# IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM II

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| **Q1** \| | **Q2** \| | **Student ID:** | **Row No:** |
| **Q3** \| | | | |
| **Last Name:** | | **First Name:** | |

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| I pledge my honour that I have not violated the honour code during this examination.<br><br>Bu sınavda onur yasamızı ihlal etmediğime şerefim üzerine yemin ederim. | **Signature :** |

1. (14 points)

   i. The times of the finishers in the Istanbul 10-km marathon are normally distributed with mean 61 minutes and standard deviation 9 minutes. Determine the percentage of finishers with times between 50 and 70 minutes.

   ii. $X$ is a random variable uniformly distributed over the interval [2,7]. What is the probability that $\mathbb{P}(0 < X < 4)$?

2. ( 10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let $A$, $B$, and $A_i$ denote events in a sample space $S$ and let $\mathbb{P}(.)$ denote a probability measure on $S$.
( *Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

*i.* If, for two random variables $X$ and $Y$, $\mathbb{E}(X) = \mathbb{E}(Y)$, then $X = Y$. *T* *F*

*ii.* For any random variable $X$, $Var(X) \geq 0$ . *T* *F*

*iii.* Cumulative distribution function is increasing (non-decreasing). *T* *F*

*iv.* PDF is a positive (non-negative) function. *T* *F*

*v.* For any random variable $X$, $\mathbb{E}(X - \mathbb{E}(X)) = 0$. *T* *F*

*vi.* We can define a uniform random variable on the interval $[0, \infty)$. *T* *F*

*vii.* Exponential random variable is a discrete random variable. *T* *F*

*viii.* You play the dart game, and $X$ is the first time you hit the board. $X$ is a Poisson random variable. *T* *F*

*ix.* If $X$ is a continuous random variable, then for any point a, $\mathbb{P}(X = a) = 0$. *T* *F*

*x.* If $X$ is a Gaussian, then $Y = 2X$ is also a Gaussian with the same mean as $X$. *T* *F*

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3. (28 points) Let $X$ be a continuous random variable with the cumulative distribution function (CDF)

$$F(x) = \begin{cases} 0 & , x < -1, \\ \dfrac{x^2}{2} + x + \dfrac{1}{2} & , -1 \leq x < 0 \\ -\dfrac{x^2}{2} + x + \dfrac{1}{2} & , 0 \leq x < 1, \\ 1 & , x \geq 1 \end{cases}$$

i. Find the probability density function (PDF) of $X$.

ii. Find the probability $\mathbb{P}(-1/2 < X < 2)$.

iii. Find its expectation $\mathbb{E}(X)$.

iii. Find its variance $Var(X)$.

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| Q6 \| | Q7 \| | | |
| Last Name: | | First Name: | |



4. (12 points) In the experiment of rolling a fair die twice, let $X$ be the maximum of the two numbers obtained.

   i. Determine and sketch the probability mass function (PMF).

   ii. Determine and sketch the cumulative distribution function (CDF).

5. (12 points) The probability mass function of a discrete random variable $X$ is given by

$$p(x) = \begin{cases} \dfrac{x}{15} & , x = 1, 2, 3, 4, 5, \\ \\ 0 & , otherwise. \end{cases}$$

Calculate the expected value of $X(6 - X)$.

6. (12 points) The probability mass function of a discrete random variable $X$ is given by

$$p(x) = \begin{cases} \dfrac{(|x|+1)^2}{27} & , x = -2, -1, 0, 1, 2, \\ & \\ 0 & , otherwise. \end{cases}$$

Find the variance of $X$.

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7. (12 points)

   i. Two fair dice are rolled 10 times. What is the probability that at least one 6 (on either die) shows in exactly five of these 10 rolls?

   ii. Suppose that $X$ is a Poisson random variable with

$$\mathbb{P}(X = 1) = \mathbb{P}(X = 3).$$

   Find $\mathbb{P}(X = 5)$.