April 20, 2015

IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM II

Q1 |	**Q2** |	**Student ID:**	**Row No:**
Q3 |	**Q4** |		
Last Name:		**First Name:**	

1. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let $\mathbb{P}(.)$ denote a probability measure on S.
(*Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

i. If f is the PDF of a random variable, then $f(x) \geq 0$ for all x. T F

ii. If f is the PDF of a random variable, then $\int_{-\infty}^{\infty} f(x)dx = 1$. T F

iii. If X is a continuous random variable, then $\mathbb{P}(X = a) = 0$ for any a. T F

iv. If X is a discrete random variable, then $\mathbb{P}(X = a) = 0$ for any a. T F

v. Binomial random variables can be approximated by Poisson random variables T F

vi. For any random variable X, we have $Var(X) \geq 0$. T F

vii. For any random variable X, we have $\mathbb{E}(X) \geq 0$. T F

viii. For any random variable X, we have $\mathbb{E}(X - \mathbb{E}(X)) = 0$. T F

ix. For any random variable X, we have $\mathbb{E}(aX + b) = a\mathbb{E}(X) + b$. T F

x. For any random variable X, we have $Var(aX + b) = a^2Var(X) + b$.. T F

2. (15 points) X is a continuous random variable with the probability density function (PDF)

$$f(x) = \begin{cases} c(x - x^2) & \text{if } 0 \le x \le 1 \\ 0 & \text{else} \end{cases}$$

i. Find c.

ii. What is $\mathbb{E}(X)$?

iii. What is $Var(X)$?

3. (16 points) X is a continuous random variable with the cumulative distribution function (CDF)

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 - e^{-x} & \text{if } x \geq 0. \end{cases}$$

i. What is the PDF of X?

ii. What is $\mathbb{P}(-2 \leq X \leq 2)$?

4. (14 points)

In a chocolate factory, each box of chocolate is filled with both white and dark chocolate balls. The number of chocolate balls are random for each box. It is known that the number of white balls are $Poisson(5)$ and the number of dark balls are $Poisson(7)$. Moreover each white ball weighs 2 gr, whereas each dark ball weighs 3 gr.

What is the expected weight of a full box if an empty box weighs 10 gr?
(**You have to set up all random variables to receive credit.**)

Q5 |	Q6 |	Student ID:	Row No:
Q7 |	Bonus |		
Last Name:		First Name:	

5. (16 points)

   a. The probability of a student passing this course is 0.82. If there are 8 students in the class, find the probability that

      i. all 8 students pass,

      ii. no student passes,

      ii. at least of them 6 pass this course.

   b. Suppose that 2.5% of the population in a town are foreigners. Using Poisson random variable, find the probability that, in a theater of this town with 80 random viewers, there are at least two foreigners.

6. (14 points) Suppose the sample space of a discrete random variable is $\{0, 1, 2, 3, 4\}$ and its probability mass function is

$$f(x) = \begin{cases} 0.1 & \text{if } x = 1 \\ 0.2 & \text{if } x = 2 \\ 0.3 & \text{if } x = 3 \\ 0.4 & \text{if } x = 4 \\ 0 & \text{else.} \end{cases}$$

What is its cumulative distribution function?

7. (15 points) Let X have the following PMF:

$$p(n) = \binom{5}{n}(1/2)^5.$$

Calculate the variance of X. (Hint: X takes only a certain number of values because of the combination term)

BONUS (10 points) Suppose you go to a casino with exactly \$63. At this casino, the only game is roulette and the only bets allowed are red and green. In addition, the wheel is fair so that P[red] = P[green] = 1/2. You have the following strategy: First, you bet \$1. If you win the bet, you quit and leave the casino with \$64. If you lose, you then bet \$2. If you win, you quit and go home. If you lose, you bet \$4. In fact, whenever you lose, you double your bet until either you win a bet or you lose all of your money. However, as soon as you win a bet, you quit and go home. Let Y equal the amount of money that you take home. Find the PMF and the expected value of Y.

Hint: Think about how many different values Y can take: very few actually.