May 16, 2013

IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM II

Q1 |	**Q2** |	**Student ID:**	**Row No:**
Q3 |	**Q4** |		
Last Name:		**First Name:**	

1. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let $\mathbb{P}(.)$ denote a probability measure on S. Let a, b, c denote some constant and X, Y, Z be random variables.
(*Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

   i. If X is a discrete random variable then $\mathbb{P}(X \leq c) = \mathbb{P}(X < c)$. **T** **F**

   ii. If X is a continuous random variable then $\mathbb{P}(X \leq c) = \mathbb{P}(X < c)$. **T** **F**

   iii. If $\mathbb{E}(X) = \mathbb{E}(Y)$ then $X = Y$. **T** **F**

   iv. $\mathbb{E}(X - \mathbb{E}(X)) = 0$ for any random variable X. **T** **F**

   v. $Var(aX + b) = a^2 Var(X) + b$ **T** **F**

   vi. $\mathbb{E}(aX + b) = a\mathbb{E}(X) + b$ **T** **F**

   vii. If X and Y are Gaussian then $X + Y$ is also Gaussian. **T** **F**

   viii. If X is a random variable with probability density function f_X,

   then for any a we have $\int_{-\infty}^{a} f_X(x)dx = 1 - \int_{a}^{\infty} f_X(x)dx$. **T** **F**

   ix. If $f(x)$ denotes a probability density function,

   then it is possible $f(a) > 1$ for some a. **T** **F**

   x. If F_X denotes the cumulative distribution function of X, then

   $$\mathbb{P}(X = a) = F_X(a) - \lim_{x \to a^-} F_X(x).$$ **T** **F**

2. (15 pts) In a game of throwing a single dice, you loose 2 TL if you throw an even number, you win 3 TL if you throw 1 or 3 and you win 1 TL if you throw a 5. If X is the random variable denoting your earnings/loss ("kazanç/kayıp") after a single game, then

   i. write the probability mass function (PMF) of X,

   ii. find your expected earnings,

   iii. find the variance of your earnings,

   iv. find the standard deviation of your earnings,

   v. find the expected value, $\mathbb{E}(|X|)$.

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3. (15 points)

   i. The number people entering a certain building is a Poisson distribution. Assume that, on average, 1 person enters the building every 2 minutes. What is the probability that no one enters the building between 12:00 pm and 12:05 pm?

   ii. On a multiple choice exam with 3 possible answers for each of the 5 questions, a) what is the expected number of correct answers of a student who answers questions just by guessing? b) What is the probability that he/she answers exactly 2 questions correctly?

   iii. In a game of dice, you throw a pair of dice continually until the sum of two dice shows a 10 and then you stop playing. What is the probability that this game ends on 12^{th} trial?

4. (10 points) In a factory producing study desks, it is required 2 kg of iron, 3 kg of wood and 1 kg of plastic to build a single desk. We know that the cost of each kg of iron is Poisson with parameter $\lambda = 1/10$, the cost of each kg of wood is Poisson with parameter $\lambda = 1/5$ and the cost of each kg of plastic is Geometric with parameter $p = 1/4$. What is the expected cost of a single desk produced?

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Q7 |			
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5. (15 points) If the PDF of a continuous random variable is given as:

$$f(x) = \begin{cases} 0, & x \leq 10 \\ \dfrac{c}{x^4}, & x > 10 \end{cases}$$

i. Find the value of the constant c?

ii. Calculate the expected value of the random variable.

iii. Calculate the variance of the random variable.

6. (15 points) If X is a normal random variable with the mean 10 and variance 36

   i. What is $\mathbb{P}(X > 5)$?

   ii. What is $\mathbb{P}(4 < X < 16)$?

   iii. Find the expectation and variance of the random variable $Y = 2X - 17$?

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7. (15 points) The time between jobs sent to a printer is an exponential distribution. It is known that an average of 3 jobs per hour are sent to the printer.

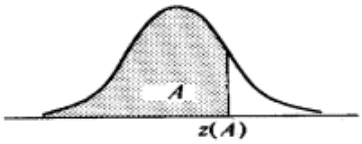
   i. What is the variance of time between jobs?

   ii. What is the probability that the next job is sent within 5 minutes?

Below, you can find the values of the CDF $\Phi(z)$ of a standard normal (Gaussian) random variable.

Useful Reminder: The density of a Gaussian random variable X with mean μ and variance σ^2 is $f(x) = \dfrac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2/(2\sigma^2)}$.

Entry is area A under the standard normal curve from $-\infty$ to $z(A)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998