.

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7. (10pts) In an exam a student is asked a total of 10 questions. His probability of answering any one of the question correctly is 0.2, independently of any of the other questions. Find the probability that he answers 8 or more questions correctly. If X is the number of correct answers, what is the expected value of X?

8. (10pts) The number of flaws in a fibre optic cable follows a Poisson distribution. The average number of flaws in 50m of cable is 1.2. What is the probability of exactly three flaws in 150m of cable?