Işık University   **Math 230**   **Exam I**

Exam Duration : 1 hr 30 min   **Nov. 12, 2012**

Department of Mathematics, Işık University

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Last Name :———————————

First Name :———————————

Student Number :———————————

Section :   ①   ②   ③

Instructor : Deniz Karlı ☐ / Sinan Özeren ☐

Row #:———————————

Directions. *Please read each question carefully. Show all work clearly in the space provided. For full credit, solution methods must be complete, logical, and understandable. Answers must be clearly labeled in the spaces provided after each question. Please mark out or fully erase any work that you do not want graded. The point value of each question is indicated before its statement. No books or other references are permitted. Calculators are not allowed. You must show all your work to receive full credit on a problem.*

GRADES			
Q 1	/10	Q 5	/15
Q 2	/12	Q 6	/15
Q 3	/18	Q 7	/15
Q 4	/15		
Total			/100

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1. (10 points) Determine whether the following statements are True or False. Circle **T** or **F**. No explanation is required. Let A, B, E, F and A_i denote events in a sample space S and let $P(.)$ denote a probability measure on S. (*Note: A statement is assumed to be true if it is true* **in any possible case**, *and it is assumed to be false if it fails in at least one case.*):

   i. $P(A \cup B) = P(A) + P(B)$.   *T   F*

   ii. $P(\emptyset) = 0$.   *T   F*

   iii. $P(\bigcap_{i=1}^{n} A_i^c) = 1 - P(\bigcup_{i=1}^{n} A_i)$.   *T   F*

   iv.   If $P(E \mid S) = P(E)$.   *T   F*

   v.   If $P(A) = 0$ then A is an empty set.   *T   F*

   vi.   If A and B are independent, then $P(A|B) = 1 - P(A^c|B^c)$.   *T   F*

   vii. If $P(E) > 0$ then $P(E \cap F \mid E) \geq P(E \cap F \mid E \cup F)$.   *T   F*

   viii. If $P(E) < P(F)$ then $E \subset F$.   *T   F*

   ix.   If A and B are two mutually exclusive events,

         then they are independent.   *T   F*

   x.   If X is a discrete random variable,

         then $P(X \leq a) = P(X < a)$ for any constant a.   *T   F*

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2. (12 points) Determine which of the following functions are PMFs of random variables and which are not. State your answer and your reason clearly in the provided boxes.

(a) $f(x) = \begin{cases} 1/2, & x = 0 \\ 1/3, & x = 1 \\ 1/6, & x = 2 \\ 0, & \text{otherwise} \end{cases}$

vfill

(b) $g(x) = \begin{cases} -1/2, & x = 0 \\ 1/2, & x = 1 \\ 1/2, & x = 2 \\ 0, & \text{otherwise} \end{cases}$

(c) $h(x) = \begin{cases} 0.2, & x = 0 \\ 0.7, & x = 1 \\ 0.05, & x = 2 \\ 0, & \text{otherwise} \end{cases}$

Determine which of the following functions are CDFs of random variables and which are not. State your answer and your reason clearly in the provided boxes.

(d) $f(x) = \begin{cases} 1/4, & x = 0 \\ 1/2, & x = 1 \\ 1/4, & x = 2 \end{cases}$

(e) $g(x) = \begin{cases} 1/2, & x = 1 \\ 1/2, & x = 2 \\ 0, & \text{otherwise} \end{cases}$

(f) $h(x) = \begin{cases} 0, & x < 0 \\ 0.5, & 0 \le x < 10 \\ 1, & x \ge 10 \end{cases}$

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3. (18 points)

    i. Suppose that, on average, in every three pages of a book there is one mistake. If the number of mistakes on a single page of the book is a Poisson random variable, what is the probability of at least one error on a specific page of the book? What is the expected value of the number of errors?

    ii. You are sitting in front of your house and observing the plate registrations of the cars that are passing by. You observe a total of 100 cars. If the probability of observing a car with Istanbul plate is 70%, Ankara plate 20% and Izmit plate 10%, what is the probability that out of 100 cars you observe 50 of them have Istanbul plate, 30 of them Ankara plate and 20 of them Izmir plate ?

    iii. A restaurant serves 8 different dishes of fish, 12 of beef, and 10 of poultry. If customers select from these dishes randomly, what is the probability that two of the next four customers order fish entrées?

4. (15 points) Let A, B and C be three events with the following probabilities:

$$P(A) = 0.2 \qquad P(A \cap B) = 0.1 \qquad P(A \cap B \cap C) = 0.08$$
$$P(B) = 0.5 \qquad P(A \cap C) = 0.16$$
$$P(C) = 0.8 \qquad P(B \cap C) = 0.4$$

   i. Are the events A and C^c independent?

   ii. Are the events B^c and C^c independent?

   iii. Are the events A, B and C independent?

5. (15 points) An urn contains 6 green and 2 blue balls. The balls are randomly selected from the urn one after another. If a green ball is selected, it is replaced by another green ball. If a blue ball is selected then it is not replaced and the blue ball is kept outside the urn. The experiment ends when both blue balls are outside the urn. Let X be the random variable recording the number of trial at which the experiment ends. (For example, if the first ball and the second ball selected are blue, the experiment ends and $X = 2$.)

   i. Find $P_X(3)$ where P_X denotes the PMF of the random variable X.

  ii. Find $F_X(3)$ where F_X denotes the CDF of the random variable X.

6. (15 points) In a city in northwestern Turkey, 25% of the cars emit too much pollution (above the safe level). Every year, each car has to be checked for its engine pollution output. The probability that a bad car (a car that procudes excessive pollution) will fail the pollution test is 99%. On the other hand a good car (a car producing little or no pollution) has 17% probability of failing the test. What is the probability that a car failing the test is producing excessive pollution ?

7. (15 points) A game is played with a pair of dice as follows; one rolls the two dice, and if the sum of the dice S is equal 7 then he/she wins, if S bigger or equal to 10, he/she losses. if S is anything else, the dice is rolled again (that is, the game starts over.). What is the probability of wining?