IŞIK UNIVERSITY, MATH 230 Final EXAM

Q1	Q2	Student ID:	Row No:
Q3	$ \mathbf{Q4} $		
Last Name:		First Name:	

1. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let X and Y denote random variables, $\mathbb{P}(.)$ denote the probability, $\mathbb{E}(.)$ denote expected value and Var(.) denote variance.

(Note: A statement is assumed to be true if it is true in any case, and it is assumed to be false if it fails in at least one case.):

<i>i</i> .	$\mathbb{E}(X - \mathbb{E}(X)) = 0$ for any random variable X.	Т	F
ii.	If $\mathbb{E}(X) = 0$ then $X = 0$.	T	F
iii.	If $\mathbb{P}(A B) = \mathbb{P}(A)$ then $\mathbb{P}(B A) = \mathbb{P}(B)$.	T	F
iv.	If X is a continuous random variable and x_0 is any point in the range of X, then $\mathbb{P}(X = x_0) = 0$	T	F
v.	If A and B are independent, then $\mathbb{P}(A B) = \mathbb{P}(A B^c)$.	T	F
vi.	$\mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B)$ for any A and B.	T	F
vii.	If A and B are two mutually exclusive events,		
	then they are independent.	T	F
viii.	If $\mathbb{P}(A) = 0$ then A is an empty set.	T	F
ix.	If $\mathbb{P}(E) < \mathbb{P}(F)$ then $E \subset F$.	T	F
<i>x</i> .	For any random variable X, we have $Var(X) \ge 0$.	Т	F

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2. (15 points)

i. On a multiple choice exam, each question has 3 possible answers. What is the probability that a student gets at least 4 correct answers just by guessing, if the test consists of 5 questions in total?

ii. The **monthly** worldwide average number of airplane crashes of commercial airlines is 3.5. What is the probability that at most 1 accident will occur in next 2 moths?

Student's Name :____

3. (15 points)A random variable X has the probability distribution function (CDF)

$$F(x) = \begin{cases} 0 & , x < 1 \\ 0.1 & , 1 \le x < 2, \\ 0.3 & , 2 \le x < 3, \\ c & , 3 \le x < 4, \\ 1 & , x \ge 4. \end{cases}$$

where c is a constant. It is also known that $\mathbb{P}(2.5 < X < 3.5) = 0.4$

i. Find c.

ii. Write the probability mass function, PMF.

iii. Find $\mathbb{E}(X)$.

iv. Find Var(X).

v. Find the standard deviation σ_X .

4. (15 points) Let A, B and C be 3 independent events with the probabilities

 $\mathbb{P}(A) = 0.4 \qquad \mathbb{P}(B) = 0.3 \qquad \mathbb{P}(C) = 0.2$

i. What is the probability $\mathbb{P}(A \cup B \cup C)$?

ii. What is the probability $\mathbb{P}(A|B)$?

iii. What is the probability $\mathbb{P}(A^c \cup B^c \cup C^c)$?

Q5	Q6	Student ID:	Row No:
Q7	Q8		
Last Name:		First Name:	

5. (10 points)A box contains seven red and 13 blue balls. Two balls are selected at random and are discarded without their colors being seen. If a third ball is drawn randomly and observed to be red, what is the probability that both of the discarded balls were blue? (discard: atmak, elemek)

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- 6. (10 points) From an ordinary deck of 52 cards we draw cards at random, with replacement, and successively until an ace is drawn. What is the probability that at least 10 draws are needed?

Department of Mathematics, Işık University

Student's Name :_____

7. (15 points) The joint PDF of X and Y is given as

$$f(x) = \begin{cases} x + y &, 0 < x < 1, 0 < y < 1 \\ 0 &, otherwise \end{cases}$$

i. Are X and Y independent ? Why ?

ii. Find the density function of X.

iii. Find the probability $\mathbb{P}(Y + X < 1)$.

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- 8. (10 points) The scores on an achievement test given to 100,000 students are normally distributed with mean 500 and standard deviation 100. What should the score of a student be to place him among the top 10% of all students?

Below, you can find the values of the CDF $\Phi(z)$ of a standard normal (Gaussian) random variable.

Useful Reminder: The density of a Gaussian random variable X with mean μ and variance σ^2 is $f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/(2\sigma^2)}$.

Entry is area A under the standard normal curve from $-\infty$ to z(A)



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	,9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	