IŞIK UNIVERSITY,   MATH 230 FINAL EXAM

Q1		Q2		Student ID:		Row No:
Q3		**Q4**				
Last Name:			**First Name:**			

1. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let $\mathbb{P}(.)$ denote a probability measure on S.
(*Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

   i.   For any two events A and B, we always have $\mathbb{P}(A|B) \leq \mathbb{P}(A)$.   *T   F*

   ii.   If $\mathbb{P}(A) = 0$ then A is the empty event.   *T   F*

   iii.   The number of r-permutations of n objects is greater then
            the number of r-combinations of n objects.   *T   F*

   iv.   For a continuous random variable X and a real number a,
            we have $\mathbb{P}(X \geq a) = \mathbb{P}(X > a)$.   *T   F*

   v.   For any random variable X, we have $\mathbb{E}(X^2) - \mathbb{E}(2X) + 1 \geq 0$.   *T   F*

   vi.   If X and Y are two independent random variables,
            then the events $(X > 1)$ and $(Y > 1)$ are independent.   *T   F*

   vii.   Variance of a random variable may equal zero.   *T   F*

   viii.   A normal random variable has a density curve which is
            symetric about its mean.   *T   F*

   ix.   If $\mathbb{P}(A \cup B) + \mathbb{P}(A \cap B) > 1$ then A and B cannot be disjoint.   *T   F*

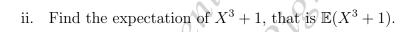   *x.*   If $A \subset B$ and $\mathbb{P}(A) = 1$ then $\mathbb{P}(B^c) = 0$.   *T   F*

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2. (14 points) You are visiting a small town in a foreign country with 3 major cell phone carriers, namely A, B and C. It is known that when a tourist dials a number on his/her phone, it will randomly connects to one of these carriers with 40%, 25% and 35% of the time, respectively. It is also known that 90% of the calls through carrier A, 95% of the calls through carrier B and 80% of the calls through carrier C are successful in general.

   i. If you dial a number on your phone in this town, what is the probability that your call will be successful? (Define events and show details of your work to receive full credit.)

   ii. If you make a successful call on your phone, what is the probability that the call goes through carrier A?

3. (12 points) Consider the following cumulative distribution function of a random variable X:

$$F(x) = \begin{cases} 0 & , x < -2, \\ 0.2 & , -2 \leq x < -1, \\ 0.5 & , -1 \leq x < 1, \\ 1 & , 1 \leq x, \end{cases}$$

i. Find the expectation of X.

ii. Find the expectation of $X^3 + 1$, that is $\mathbb{E}(X^3 + 1)$.

4. (14 points)

    i. Suppose that a server fails if at least 2 power outages occur in one hour. Moreover, suppose that in this region power outages happen according to a Poisson distribution with an average of 1 in every 5 hours. What is the probability that the server functions properly during a one hour period?

    ii. Suppose that people have problems logging onto a particular website with probability 0.2, on each attempt. Assuming the attempts are independent, what is the probability that an individual will be able to log on to the site for the first time on his 4th attempt? (Define a random variable and state its type to receive full credit.)

Q5		Q6		Student ID:	Row No:
Q7		Q8			
Last Name:		First Name:	

5. (12pts) Random variables X and Y have the joint probability density function (PDF)

$$f_{X,Y}(x,y) = \begin{cases} 9x^2y^2 & ,0 \leq x \leq 1; 0 \leq y \leq 1 \\ 0 & ,\text{otherwise.} \end{cases}$$

(a) (6pts) Find the marginal PDFs $f_X(x)$ and $f_Y(y)$.

(b) (6pts) Are X and Y independent random variables?

6. (15pts) The joint probability mass function (PMF) of two random variables X and Y is given below:

$p_{X,Y}(x,y)$	$x = -2$	$x = 0$	$x = 2$
$y = -1$	1/16	1/16	1/8
$y = 1$	1/4	1/4	1/4

(a) (5pts) Find the marginal PMF of X, $p_X(x)$, and the marginal PMF of Y, $p_Y(y)$.

(b) (5pts) Find the conditional PMF of X given $Y = -1$, $p_{X|Y}(x|-1)$.

(c) (5pts) Find $\mathbb{P}(X > Y)$.

7. (10pts) Let A be a normal random variable with mean 0 and variance 9. Let B be a normal random variable with mean 1 and variance 4. Which probability is larger: $\mathbb{P}(A > 1)$ or $\mathbb{P}(B > 1)$? Show your reasoning for credit.

8. (13pts) Let X be an exponential random variable with $\mathbb{E}[X] = 1$. Define a new random variable $Y = e^X$. Find the pdf of Y, $f_Y(y)$.