January 4, 2019

# IŞIK UNIVERSITY, MATH 230 FINAL EXAM

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|---|---|---|---|
| **Q1** \| | **Q2** \| | **Student ID:** | **Row No:** |
| **Q3** \| | **Bonus** \| | | |
| **Last Name:** | | **First Name:** | |

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|---|---|
| I pledge my honour that I have not violated the honour code during this examination.<br><br>Bu sınavda onur yasamızı ihlal etmediğime şerefim üzerine yemin ederim. | **Signature :** |



1. (18 points) There is a toll bridge where the vehicles are charged by an automated license-plate-reading system. The system scan each vehicle's plate and charge the toll from their accounts. However the system may fail and may not read the license plate 1 of every 50 vehicles. For a quality check, we observe the next 20 vehicles passing through the bridge.

   i. What is the exact probability that the system fails at most 2 times, that is, it won't be able to read the plates of at most 2 of the vehicles?

   ii. Find the probability that the system fails at most 2 times by a Poisson approximation.

2. ( 10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let $A$, $B$, and $A_i$ denote events in a sample space $S$, let $\mathbb{P}(.)$ denote a probability measure and $\mathbb{E}(.)$ denote the expectation on $S$.
( *Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

*i.* If $X$ is a continuous random variable and $x_0$ is any point in the range of $X$, then $\mathbb{P}(X = x_0) = 0$. *T F*

*ii.* If $X$ is a discrete random variable, then $P(X \leq a) = P(X < a)$ for any constant $a$. *T F*

*iii.* $\mathbb{E}(X - \mathbb{E}(X)) = 0$ for any random variable $X$. *T F*

*iv.* If, for two random variables $X$ and $Y$, $X \geq Y$, then $\mathbb{E}(X) \geq \mathbb{E}(Y)$. *T F*

*v.* If $X$ and $Y$ are two independent random variables, then the events $(X > 1)$ and $(Y > 1)$ are independent. *T F*

*vi.* If $X$ and $Y$ are 2 discrete random variables then their joint PMF, $p_{X,Y}$, is the product of their marginal PMFs. *T F*

*vii.* If $X$ and $Y$ are 2 jointly continuous random variables then their joint PDF satisfies $0 \leq f_{X,Y}(x, y) \leq 1$. *T F*

*viii.* If $X$ is a Gaussian with variance $\sigma^2$ and $\mathbb{P}(X > a) = \mathbb{P}(X < a)$ then $Y = (X - a)/\sigma^2$ is a standard Gaussian. *T F*

*ix.* If Z is a standard Gaussian then $\mathbb{P}(Z < 0) = \mathbb{P}(Z > 0)$. *T F*

*x.* Variance can be negative for some random variables. *T F*

Student's Name : ____________________

3. (20 points) Let $X$ and $Y$ be jointly continuous random variables with the joint density

$$f_{X,Y}(x,y) = \begin{cases} \lambda\, xy^2 & , 0 \leq x \leq y \leq 1, \\ 0 & , \text{otherwise} \end{cases}.$$

i. Find the value of the constant $\lambda$ ?

ii. What are the marginal probability density functions?

iii. Find the conditional probability density $f_{Y|X}(y|x)$.

iv. Calculate the probability $\mathbb{P}\left(Y > \frac{3}{4} \middle| X = \frac{1}{2}\right)$.

v. Are $X$ and $Y$ independent? State with reasons?

BONUS (10 puan) Evaluate the following integral:

$$\int_2^5 e^{\frac{(x-3)^2}{8}}\,dx$$

| Q4 \| | Q5 \| | Student ID: | Row No: |
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| Q6 \| | | | |
| **Last Name:** | | **First Name:** | |

4. (18 points) There are 10 dice and 2 of them are biased. They are biased in the following particular way: for these biased dice "6" occurs half of the times and "1, 2, 3, 4 and 5" show with equal probability. Now, we randomly chose one die out of these 10 dice and we roll it. ("biased": hileli)

   i. What is the probability that we get a 6?

   ii. If we get a 6, what is the probability that it is one of the biased dice?

   iii. If we get a 2, what is the probability that it is NOT one of the biased dice?

5. (18 points) A function is defined in the following way:

$$f(x) = \begin{cases} a \cdot 3^{x} & , x < 0, \\ a \cdot 3^{-x} & , x \geq 0. \end{cases}$$

i. What does $a$ need to be so that the function is a probability density function (PDF) of a random variable $X$?

ii. Calculate the Cumulative Distribution Function.

iii. Calculate the expected value $\mathbb{E}(X)$.

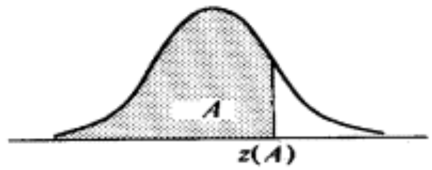
Student's Name : ____________________

6. (16 points)

   i. A robot's life time until its first technical problem (breaking down) is an exponential random variable $X$ with parameter $\lambda$ . It is known that $\mathbb{P}(X > 6) = 0.3$. What is the probability the robot breaks down within the first 4 years of production?

   ii. A real number is selected with respect to a normal distribution with mean 8 and variance 36. What is the probability that the square of the selected number is greater than 100?

**Entry is area $A$ under the standard normal curve from $-\infty$ to $z(A)$**




| z | .00 | .01 | .02 | .03 | .04 | .05 | .06 | .07 | .08 | .09 |
|---|---|---|---|---|---|---|---|---|---|---|
| .0 | .5000 | .5040 | .5080 | .5120 | .5160 | .5199 | .5239 | .5279 | .5319 | .5359 |
| .1 | .5398 | .5438 | .5478 | .5517 | .5557 | .5596 | .5636 | .5675 | .5714 | .5753 |
| .2 | .5793 | .5832 | .5871 | .5910 | .5948 | .5987 | .6026 | .6064 | .6103 | .6141 |
| .3 | .6179 | .6217 | .6255 | .6293 | .6331 | .6368 | .6406 | .6443 | .6480 | .6517 |
| .4 | .6554 | .6591 | .6628 | .6664 | .6700 | .6736 | .6772 | .6808 | .6844 | .6879 |
| .5 | .6915 | .6950 | .6985 | .7019 | .7054 | .7088 | .7123 | .7157 | .7190 | .7224 |
| .6 | .7257 | .7291 | .7324 | .7357 | .7389 | .7422 | .7454 | .7486 | .7517 | .7549 |
| .7 | .7580 | .7611 | .7642 | .7673 | .7704 | .7734 | .7764 | .7794 | .7823 | .7852 |
| .8 | .7881 | .7910 | .7939 | .7967 | .7995 | .8023 | .8051 | .8078 | .8106 | .8133 |
| .9 | .8159 | .8186 | .8212 | .8238 | .8264 | .8289 | .8315 | .8340 | .8365 | .8389 |
| 1.0 | .8413 | .8438 | .8461 | .8485 | .8508 | .8531 | .8554 | .8577 | .8599 | .8621 |
| 1.1 | .8643 | .8665 | .8686 | .8708 | .8729 | .8749 | .8770 | .8790 | .8810 | .8830 |
| 1.2 | .8849 | .8869 | .8888 | .8907 | .8925 | .8944 | .8962 | .8980 | .8997 | .9015 |
| 1.3 | .9032 | .9049 | .9066 | .9082 | .9099 | .9115 | .9131 | .9147 | .9162 | .9177 |
| 1.4 | .9192 | .9207 | .9222 | .9236 | .9251 | .9265 | .9279 | .9292 | .9306 | .9319 |
| 1.5 | .9332 | .9345 | .9357 | .9370 | .9382 | .9394 | .9406 | .9418 | .9429 | .9441 |
| 1.6 | .9452 | .9463 | .9474 | .9484 | .9495 | .9505 | .9515 | .9525 | .9535 | .9545 |
| 1.7 | .9554 | .9564 | .9573 | .9582 | .9591 | .9599 | .9608 | .9616 | .9625 | .9633 |
| 1.8 | .9641 | .9649 | .9656 | .9664 | .9671 | .9678 | .9686 | .9693 | .9699 | .9706 |
| 1.9 | .9713 | .9719 | .9726 | .9732 | .9738 | .9744 | .9750 | .9756 | .9761 | .9767 |
| 2.0 | .9772 | .9778 | .9783 | .9788 | .9793 | .9798 | .9803 | .9808 | .9812 | .9817 |
| 2.1 | .9821 | .9826 | .9830 | .9834 | .9838 | .9842 | .9846 | .9850 | .9854 | .9857 |
| 2.2 | .9861 | .9864 | .9868 | .9871 | .9875 | .9878 | .9881 | .9884 | .9887 | .9890 |
| 2.3 | .9893 | .9896 | .9898 | .9901 | .9904 | .9906 | .9909 | .9911 | .9913 | .9916 |
| 2.4 | .9918 | .9920 | .9922 | .9925 | .9927 | .9929 | .9931 | .9932 | .9934 | .9936 |
| 2.5 | .9938 | .9940 | .9941 | .9943 | .9945 | .9946 | .9948 | .9949 | .9951 | .9952 |
| 2.6 | .9953 | .9955 | .9956 | .9957 | .9959 | .9960 | .9961 | .9962 | .9963 | .9964 |
| 2.7 | .9965 | .9966 | .9967 | .9968 | .9969 | .9970 | .9971 | .9972 | .9973 | .9974 |
| 2.8 | .9974 | .9975 | .9976 | .9977 | .9977 | .9978 | .9979 | .9979 | .9980 | .9981 |
| 2.9 | .9981 | .9982 | .9982 | .9983 | .9984 | .9984 | .9985 | .9985 | .9986 | .9986 |
| 3.0 | .9987 | .9987 | .9987 | .9988 | .9988 | .9989 | .9989 | .9989 | .9990 | .9990 |
| 3.1 | .9990 | .9991 | .9991 | .9991 | .9992 | .9992 | .9992 | .9992 | .9993 | .9993 |
| 3.2 | .9993 | .9993 | .9994 | .9994 | .9994 | .9994 | .9994 | .9995 | .9995 | .9995 |
| 3.3 | .9995 | .9995 | .9995 | .9996 | .9996 | .9996 | .9996 | .9996 | .9996 | .9997 |
| 3.4 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9997 | .9998 |

NOTE: The density of a Gaussian random variable is $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(x-\mu)^2}{2\sigma^2}}$.