

IŞIK UNIVERSITY, MATH 230 FINAL EXAM

Q1	Q2	Student ID:	Row No:
Q3	Q4		
Last Name:		First Name:	

1. (16 points) Let X be random variable with cumulative distribution function (CDF)

$$F(x) = \begin{cases} 0 & \text{if } x < -1 \\ 0.2 & \text{if } -1 \leq x < 2 \\ 0.7 & \text{if } 2 \leq x < 3 \\ 1 & \text{if } x \geq 3. \end{cases}$$

- i. Write its PMF.
- ii. Find its expectation.
- iii. Find its variance.
- iv. Find $\mathbb{E}(|X|)$.

2. (12 points) A student, who lives in Kadıköy, has to take an exam in Şile Campus. The probabilities of dry (“kuru”) weather (D), rain (R) or snow (S) on the exam day are

$$\mathbb{P}(D) = 0.1, \quad \mathbb{P}(R) = 0.4, \quad \mathbb{P}(S) = 0.5.$$

If it is dry the probability that he will arrive in time for the exam is 0.8.

If it rains the probability that he will arrive in time for the exam is 0.6.

If it snows the probability that he will arrive in time for the exam is 0.2.

- i. Calculate the probability that he is late for the exam.
- ii. Does the event “the student being late to the exam” depend on the event “it snows”? (**Only mathematical (probabilistic) solution will get credit. Other solutions will receive zero point.**)

i. At the end of a soccer match, the teams go for penalty shootouts. If a team scores at least 4 goals in 5 trials then that team wins. If each of the players scores a goal with equal probability 0.7, what is the probability that this team wins by scoring 4 or more of 5 shootouts?





4. (12 points)

- i. In how many arrangements can a teacher seat 3 girls and 3 boys in a row of 6 if the boys are to be together?
- ii. A three-person committee must be chosen from a group of 7 professors and 10 graduate students. If at least one of the people on the committee must be a professor, how many different groups of people could be chosen for the committee?
- iii. How many different codes can be formed by just switching the orders of numbers of 1100100?



Q5	Q6	Student ID:	Row No:
Q7	Q8		
Last Name:		First Name:	

5. (10 points) I have a wallet that contains either a 10 TL or a 20 TL banknote (there is **only 1 of them** with equal likelihood), but I don't know which one. I add a 10 TL banknote into the wallet.
Later, I reach into my wallet (without looking) and remove a bill ("kağıt para"). It's a 10 TL banknote. There's one bill remaining in the wallet.
What are the chances that it (the remaining one) is a 10 TL banknote ?

6. (14 points) The lifetime of a machine part has a continuous distribution on the interval $(0, 40)$ with probability density function f , where $f(x)$ is given as

$$f(x) = \begin{cases} \frac{c}{(1+x)^2} & , \text{if } 0 \leq x \leq 40 \\ 0 & , \text{otherwise.} \end{cases}$$

- i. Find the value of c .
- ii. Calculate the probability that the lifetime of the machine part is less than 6.



7. (10 points) The lifetime (T) of a printer costing 200 TL is exponentially distributed with mean 2 years. The manufacturer agrees to pay a 200 TL refund (“para iadesi”) to a buyer if the printer fails during the first year following its purchase, and a 100 TL refund if it fails during the second year.

If the manufacturer sells 100 printers, how much should it expect to pay in refunds?

Hint: First calculate $\mathbb{P}(T \leq 1)$ and $\mathbb{P}(1 \leq T \leq 2)$ and then calculate the expected value of the refund (the refund takes two separate values).



8. (12 points) X and Y are jointly continuous with joint pdf

$$f_{X,Y}(x,y) = \begin{cases} cx^2 + \frac{xy}{3} & \text{if } 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 2 \\ 0 & \text{otherwise.} \end{cases}$$

- i. Find c .
- ii. Find $\mathbb{P}(X + Y \geq 1)$ (Calculate the integral, do not use basic geometry!!)
- iii. Find marginal PDFs of X and of Y .
- iv. Are X and Y independent ? (Justify your answer. Show your work!).