## IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM II

Q1	$\mathbf{Q2}$	Student ID:	Row No:		
Q3	$\mathbf{Q4}$	Bonus Question			
Last Name:		First Name:			

 (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let X and Y denote random variables, ℙ(.) denote the probability, 𝔼(.) denote expected value and Var(.) denote variance.

(Note: A statement is assumed to be true if it is true in any case, and it is assumed to be false if it fails in at least one case.):

i.	$\mathbb{E}(X - \mathbb{E}(X)) = 0$ for any random variable X.	Т	F
ii.	If $\mathbb{E}(X) = 0$ then $X = 0$ .	T	F
iii.	If $X = 0$ (means X equals zero for any sample point) then $\mathbb{E}(X) = 0$ .	T	F
iv.	If X is a continuous random variable and $x_0$ is any point in the range of X, then $\mathbb{P}(X = x_0) = 0$ .	T	F
v.	If X is a continuous random variable and $f_X$ denotes its PDF, then $f_X$ can take values greater then 1.	Т	F
vi.	If X is a discrete random variable and $P_X$ denotes its PMF, then $P_X$ can take values greater then 1.	Т	F
vii.	$Var(aX+b) = a^2 Var(X) + b.$	T	F
viii.	If X is a standard Gaussian, then $\mathbb{P}(X \ge 0) = 1/2$ .	T	F
ix.	If X is a continuous random variable then $\mathbb{P}(X \leq c) = \mathbb{P}(X < c)$ .	T	F
x.	For any random variable X, we have $Var(X) \ge 0$ .	Т	F

## 2. (16 points)

i. The total number of goals in a soccer match is distributed as a Poisson random variable with an expected 4 goals per game. What is the probability that the total number of goals during the game exceeds 2? (exceed: aşmak)

ii. Heart Association says only 10% of adults can pass the fitness test. Suppose 20 people are selected at random . What is the probability that only 4 of them will pass the test?

iii. You decide to play the lottery. If the probability of winning of each ticket is 1/100, what is the probability that you win the first time at your  $40^{th}$  ticket?(lottery: piyango)

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3. (16 points) The random variable X has  $\mathbb{E}(X) = 1.3$  and probability mass function (PMF)

$$p(x) = \begin{cases} a & , x = 1, \\ b & , x = 2, \\ 0 & , otherwise \end{cases}$$

where a and b are two constants.

i. Find a and b.

ii. Find  $\mathbb{P}(X < 3/2)$ .

iii. Find Var(X).

iv. Find the standard deviation  $\sigma_X$ .

## 4. (12 points)

- i. List all types discrete random variables that you learned in class.
- ii. List all types continuous random variables that you learned in class.
- iii. Determine, in each case, which type of random variable fits the model the best.
  - a. Lois sends her resume to 1000 companies. Each company responds with probability 3/1000 (independently of what all other companies do). Let X be the number of companies that respond.

X is a ..... random variable.

b. You arrive at the bus stop sometime between 10am and 10:30am, and your arrival time is equally distributed between 10am and 10:30am. Let Y be the random variable denoting the time you arrive at the stop.

c. The expected number of typographical errors on a page of a certain magazine is 0.2. Let Z denote the number of errors on the next page.

d. A hospital keep records of the gender of each new-born baby. If it is a girl they assign the number 1, and if it is a boy they assign number 0.

This is a ..... random variable.

e. You try to open a door by using the keys in your pocket. You continue to try each key until one of them unlocks the door. R is the random variable recording how many keys you try until opening the door.

R is a ..... random variable.

f. S is a random variable with the probability density function  $f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ .

S is a ..... random variable.

$\mathbf{Q5}$	Q6	Student ID:	Row No:		
$\mathbf{Q7}$					
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- 5. (16 points) The thickness X of a protective coating applied to an astronaut suit follows a uniform distribution over the interval [20, 40] microns. (coating: kaplama, suit: elbise, thickness: kalınlık)
  - i. Find the mean,

ii. find the standard deviation,

iii. and find the probability cumulative distribution function (CDF) of the thickness of the protective coating?

iv. Find the probability that the coating is less than 35 microns thick.

6. (15 points) Let T be the lifetime in years of new laptop computers of a particular brand. Suppose that T is continuous with probability density function (PDF)

$$f_T(x) = \begin{cases} 0, x < 1, \\ \\ \frac{c}{x^3}, x > 1. \end{cases}$$

i. Find the value of c?

ii. Find the mean of T?

iii. Find the probability that a laptop will last 4 years or more?

7. (15 points) Assume that the test grades for a given test in a particular class are normally distributed with mean (i.e. expected value, or class average")  $\mu$  and standard deviation  $\sigma$ . The usual (though not universal) method for grading on a curve" means adjusting the overall scores so that they fit a normal distribution with mean  $\mu$  and variance  $\sigma^2$ . Generally, one assigns a grade of A to all scores that are greater than one standard deviation away from the mean, a grade of B to all scores that are above  $\mu + \frac{\sigma}{2}$  and lower than  $\mu + \sigma$ , a grade of C to all scores that are between  $\mu - \frac{\sigma}{2}$  and  $\mu + \frac{\sigma}{2}$ , a grade of D to all scores that are between  $\mu - \sigma$ . If this type of curve is used, determine the percentage of students that make each grade. (To solve this problem you do not need to know the actual values of the mean and standard deviation.)

<u>Türkçe Açıklaması</u>: Farzedelim ki bir sınavın sonucları normal dağılıma (çan eğrisine) uygun olarak dağılmış olsun ve bu dağılımın beklenen değeri  $\mu$ , standart sapması ise  $\sigma$  olsun. Bu durumda "çan eğrisine göre notlama" çoğunlukla şöyle yapılır: (Notların ortalaması  $\mu$  ve varyansı  $\sigma^2$  olacak şekilde ayarlandıktan sonra) eğer not  $\mu + \sigma$ 'den büyükse A,  $\mu + \frac{\sigma}{2}$  ile  $\mu + \sigma$  arasında ise B,  $\mu - \frac{\sigma}{2}$ ile  $\mu + \frac{\sigma}{2}$  arasında ise C,  $\mu - \sigma$  ve  $\mu - \frac{\sigma}{2}$  arasında ise D ve  $\mu - \sigma$  altında ise F ile değerlendirilir. Bu durumda Çan Eğrisi kullanıldığında öğrencilerin yüzde kaçının A, yüzde kaçının B, yüzde kaçının C, yüzde kaçının D ve yüzde kaçının F alacagini bulun. (Bu soruyu çözmek için  $\mu$  ve  $\sigma$ 'nın değerlerine ihtiyacınız yok.) Below, you can find the values of the CDF  $\Phi(z)$  of a standard normal (Gaussian) random variable.

Useful Reminder: The density of a Gaussian random variable X with mean  $\mu$  and variance  $\sigma^2$  is  $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/(2\sigma^2)}$ .

Entry is area A under the standard normal curve from  $-\infty$  to z(A)



	2(1)									
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	,9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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BONUS Question (10 points) What is the value of the integral

$$\int_{-2}^4 \frac{1}{\sqrt{18\pi}} e^{-(x-1)^2/18} dx$$

(Just the result will not receive any credit. Explain your answer for full credit.)