IŞIK UNIVERSITY,   MATH 230 MIDTERM EXAM

Q1		Q2		Student ID:		Row No:
Q3						
Last Name:			**First Name:**			

I pledge my honour that I have not violated the honour code during this examination. Bu sınavda onur yasamızı ihlal etmediğime şerefim üzerine yemin ederim.	**Signature :**

1. (20 points) An urn contains 3 red and 2 green balls. Two balls are randomly drawn <u>without replacement</u>. Let A be the event that the first ball is red, and let B be the event that the second ball is red. To receive credit you need to verify your answer with probabilistic methods.

    i.   Find the probability of B.

    ii. Are A and B independent?

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2. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let $\mathbb{P}(.)$ denote a probability measure on S.
(*Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

i.   If A and B are mutually exclusive then $\mathbb{P}(A \cup B|F) = \mathbb{P}(A|F) + \mathbb{P}(B|F)$ T F

ii.   If $\mathbb{P}(A) = 0$ and $B \subseteq A$ then $\mathbb{P}(B) = 0$. T F

iii. $\mathbb{P}(A|B^c) = 1 - \mathbb{P}(A|B)$. T F

iv. $\mathbb{P}(A^c|B) = 1 - \mathbb{P}(A|B)$. T F

v.   CDF is a decreasing function. T F

vi.   If A and B are independent, then $\mathbb{P}(A|B) = \mathbb{P}(A|B^c)$. T F

vii.   Variance can be a negative number. T F

viii.   The sum of all coefficients in the expansion of $(x + y)^{10}$ is 2^{10}. T F

ix.   The coefficient of $x^3 y z^5$ in the expansion of $(x + y + z)^9$ is 72. T F

x.   If $p(\cdot)$ is a PMF, then $p(x) < 0$ is possibe for some x. T F

2

3. (20 points) An urn contains 3 red and 2 green balls. We draw 2 balls at random with replacement. (We place the first ball back before the second draw.) Let X be the random variable denoting the number of red balls in our selection.

   i.   Write the probability mass function (PMF) of X.

   ii.   Compute the expectation $E(X)$.

   iii.   Compute the variance $Var(X)$.

   iv.   Find the probability $\mathbb{P}(\ln(X) \geq 0)$.

Q4	Q5	Student ID:	Row No:
Q6			
Last Name:		First Name:	

4. (18 points) In a study conducted three years ago, 82% of the people in a randomly selected sample were found to have "good" financial credit ratings, while the remaining 18% were found to have "bad" financial credit ratings. Current records of the people from that sample show that 30% of those with bad credit ratings have since improved their ratings to good, while 15% of those with good credit ratings have since changed to having a bad credit rating. What percentage of people, who has good credit ratings now, had bad ratings three years ago?

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5. (16 points)

   i. In how many ways can 10 people be divided into 3 groups of 5, 3 and 2 people ?

   ii. A committee of 5 is to be chosen from a club which has 10 men and 12 women members. How many ways can the committee be formed if it has to have at least 2 women? How many ways if, **<u>in addition</u>**, one particular man and one particular woman who are members of the club, refuse to serve together on the committee?

6. (16 points) In a school, 11 of the 25 schoolteachers are against teaching drama classes, eight are for teaching drama classes, and the rest are indifferent. A random sample of five schoolteachers is selected for an interview. What is the probability that

   i. all of them are for teaching drama classes ?

   ii. all of them have the same opinion?