IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM I

$\mathbf{Q1}$	Q2	Student ID:	Row No:
Q3	$\mathbf{Q4}$		
Last Name:		First Name:	

 (10 points) Determine whether the following statements are True or False. Circle T or F. No explanation is required. Let A, B, and A_i denote events in a sample space S and let P(.) denote a probability measure on S.
(Note: A statement is assumed to be true if it is true in any possible case, and it is assumed to be false if it fails in at least one case.):

i. If A and B are independent, then $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$. T F

ii.
$$F(x) = \begin{cases} 0, & x < 0\\ 0.8, & 0 \le x < 2 \text{ is a cumulative distibution function.} & T & F\\ 0.9, & 2 \le x \end{cases}$$

iii.
$$\mathbb{P}(A_1^c \cup A_2^c \cup \ldots \cup A_n^c) = 1 - \mathbb{P}(A_1 \cap A_2 \cap \ldots \cap A_n).$$
 $T \in F$

$$iv. \quad p(x) = \begin{cases} 0.1, & x = 0\\ 0.2, & x = 2\\ 0.3, & x = 3\\ 0, & otherwise \end{cases}$$
 is a probability mass function.
$$T \quad F$$

v. If
$$\mathbb{P}(A \cup B) = \mathbb{P}(A)$$
 then B is an empty set. T F

vi. If A and B are independent, then
$$\mathbb{P}(A|B) = \mathbb{P}(A|B^c)$$
. T F

vii.
$$\mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B)$$
 for any A and B. $T \in F$

viii. If
$$\mathbb{P}(E) < \mathbb{P}(F)$$
 then $E \subset F$. $T \in F$

$$ix.$$
If A and B are two mutually exclusive events,
then they are independent. T F

$$x.$$
The number of r -permutations of n objects is greater then
the number of r -combinations of n objects. T F

2. (15 pts) Let X be a discrete random variable with the cumulative distribution function (CDF), F(x), given by

$$F(x) = \begin{cases} 0, & x < 1\\ 0.2, & 1 \le x < 2\\ c, & 2 \le x < 3\\ 1, & x \ge 3 \end{cases}$$

where c is a constant. It is also given that $\mathbb{P}(X = 3) = 0.3$. Under these assumptions, answer the following questions:

i. What is the value of c?

ii. Write the probability mass function (PMF) of X.

iii. Compute the probability $\mathbb{P}(X > 1)$.

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- 3. (15 points)
 - i. In a class of 5 students, what is the probability that at least 2 students have the same birthday, i.e. at least 2 of them were born on the same day of the same month (but maybe in different years)?

ii. In a deck of 52 cards, each suit of cards are labeled as 1, 2, ..., 9, 10, J, Q, K and there are 4 of these suits, namely clubs ♣, spades ♠, hearts ♥ and diamonds ♠. Find the probability that a 13 card hand chosen randomly from a standard deck of 52 cards contains exactly 5 hearts.

iii. Each path in the map of part of Işık campus below is under construction (closed) with probability p and open with probability q = 1 - p, independently of all others. What is the probability that it is possible to get from the Maroon Dorms (A) to the Dining Hall (B)?(Assume that you walk through a path at most once.)



4. (15 points) Let A, B and C be three independent events with the probabilities

 $\mathbb{P}(A) = 0.5 \qquad \mathbb{P}(B) = 0.4, \qquad \mathbb{P}(C) = 0.2$

i. What is the probability $\mathbb{P}(A \cup B \cup C)$?

ii. What is the probability $\mathbb{P}(A^c \cap B^c \cap C^c)$?

iii. Are two events $A \cup B$ and C independent? Why or Why not? Explain using probabilistic methods.

$\mathbf{Q5}$	Q6	Student ID:	Row No:
$\mathbf{Q7}$			
Last Name:		First Name:	

5. (15 points)

i. If 8 new nurses are to be divided among four hospitals (assuming of course the nurses are distinct) and if each hospital must receive 2 nurses, how many divisions are possible ?

ii. There are 4 girls and 4 boys, in how many ways can they sit in a row if only the boys must sit together but the girls do not need to ?

iii. From an ordinary deck of 52 cards, seven cards are drawn at random without replacement. How many 7-card hands can be drawn if at least one of the cards has to be a king?

- Department of Mathematics, Işık University
- 6. (15 points) In a certain region of Turkey, the probability that a person lives at least 80 years is 0.75, and the probability that he or she lives at least 90 years is 0.63. What is the probability that a randomly selected 80-year-old person from this region will survive to become 90?

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7. (15 points) In a study conducted by a bank three years ago, 82% of the people in a randomly selected sample were found to have good financial credit ratings, while the remaining 18% were found to have bad financial credit ratings. Current records of the people from that sample show that 30% of those with bad credit ratings have since improved their ratings to good, while 15% of those with good credit ratings have since changed to having a bad credit rating. What percentage of people with good credit ratings now had bad ratings three years ago?