January 10, 2018

IŞIK UNIVERSITY, MATH 230 FINAL EXAM

Q1 |	**Q2** |	**Student ID:**	**Row No:**
Q3 |	**Q4** |		
Last Name:		**First Name:**	

I pledge my honour that I have not violated the honour code during this examination. Bu sınavda onur yasamızı ihlal etmediğime şerefim üzerine yemin ederim.	**Signature :**

1. (12 points)

   i. A certain type of curcuit has an exponential lifetime with an average of 500 hours. What is the probability that a curcuit of this type will survive at least 2000 hours?

   ii. A real number is chosen randomly according to a normal distribution with mean 40 and variance 360. What is the probability that the square of this number is greater than 250?

2. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let $\mathbb{P}(.)$ denote a probability measure on S.
(*Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

i. Variance of a random variable may be a negative number. T F

ii. Expectation of a random variable may be a negative number. T F

iii. If X and Y are independent then $\mathbb{P}(X < 2, Y < 3) = \mathbb{P}(X < 2)\mathbb{P}(Y < 3)$. T F

iv. If X is a constant random variable then its variance is zero. T F

v. If the joint density, $f_{X,Y}$, of X and Y is given, then one can find $\mathbb{E}(X)$. T F

vi. If X is a Gaussian with variance σ^2 and $\mathbb{P}(X > a) = \mathbb{P}(X < a)$ then $Y = (X - a)/\sigma^2$ is a standard Gaussian. T F

vii. If $A \subset B$ and $\mathbb{P}(A) = 1$ then $\mathbb{P}(B^c) = 0$. T F

viii. If $\mathbb{P}(A \cup B) + \mathbb{P}(A \cap B) > 1$ then A and B cannot be disjoint. T F

ix. If marginal densities of X and Y are known then their joint density can be written. T F

x. For any random variable X, we have $\mathbb{E}(X^2) - \mathbb{E}(2X) + 1 \geq 0$. T F

Student's Name : ____________________

3. (14 points) Let X be a random variable with **Cumulative Distribution Function (CDF)**

$$F(x) = \begin{cases} 0 & x < 2 \\ 1 - e^{2-x} & x \geq 2. \end{cases}$$

i. Find the probability $\mathbb{P}(X > 10)$.

ii. What is the expectation of X, that is $\mathbb{E}(X)$?

4. (14 points) Let X and Y be jointly continuous random variables with the joint pdf

$$f_{X,Y}(x,y) = \begin{cases} 12x^2 & , 0 \le y \le 1-x, \quad x \ge 0, \\ 0 & , \text{otherwise.} \end{cases}$$

i. Find the conditional density $f_{Y|X}(y|x)$.

ii. Find the probability $\mathbb{P}\left(Y > \frac{1}{4} \middle| X = \frac{1}{2}\right)$.

Q5 |	**Q6** |	**Student ID:**	**Row No:**
Q7 |	**Q8** |		
Last Name:		**First Name:**	

5. (12 points) The water company analyse the number of complaint filed by people during a year. They notice that the probability to file two complaints is 3 times the probability to file four complaints at a given year. The number of claims filed has a Poisson distribution. Calculate the variance of the number of claims filed.

6. (10 points) X is a discrete random variable and its expected value is 1 and its variance is 5.

   i. Find the value $\mathbb{E}(2 + X^2)$.

   ii. What is the value of $Var(4 + 3X)$?

7. (14 points) A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers. Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers. A randomly selected participant from the study died during the five-year period. Calculate the probability that the participant was a heavy smoker. (Hint: write everything in terms of conditional probability $\mathbb{P}(\text{death} \mid \text{light smoker})$.

8. (14 points) The joint probability density function of random variables X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} \lambda\, xy^2 & , 0 \le x \le y \le 1, \\ 0 & , \text{otherwise.} \end{cases}$$

i. Find the value of λ.

ii. Find the marginal PDFs of X and Y.