IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM II

Q1	Q2	Student ID:	Row No:					
Q3								
Last Name:		First Name:						
 (10 points) Determine whether the following statements are True or False. Circle T or F. No explanation is required. Let A, B, and A_i denote events in a sample space S and let P(.) denote a probability measure on S. (Note: A statement is assumed to be true if it is true in any possible case, and it is assumed to be false if it fails in at least one case.): 								
i. If	X is a continuous ra	ndom variable then we have $\mathbb{P}(X \ge c)$	$=\mathbb{P}(X>c).$	Т	F			
ii. If	If X is a discrete random variable then we have $\mathbb{P}(X \ge c) = \mathbb{P}(X > c)$.				F			
iii. A	A random variable with higher variance is more spread around its mean.							
iv. If	iv. If $p(x)$ is the PMF of a discrete r.v. X then $p(x) \le 1$ for any x.							
v. If	v. If $f(x)$ is the PDF of a continuous r.v. X then $f(x) \leq 1$ for any x.							
vi. V	$Var(X) \ge 0$ for any random variable X.							
vii. I	If $\mathbb{E}(X^2) = \mathbb{E}^2(X)$ then X is a constant random variable (X=c).				F			
viii.	$\mathbb{E}(X - \mathbb{E}(X)) > 0 \text{ for}$	or any random variable.		Т	F			
ix. E	$\mathbb{E}(aX+b) = a\mathbb{E}(X) + b \text{ for any } a, b \in \mathbb{R}.$				F			
x. V	$Var(aX) = aVar(X)$ for any $a \in \mathbb{R}$.							

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- 2. (12 points) A charity organisation decides to give boxes of books to kids this new years. So a box is prepared as a mix of novels and science books. But they have different weights. A science book has weight given by a Poisson distribution with parameter 400 and a novel has weight given by a Poisson variable with parameter 200.

Each box includes 3 novels and 5 science books. If we assume that an empty box weighs 50 gr, what is the expected weight of each box filled with books? (You have to set up your random variables and show each step clearly to get the full credit. If you use just basic algebra, you get only a few points.)

3. (30 points) Suppose X is a continuous random variable with the probability density function

$$f(x) = \begin{cases} c(x - x^2) & \text{if } 0 \le x \le 1\\ 0 & \text{else.} \end{cases}$$

i. Find the value of c.

ii. Find the probability $\mathbb{P}(X > 1/2)$.

ii. What is the cumulative distribution function of X?

ii. What is the expectation of X?

ii. What is the variance of X?

ii. What is the standard deviation of X?

$ \mathbf{Q4} $	Q5	Student ID:	Row No:
$\mathbf{Q6}$	Bonus		
Last Name:		First Name:	

- 4. (20 points)
 - i. Roll a dice twice and let X be the sum of the two outcomes. Then X is a discrete random variable. Find the CDF of X.

ii. A biased coin is tossed 6 times. The probability of heads on any toss is 0.3. Let X denote the number of heads that come up. What is $\mathbb{P}(1 < X \leq 5)$?

- 5. (14 points) 4) The number of accidents per working week in a particular shipyard is Poisson distributed with mean 0.5. Find the probability that:
 - i. In a particular week there will be at least 2 accidents.

ii. In a particular two week period there will be exactly 5 accidents.

ii. In a particular month (i.e. 4 week period) there will be exactly 2 accidents.

6. (14 points) In an army unit it is known that 11% of the soldiers in the unit has Type B blood but it is NOT known which soldiers have which blood type individually. An accident occurs and they start randomly test soldiers for their blood type .What is the probability that the first Type B blood donor is among the first four people in line? (That is, the first donor with Type B is either the first or the second or the third or the fourth person in line.)

Bonus Question (10 points) The roulette game consists of a small ball and a wheel with 38 numbered pockets around the edge.



As the wheel is spun, the ball bounces around randomly until it settles down in one of the pockets. Suppose you bet on a single number and random variable X represents the (monetary) outcome (the money you win or lose). If the bet wins, the payoff is \$35; otherwise the player loses the bet and gets no money back. What is the expected profit ?