## IŞIK UNIVERSITY, MATH 230 FINAL EXAM

Q1	Q2	Student ID:			Row No:
Q3	Q4				
Last Name:		First Name:			
I pledge my hon- the honour code	Signature	:			
Bu sınavda onur yasamızı ihlal etmediğime serefim üzerine vemin ederim			-		
Last Name: I pledge my hone the honour code Bu sınavda onur şerefim üzerine y	Signature	:			

1. (15 points) 70% of the light aircraft that disappear while in flight are discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 90% of the aircraft **not** discovered do **not have** any emergency locator. Suppose that a light aircraft has disappeared. If it has an emergency locator, what is the probability that it will be discovered?

Circle sample ( <i>Note</i> and it	<b>T</b> or <b>F</b> . No explanation is required. Let $A$ , $B$ , and $A_i$ denote events in a e space $S$ and let $\mathbb{P}(.)$ denote a probability measure on $S$ . : A statement is assumed to be true if it is true <b>in any possible case</b> , is assumed to be false if it fails in at least one case.):		
i.	If the joint density function of two dependent random variables is given, then the marginal densities can be calculated.	T	F
ii.	If two random variables X and Y are independent, then for the densities we have $f_X(x)f_Y(y) = f_{X,Y}(x,y)$ .	T	F
iii.	Joint CDF of two random variables can be negative.	T	F
iv.	If X and Y are 2 jointly continuous random variables then their joint PDF satisfies $0 \le f_{X,Y}(x,y) \le 1$ .	T	F
v.	If X and Y are 2 discrete random variables then their joint PMF satisfies $0 \le p_{X,Y}(x,y) \le 1$ .	Т	F
vi.	If X and Y are 2 discrete random variables then their joint PMF $p_{X,Y}$ is the product of their marginal PMFs.	Т	F
vii.	For any two jointly continuous X and Y, we have $f_{X Y} = f_X$ .	Т	F
viii.	For any two jointly continuous X and Y, we have $\int_{-\infty}^{\infty} f_{X,Y}(x,y) dy = f_X(x).$	Т	F
ix.	If the joint CDF of X and Y is given by $F_{X,Y} = (1 - e^{-x})(1 - e^{-y})$ for $x > 0, y > 0$ , then the marginal PDF of X is $f_X(x) = e^{-x}$ for $x > 0$ .	Т	F
x.	If X and Y are 2 discrete random variables then $\sum_{x} \sum_{y} p_{X,Y}(x,y) = 1.$	Т	F

2. (10 points) Determine whether the following statements are True or False.

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3. (15 points) Suppose

$$f_{X,Y}(x,y) = \begin{cases} 3xy & , 0 < y < -\frac{x}{2} + 1, \quad x > 0, \\ 0 & \text{else} \end{cases}$$

is the joint density function of the random variables X and Y.

i. Find the probability  $\mathbb{P}(X > Y)$ .

ii. Find the marginal density,  $f_X(x)$ , of X.

iii. Find the conditional density  $f_{Y|X}(y|1)$ .

i. The height of pine trees of age 10 are estimated to be a normal distribution with the mean 200 cm and variance 64 cm. What is the probability a pine tree of age 10 grows more than 210 cm?

ii. Suppose that the amount of time a repairman needs to fix a furnace is uniformly distributed between 1.5 and 4 hours. What is the probability that fixing a certain furnace will last less than 2 hours?

$\mathbf{Q5}$	Q6	Student ID:	Row No:	
$\mathbf{Q7}$	Q8	Bonus		
Last Name:		First Name:		

## 5. (12 points)

i. A random sample of 45 students from different universities were selected randomly and asked whether they are happy with their social activities on campus. The responses of 32 were negative. If Ahmet, Ali and Zeynep were among those questioned, what is the probability that all three of them gave negative responses?

ii. In a class, 11 of the 25 students dislike rock music, eight like rock music, and the rest are indifferent. A random sample of five students is selected for an interview. What is the probability that all of them have the same opinion about rock music ?

6. (15 points) Let X be a continuous random variable with probability density

$$f(x) = c e^{-3x} , x > 0$$

for some constant c.

i. Find the value of c.

ii. What is the probability  $\mathbb{P}(1 < X < 3)$ ?

iii. Fin the expectation of X.

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7. (10 points) Urn I contains three "25 kuruş" and four "50 kuruş", urn II contains two "25 kuruş" and five "50 kuruş", and urn III contains three "25 kuruş" and one "50 kuruş". One coin is selected at random from each urn. If two of the three coins are "50 kuruş" coins, what is the probability that the coin selected from urn I is a "50 kuruş" coin ?

## 8. (11 points)

i. Suppose you and your friend flip coins and play a game. Each of you flip a fair coin. If the upper faces match (they are the same), and they are tails then you win \$1. If the upper faces match and they are heads, you win \$2. You loose \$1.50 if the coins do not match (if one is head and the other is tail). What is your expected winnings ?

DNUS (10 points) Julia is interested in two games, Beno and Kolita. To play Kolita, she buys a ticket for \$1, draws a ball at random from a box of 100 balls numbered 1 to 100. If the ball drawn matches the number on her ticket, she wins \$75; otherwise, she loses. To play Beno, Julia bets \$1 on a single number that has a 25% chance to win. If she wins, they will return her dollar plus two dollars more; otherwise, they keep the dollar. Calculate the expected value and variances of both games.