IŞIK UNIVERSITY, MATH 230 Final EXAM-2

Q1	Q2	Student ID:	Row No:
Q3	$\mathbf{Q4}$		
Last Name:	-	First Name:	

1. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let P(.) denote a probability measure on S. Let a, b, c denote some constant and X, Y, Z be random variables. (Note: A statement is assumed to be true if it is true in any possible case, and it is assumed to be false if it fails in at least one case.):
i. If X is a discrete random variable then P(X ≥ c) = P(X > c). T F
ii. If X and Y are independent, then E(XY) = E(X) · E(Y). T F

iii.	If $\mathbb{P}(A \cup B) = \mathbb{P}(A)$ then B is an empty set.	T	F
iv.	$\mathbb{E}(X - \mathbb{E}(X)) = 0$ for any random variable X.	T	F
v.	If A and B are independent, then $\mathbb{P}(A B) = \mathbb{P}(A B^c)$.	T	F
vi.	If $\mathbb{P}(E) < \mathbb{P}(F)$ then $E \subset F$.	T	F
vii.	If X and Y are Gaussian then $X + Y$ is also Gaussian.	T	F
viit	If A and B are two mutually exclusive events, then they are independent.	T	F
ix.	If A and B are independent, then $\mathbb{P}(A \cap B) = \mathbb{P}(A) + \mathbb{P}(B)$.	T	F
x.	If X is a continuous random variable then $\mathbb{P}(X = c) = 0$ for any $c \in \mathbb{R}$.	T	F

2. (12 pts)



The graph of the probability density function (PDF) of a Gaussian random variable X is given above. The variance of X is 16.

i. What is the expected value $\mathbb{E}(X)$?

ii. What is the area of the shaded region?

iii. What is the probability $\mathbb{P}(X > 4)$?

iv. What is the probability $\mathbb{P}(X^2 - 4X + 4 < 1)$?

Student's Name :_____

- 3. (14 points)
 - i. A man and a woman agree to meet at a certain location. If the man arrives at 12:45 P.M., and if the woman independently arrives at a time uniformly distributed between 12:00 and 1 P.M., find the probability that the first to arrive waits less than 10 minutes?

ii. X is a continuous random variable with the probability distribution

$$f(x) = \begin{cases} \frac{6}{5}(x^2 + x), & 0 < x < 1\\ 0, & otherwise \end{cases}$$

Find the standard deviation of X.

4. (15 points) The joint cumulative distribution function of two random variables X and Y is given by

$$F_{X,Y}(x,y) = \begin{cases} (1 - e^{-x})(1 - e^{-y}), & x > 0, y > 0\\\\0, & otherwise \end{cases}$$

i. Find the joint probability density $f_{X,Y}$ of X and Y.

ii. Write the marginal densities f_X and f_Y .

iii. Are X and Y independent? Why?

iv. What is the probability $\mathbb{P}(X > 3Y)$?

v. What is the conditional probability density $f_{X|Y}(x|2)$?

$ \mathbf{Q5} $	Q6	Student ID:	Row No:
Q7	Q8		
Last Name:		First Name:	

5. (12 points)

i. The function given by

$$p(x) = \begin{cases} c \left(\frac{3}{4}\right)^x, & x = 1, 2, 3, \dots \\ 0, & otherwise \end{cases}$$

is a probability mass function (PMF) only for one particular value of c, what is this value? Hint: remember the three conditions the PMF has to satisfy and also remember from calculus that for |r| < 1, $\sum_{i=1}^{\infty} r^i = \frac{r}{1-r}$

ii. If the weather is good (with probability 0.6), Ali walks the 2 miles to class at a speed of V=5 miles per hour, and otherwise rides her motorcycle at a speech of V=30 miles per hour. What is the expected time E[T] to get to the class ? What is the variance ?

6. (13 points) A box contains 7 red and 13 blue balls. Two balls are selected at random and are discarded without their colors being seen. If a third ball is drawn randomly and observed to be red, what is the probability that both of the discarded balls were blue?

HINT: The two balls selected, even though we do not know which colour they were, they should be one of the following (BB,RR,BR,RB). *(be discarded: atılmak, çıkarılmak)*

Student's Name :_____

7. (12 points) In a small class, 10 of the 25 students are against the idea of taking extra arts classes, 7 support the idea of extra arts classes , and the rest are indifferent . A random sample of six students is selected for an interview. What is the probability that

(against: karşı; indifferent : kararsız, belirli fikri olmayan

i. all of them support the idea of taking extra arts classes ;

ii. all of them have the same opinion?

Student's Name :_____

8. (12 points) A student is selling books by making random calls to people and trying to convince them to buy books from him. The probability is 0.038 that a person reached by the student will make a purchase (assume that calls are independent and for each call this is the probability). If the student calls 50 people, what is the probability that at least one sale will result? *(result : gerçekleşmek; convince: ikna etmek; reach: ulaşmak)*

Below, you can find the values of the CDF $\Phi(z)$ of a standard normal (Gaussian) random variable.

Useful Reminder: The density of a Gaussian random variable X with mean μ and variance σ^2 is $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/(2\sigma^2)}$.

Entry is area A under the standard normal curve from $-\infty$ to z(A)



	2(1)									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	,9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	