May 16, 2014

IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM II

Q1 |	**Q2** |	**Student ID:**	**Row No:**
Q3 |		**Bonus #1** |	
Last Name:		**First Name:**	

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1. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let $\mathbb{P}(.)$ denote a probability measure on S.
(*Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

i. If X is a continuous random variable and a is any point in the range of X, then $\mathbb{P}(X = a) = 0$.. $T \quad F$

ii. If X is a continuous random variable and f_X denotes its PDF, then f_X can take values greater then 1. $T \quad F$

iii. If X is a discrete random variable and P_X denotes its PMF, then P_X can take values greater then 1. $T \quad F$

iv. If X is a standard Gaussian, then $\mathbb{P}(X \geq 0) = 1/2$. $T \quad F$

v. $Var(X)$ is always non-negative. $T \quad F$

vi. $Var(aX + b) = a^2 Var(X) + b$. $T \quad F$

vii. If X is a continuous random variable then $\mathbb{P}(X \leq c) = \mathbb{P}(X < c)$. $T \quad F$

viii. If $\mathbb{E}(X) = \mathbb{E}(Y)$ then X and Y are the same random variable. $T \quad F$

ix. $\displaystyle\int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{2\pi}$ $T \quad F$

x. If two random variables have the same PDF, then their expectations and variances are the same. $T \quad F$

2. (20 points)

   i. The daily number of arrivals to an emergency room is a Poisson random variable with a mean of 100 people per day. Find probability that 112 or more people arrive in 3 days.

   ii. On average, every 3 out of 100 phone calls turn out be wrong numbers. What is the probability that if you receive 10 phone calls, 4 of them will be wrong numbers? (receive: "almak, erişmek")

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Student's Name :______________________

3. (20 points) The random variable X has $\mathbb{E}(X) = 1.6$ and probability mass function (PMF)

$$p(x) = \begin{cases} a & , x = 0, \\ b & , x = 2, \\ 0 & , otherwise \end{cases}$$

where a and b are two constants.

i. Find a and b.

ii. Write its cumulative distribution function (CDF).

iii. Find $Var(X)$.

iv. Find the standard deviation σ_X.

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Bonus #1 : (25 points) Red Kid questions Dalton brothers; Joe, William, Jack and Averell.

If you ask a question to Joe, he tells the truth with probability 1/4.
If you ask a question to William, he tells the truth with probability 2/4.
If you ask a question to Jack, he tells the truth with probability 3/4.
If you ask a question to Averell, he always tells the truth.

Red Kid randomly picks one of them and asks a question. If the answer is the truth, what is the probability that it was Joe Dalton?

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Q4 |	Q5 |	Student ID:	Row No:
Q6 |		Bonus #2 |	
Last Name:		First Name:	

4. (14 points) As you know from the classes, the PDF for the exponential random variable is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

If X is an exponential random variable and $Y = X^3$, then what is the PDF of Y ?

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5. (20 points)

i. The duration of a certain type of integrated circuits is modeled by an exponential distribution with mean 2000 days. What is the probability such an integrated circuit lasts more than 10 years?
(duration: ”süre”, integrated circuit: ”entegre devre”)

ii. A collection of thermometers have an average reading of 0 degrees and a standard deviation of 2 degree when inserted into freezing water. Assuming that these readings are normally distributed and one thermometer is randomly selected and tested, find the probability that it reads (at the freezing point of water) above -1.22 degrees.
(freezing point:”donma noktası”, insert: ”sokmak”)

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6. (16 points) Starting at 5:00 A.M. (morning), every half hour there is a flight from Istanbul airport to Ankara airport. A person who wants to fly to Ankara arrives at the airport at a random time between 8:45 A.M. and 9:45 A.M with equal probability. Find the probability that she waits

   i. at most 10 minutes;

   ii. at least 15 minutes.

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Bonus #2 (25 points)

i. Sketch the graph of the following function:

$$f(x) = \begin{cases} \frac{1}{2} - \frac{1}{4}|x-3| & \text{if } 1 \leq x \leq 5 \\ 0 & \text{otherwise.} \end{cases}$$

Mathematically prove that this is a probability density function (PDF). If you use an integral, explicitly show the steps in the calculation of this integral.

ii. Calculate the Cumulative Distribution Function (CDF).

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