## IŞIK UNIVERSITY, MATH 230 FINAL EXAM

Q1	Q2	Student ID:		Row No:
Q3	Q4			
Last Name:		First Name:		
I pledge my honour that I have not violated the honour code during this examination.			<b>d</b> .	
			Signature :	
Bu sınavda onur yasamızı ıhlal etmediğime				
şerefim üzerine yemin ederim.				

1. (13 points) The joint PDF of two random variables X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} 6xy^2 & , 0 < x < 1, 0 < y < 1\\ 0 & , else. \end{cases}$$

Find the probability  $\mathbb{P}\left(\frac{Y}{X} > 2\right)$ .

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2. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and  $A_i$  denote events in a sample space S and let  $\mathbb{P}(.)$  denote a probability measure on S, X and Y be random variables.

(Note: A statement is assumed to be true if it is true in any possible case, and it is assumed to be false if it fails in at least one case.):

*i.* If, for two random variables X and Y,  $X \ge Y$ , then  $\mathbb{E}(X) \ge \mathbb{E}(Y)$ . T F

*ii.* If 
$$Var(X) = 0$$
 then  $\mathbb{E}(X) = 0$ .  $T \in F$ 

*ii.* If 
$$\mathbb{P}(A) \leq \mathbb{P}(B)$$
 then  $A \subseteq B$ . T F

iv. PDF is a negative function. T = F

v. If the joint density,  $f_{X,Y}$ , of X and Y is given, then one can find  $\mathbb{E}(X)$ . T F

vi. If X and Y are independent random variables, then 
$$f_{X,Y} = f_X \cdot f_Y$$
. T F

vii. There are events with negative probability. T - F

viii. If X is a Gaussian with 
$$\mathbb{E}(X) = \mu$$
 and  $Var(X) = \sigma^2$ , then  
 $Y = (X - \mu)/\sigma$  is a standard Gaussian. T F

*ix.* If X is a continuous random variable, then for any point a,  $\mathbb{P}(X = a) = 0$ . T F

x. If Z is a standard Gaussian, the 
$$\mathbb{P}(Z > 0) = 0.5$$
. T F

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3. (15 points) If the joint PDF of two random variables X and Y is given by

$$f_{X,Y}(x,y) = \begin{cases} \frac{5}{16}(4x - x^2y) & , 0 < x < y, 0 < y < 2\\ 0 & , else, \end{cases}$$

then

i. find the marginal PDF,  $f_X(x)$ , of X

ii. find the conditional probability density  $f_{Y|X}(y|1)$ ,

iii. find the conditional probability  $\mathbb{P}(0 < Y < 1/2 | X = 1)$ .

4. (12 points) In a game, players starts at the point Z and walk among the paths given on the map below.



At each intersection, a player chooses one of the possible paths ahead of him/her with equal probabilities, then moves forward, and the player never walks back. When the player reaches points A, D or F, he/she gets 2 gold coins, when he/she reaches the point B, he/she gets 3 gold coins, and when he/she reaches points C or E he/she gets 4 gold coins. What is the expected number of gold coins that a player gets in this game?

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Q7	Q8		
Last Name:		First Name:	

5. (14 points) At a certain gas station 40% of the customers request regular gas, 35% request unleaded gas, and 25% request premium gas. Of those customers requesting regular gas, only 30% fill their tanks. Of those customers requesting unleaded gas, 60% fill their tanks, while of those requesting premium, 50% fill their tanks. If the next customer fills the tank, what is the probability that regular gas is requested?

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  - 6. (12 points)
    - i. Two fair coins are tossed, and it is known that at least one of them was a head. Find the probability that both were heads. Formulate the problem using conditional probabilities.

ii. On a typical January day in Istanbul the probability of snow is 0.1, the probability of a traffic jam is 0.8, and the probability of snow or a traffic jam (or both) is 0.82. Are the two events it snows and a traffic jam occurs independent? Give the full mathematical analysis to justify your answer.

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7. (12 points) The sales of a convenience store on a randomly selected day are X thousand dollars, where X is a random variable with a density function of the following form:

$$f(x) = \begin{cases} 0 & , x < 0, \\ x/2 & , 0 \le x < 1 \\ c(4-2x) & , 1 \le x < 2, \\ 0 & , x \ge 2 \end{cases}$$

i. Find the value of c.

ii. Let A be the events that tomorrow the stores total sales are between 500 and 1500 dollars, and B be the event that the sales are over 1000 dollars. Find  $\mathbb{P}(A)$  and  $\mathbb{P}(B)$ .

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## 8. (12 points)

i. It is observed that every 1 out of 4 babies born in a certain town is blonde. If we observe the new-born babies this month in this town, what is the probability that the first blonde baby will be the 10th baby?

ii. A manufacturer of nails claims that only 3% of its nails are defective. A random sample of 24 nails is selected. What is the probability that 2 of them are defective?