April 3, 2014

IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM I

Q1 |	**Q2** |	**Student ID:**	**Row No:**
Q3 |	**Q4** |		
Last Name:		**First Name:**	

1. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let $\mathbb{P}(.)$ denote a probability measure on S.
(*Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

   i. $\mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B) \geq 0$. T F

   ii. If A and B are mutually exclusive then $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$. T F

   iii. $\mathbb{P}(A|B^c) = 1 - \mathbb{P}(A|B)$. T F

   iv. $\mathbb{P}(A^c|B) = 1 - \mathbb{P}(A|B)$. T F

   v. For any positive integer n, $\binom{n}{0} + \binom{n}{1} + ... + \binom{n}{n-1} + \binom{n}{n} = 2^n$. T F

   vi. If A and B are independent, then $\mathbb{P}(A|B) = \mathbb{P}(A|B^c)$. T F

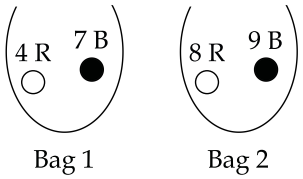
   vii. If $\mathbb{P}(A) = (\mathbb{P}(A))^2$ then A is independent of A. T F

   viii. If $\mathbb{P}(E) < \mathbb{P}(F)$ then $E \subset F$. T F

   ix. If A and B are two mutually exclusive events, then they are independent. T F

   x. The number of r-permutations of n objects is greater then the number of r-combinations of n objects. T F

2. (12 points) Suppose you have two bags with red and black balls: The first bag has 4 red and 7 black balls whereas the second has 8 red 9 black balls.



A friend picks a ball from the first bag randomly and places that ball into the second bag without seeing its colour. Then you randomly pick a ball from the second bag and it is black. What is the probability that the ball chosen by your friend from the first bag was red?

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3. (12 points) Let A, B and F be three events with the conditional probabilities

$$\mathbb{P}(A|F) = 0.3 \qquad \mathbb{P}(B|F) = 0.5, \qquad \mathbb{P}(A \cup B|F) = 0.7$$

Find the following probabilities:

i. $\mathbb{P}(A \cap B|F) =?$

ii. $\mathbb{P}(A^c \cap B^c|F) =?$

iii. $\mathbb{P}(A - B|F) =?$

4. (14 points) Consider the following two games:

   GAME I : Roll 4 dice. If **exactly one** 6 shows, then you win.

   GAME II: Roll 8 dice. If **exactly two** 6's show, then you win.

   Compute the probability of winning at each game. Which game would you play if you prefer the one in your favor?

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5. (12 points) Suppose that there are 3 chests each with 2 drawers. The first chest has a gold coin in each drawer, the second has a gold coin in one drawer and a silver coin in the other, the third has a silver coin in each drawer.

A chest is chosen at random and a drawer from that chest is randomly opened. If the drawer has a gold coin, what is the probability that the other drawer also has a gold coin?

6. (12 points) Assume A and B are independent events with $\mathbb{P}(A) = 0.2$ and $\mathbb{P}(B) = 0.3$. Let C be the event that at least one of A or B occurs, and let D be the event that exactly one (only one) of A or B occurs. Find $\mathbb{P}(C)$ and $\mathbb{P}(D)$.

7. (14 points)

   i. You draw 5 cards from a standard deck of 52 cards without replacement. Let X denote the number of aces in your hand. Write the PMF (probability mass function) of X.

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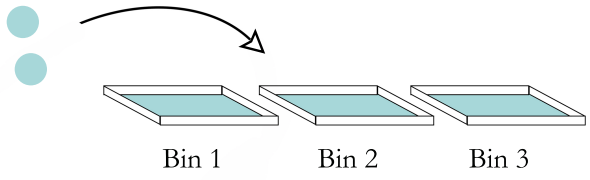
ii. Suppose that 2 balls are tossed into 3 bins so that each ball is equally likely to fall into any of the bins and that the tosses are independent.



Find the expected number and the variance of balls that land in the first bin.

8. (14 points)

i. A child has 12 blocks, of which 6 are black, 4 are red, 1 is white, and 1 is blue. If the child puts the blocks in a line, how many arrangements are possible?

ii. If 8 new teachers are to be divided among 4 schools, how many divisions are possible? What if each school must receive 2 teachers?