November 12, 2013

IŞIK UNIVERSITY, MATH 230 MIDTERM EXAM I

Q1 |	**Q2** |	**Student ID:**	**Row No:**
Q3 |	**Q4** |	**Bonus Question** |	
Last Name:		**First Name:**	

1. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, and A_i denote events in a sample space S and let $\mathbb{P}(.)$ denote a probability measure on S.
(*Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

   i. If A and B are independent, then $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B)$. T F

   ii. If $\mathbb{P}(A|B) = \mathbb{P}(A)$ then $\mathbb{P}(B|A) = \mathbb{P}(B)$. T F

   iii. $\mathbb{P}(A|B^c) = 1 - \mathbb{P}(A|B)$. T F

   iv. $\mathbb{P}(A^c|B) = 1 - \mathbb{P}(A|B)$. T F

   v. If $\mathbb{P}(A \cup B) = \mathbb{P}(A)$ then B is an empty set. T F

   vi. If A and B are independent, then $\mathbb{P}(A|B) = \mathbb{P}(A|B^c)$. T F

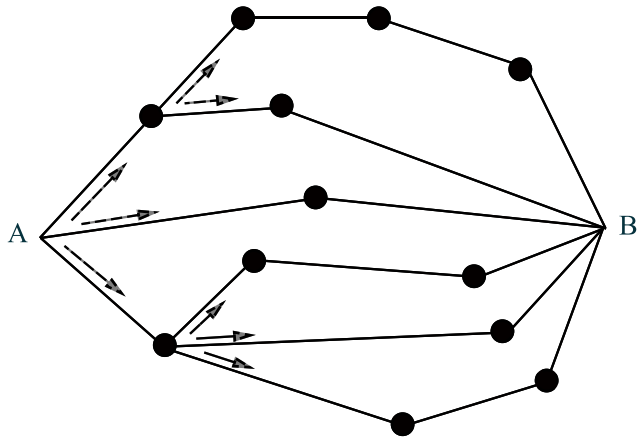
   vii. $\mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B)$ for any A and B. T F

   viii. If $\mathbb{P}(E) < \mathbb{P}(F)$ then $E \subset F$. T F

   ix. If A and B are two mutually exclusive events, then they are independent. T F

   x. The number of r-permutations of n objects is greater then the number of r-combinations of n objects. T F

2. (12 points) Consider the following map which shows all possible paths from A to B.



A hiker walks from A to B. At every intersection of roads he randomly chooses one of the possible roads. (For example, at point A, he chooses upper road with probability 1/3, the middle road with probability 1/3 and the lower road with probability 1/3.) He never turns back, and always keep walking toward B.
If X is a random variable that counts the number of circles ● the hiker passes, then what is

i. the possible values that X can take?

ii. the probability $\mathbb{P}(X = 2)$?

iii. the probability $\mathbb{P}(X \geq 2)$?

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3. (12 points) Let A, B and F be three events with the conditional probabilities

$$\mathbb{P}(A|F) = 0.5 \qquad \mathbb{P}(B|F) = 0.4, \qquad \mathbb{P}(A \cap B|F) = 0.2$$

i. What is the probability $\mathbb{P}(A \cup B|F)$?

ii. What is the probability $\mathbb{P}(A^c \cup B^c|F)$?

iii. What is the probability $\mathbb{P}(A - B|F)$?

4. (15 points) Suppose we have 3 coins such that if the i^{th} coin is flipped, heads will appear with probability $\frac{i}{3}$ for $i = 1, 2, 3$. When one of the coins is randomly selected and flipped, it shows heads. What is the probability that it was the second coin?

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Q7 |	Q8 |		
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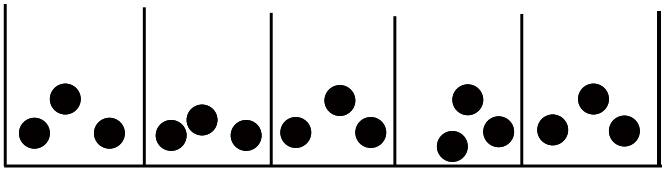
5. (13 points) In how many ways can you place 15 labelled balls in 5 boxes with 3 balls to each box ?



6. (12 points) A card is drawn from a standard deck of 52 cards. Without replacing the first card, a second card is drawn. Is the event that the second card is a diamond independent of the event that the first card is a diamond? Verify your answer. (Note: There are 13 diamonds ♦ in a standard 52-card deck.)

7. (14 points) An instructor gives her class a set of 10 problems with the information that the final exam will consist of a random selection of 5 of them. If a student has figured out how to do 7 of the problems and he or she doesn't know how to answer the remaining 3 problems, what is the probability that he or she will answer correctly

   i. all 5 problems or

   ii. at least 4 of the problems?

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8. (12 points) A bakery employs 3 bakers who all make brownies. Ayşe makes 25% of the brownies, while Zeynep makes 45% and Ali makes 30%. Everyone makes mistakes though: Ayse burns 2% of her brownies, while Zeynep and Ali burn 4% and 5% of theirs, respectively.

   i. What is the probability that a certain brownie is burned?

   ii. Given that a brownie has been burned, what is the probability that it was baked by Zeynep ?

   iii. Given that a brownie is not burned, what is the probability that it was baked by Ali ?

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BONUS Question (10 points) Is it possible that a set A is independent of itself, that is, A is independent of A? If YES, give an example, if NO, then show that it is not possible. (Just "yes"/"no" will not receive any credit.)