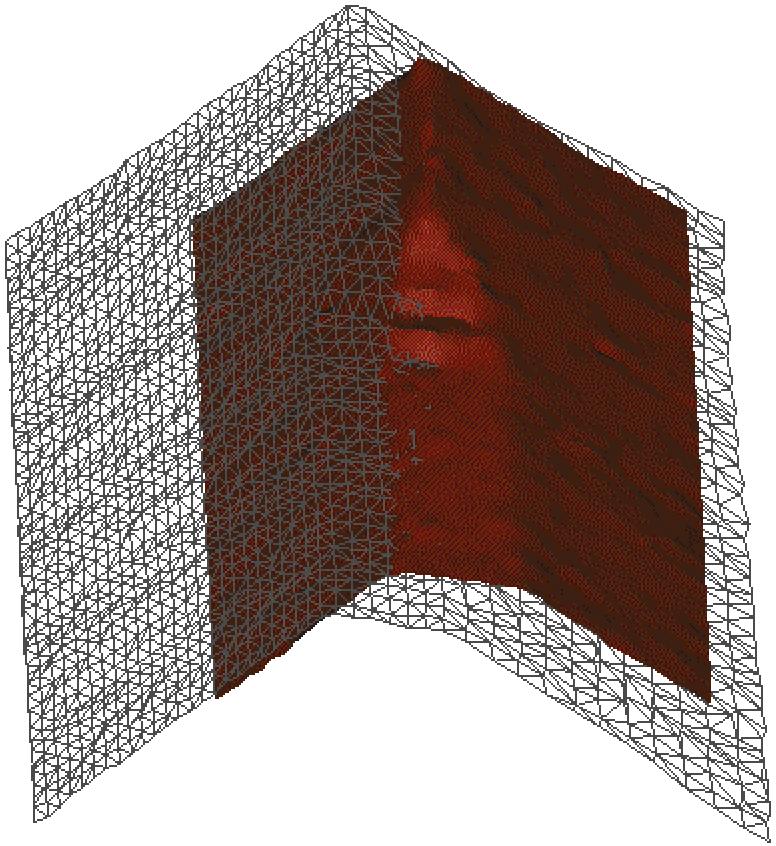
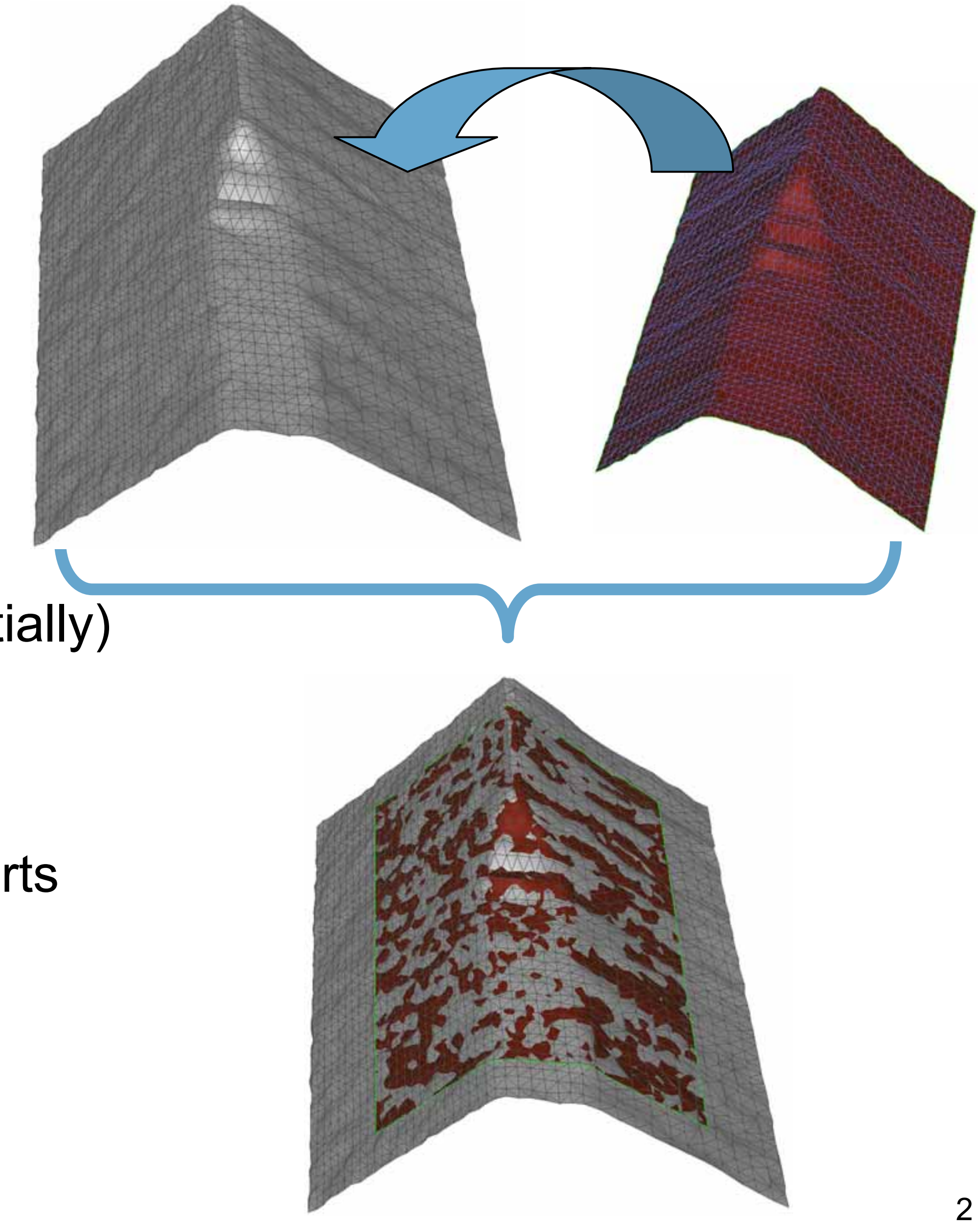


Least Squares 3D surface matching

Devrim Akca

Institute of Geodesy and Photogrammetry
Swiss Federal Institute of Technology Zurich
www.photogrammetry.ethz.ch

The Objective: Co-registration of overlapping 3D surfaces



An object surface may be:

- digitized using:
  + a laser scanner device,
  + the photogrammetric method,
  + or other techniques
- acquired:
  + from different standpoints (spatially)
  + at different times (temporally)

The goal:
Matching of the conjugate surface parts
and
estimating the 3D transformation

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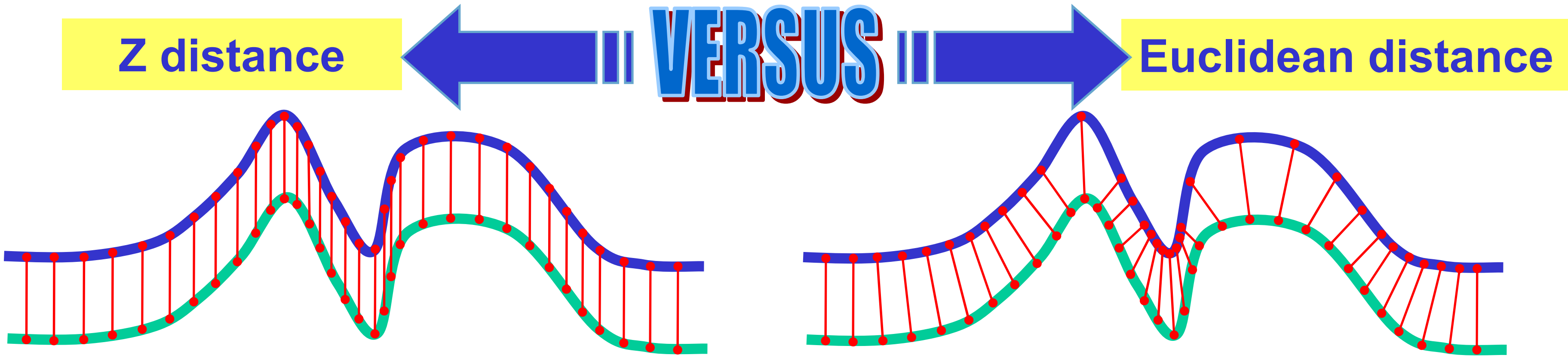
Introduction: Previous work

Least Squares Matching (LSM) (Grün, 1985)

- Surface matching first was addressed as a straight extension of LSM

DEM Matching (Ebner & Müller, 1986; Ebner & Strunz, 1988; Rosenholm & Torlegard, 1988)

- Minimizes height differences along Z-axis by LSs (corresponds to LSM)
- It has been used for:
  - + absolute orientation of stereo models
  - + block triangulation
  - + registration of airborne laser scanner strips



- Valid for **2.5D** surfaces, cannot work with **3D** surfaces

Introduction: Previous work and Motivation

Iterative Closest Point (ICP) (Besl & McKay, 1992; Chen & Medioni, 1992; Zhang, 1994)
- Iterative solution based on closed-form LS rigid transformation
- Converges slowly
- Lacks of internal quality indicators

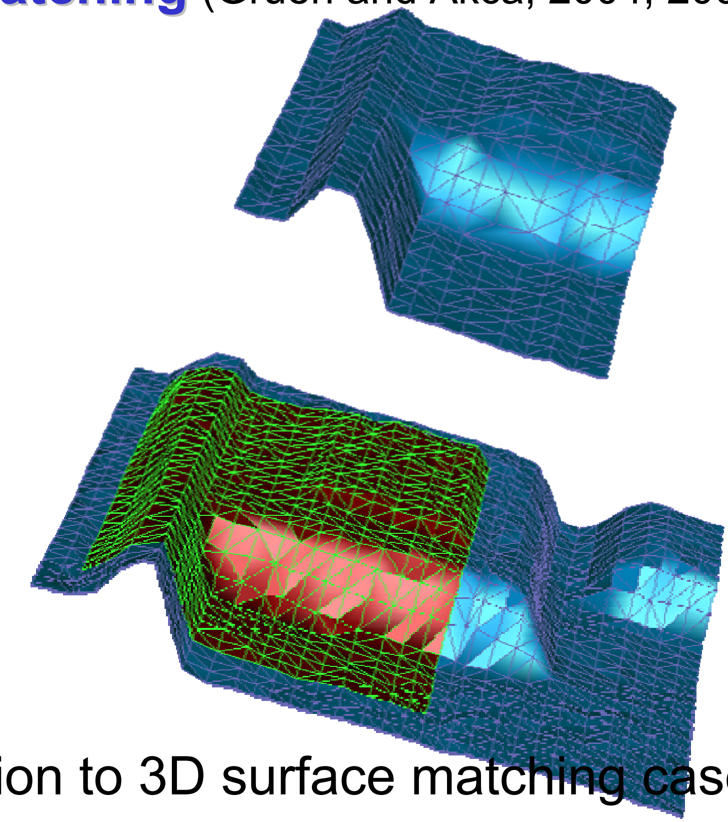
Motivation: to develop such a surface matcher,
- Matching of fully 3D surfaces (as opposed to 2.5D)
- Rigorous mathematical model for high accuracy demands
- Flexible mathematical model for further algorithmic extensions
- Mechanisms and statistical tools for internal quality control
- Capability of matching of data sets in different quality and resolution

Introduction: Our proposed method LS3D

LS Image Matching (Grün, 1984; 1985)

LS Cuboid (Voxel) Matching (Maas, 1994; Maas and Grün, 1995)

LS 3D Surface Matching (Gruen and Akca, 2004; 2005)



Generalization to 3D surface matching case

The basic estimation model: Observation equations

Two partial surfaces of an object:

- template surface f(x,y,z) and search surface g(x,y,z) (to be transformed)
- surface representation in a piecewise form
- f(x,y,z) and g(x,y,z) any surface element

3D transformation of the search surface g(x,y,z) to be estimated.
In a ideal case,

$$f(x,y,z) = g(x,y,z) \qquad (1)$$

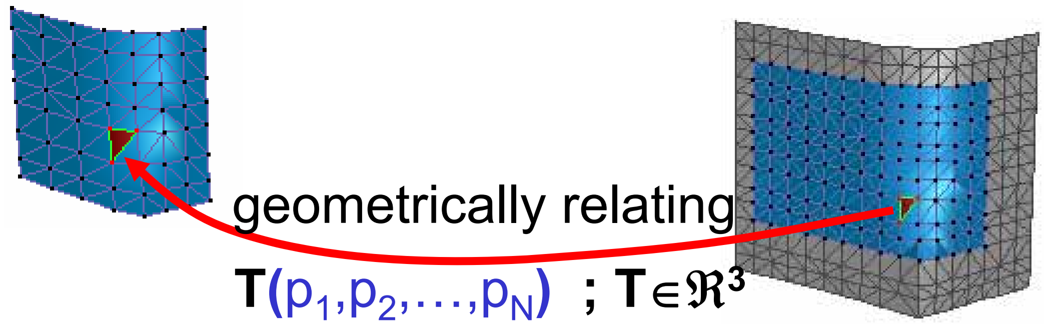
Considering the stochastic discrepancies,

$$f(x,y,z) - e(x,y,z) = g(x,y,z) \qquad (2)$$

Equation (2) is observation equations
The goal function: $[d_E d_E] = \min$
The final location of g(x,y,z) is estimated w.r.t. an initial position $g^0(x,y,z)$

The basic estimation model: Geometric relationship

f(x,y,z)



g(x,y,z)

Geometric relationship: 7-parameter 3D similarity transformation

$$x = t_x + m(r_{11}x_0 + r_{12}y_0 + r_{13}z_0)$$
$$y = t_y + m(r_{21}x_0 + r_{22}y_0 + r_{23}z_0) \quad (3)$$
$$z = t_z + m(r_{31}x_0 + r_{32}y_0 + r_{33}z_0)$$

The basic estimation model: Functional model

Non-linear functional model,

$$f(x,y,z) - e(x,y,z) = g(x,y,z) \qquad (2)$$

Linearization by Taylor Expansion,

Numerical derivatives: g_x, g_y, g_z

$$f(x,y,z) - e(x,y,z) = g^0(x,y,z) + \frac{\partial g^0(x,y,z)}{\partial x}dx + \frac{\partial g^0(x,y,z)}{\partial y}dy + \frac{\partial g^0(x,y,z)}{\partial z}dz \qquad (4)$$

Initial state (approximations)

$$dx = dt_x + a_{10}\,dm + a_{11}\,d\omega + a_{12}\,d\varphi + a_{13}\,d\kappa$$

$$dy = dt_y + a_{20}\,dm + a_{21}\,d\omega + a_{22}\,d\varphi + a_{23}\,d\kappa$$

$$dz = dt_z + a_{30}\,dm + a_{31}\,d\omega + a_{32}\,d\varphi + a_{33}\,d\kappa$$

Differentiation terms

The basic estimation model: Functional model

Final functional model in linearized form:

$$
\begin{aligned}
-e(x,y,z) = {} & g_x dt_x + g_y dt_y + g_z dt_z \\
& + (g_x a_{10} + g_y a_{20} + g_z a_{30})dm \\
& + (g_x a_{11} + g_y a_{21} + g_z a_{31})d\omega \\
& + (g_x a_{12} + g_y a_{22} + g_z a_{32})d\varphi \\
& + (g_x a_{13} + g_y a_{23} + g_z a_{33})d\kappa \\
& - (f(x,y,z) - g^0(x,y,z))
\end{aligned}
\qquad (5)
$$

The functional model in matrix notation:

$$-\mathbf{e} = \mathbf{A}\,\mathbf{x} - \ell \quad , \quad \mathbf{P} \qquad (6)$$

weight matrix	**P**
design matrix	**A**
parameter vector	**x**T = [dtx dty dtz dm dω dφ dκ]
discrepancies vector	ℓ = f(x,y,z)–g0(x,y,z)

The unknown parameters as stochastic quantities,

$$-\mathbf{e}_b = \mathbf{I}\mathbf{x} - \ell_b \quad , \quad \mathbf{P}_b \qquad (7)$$

The basic estimation model: Mathematical model

The total system is a Generalized Gauss-Markoff model:

$$-\mathbf{e} = \mathbf{A}\,\mathbf{x} - \ell \quad , \quad \mathbf{P} \qquad (8)$$

$$-\mathbf{e}_b = \mathbf{I}\mathbf{x} - \ell_b \quad , \quad \mathbf{P}_b \qquad (9)$$

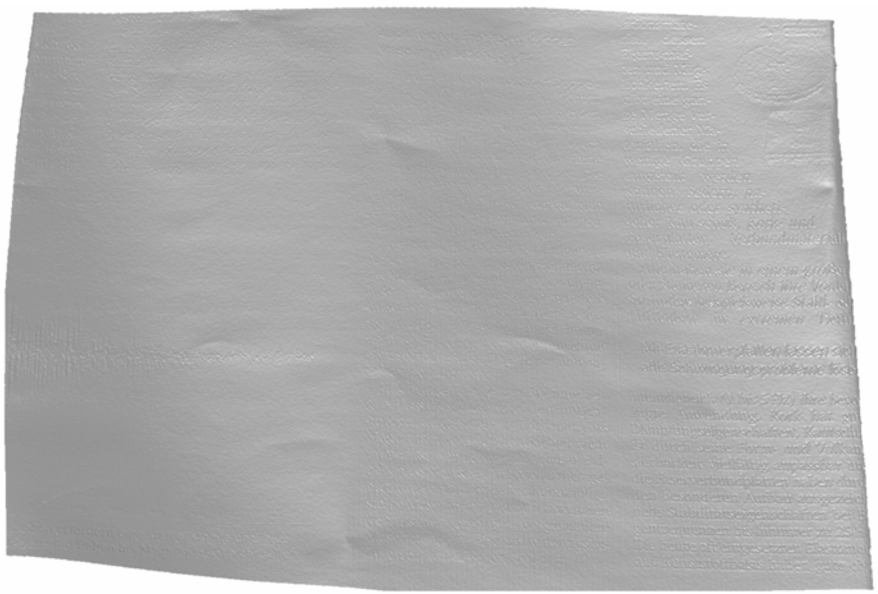
The Least Squares solution of the joint system gives as:

Solution vector: $\hat{\mathbf{x}} = (\mathbf{A}^T\mathbf{P}\mathbf{A} + \mathbf{P}_b)^{-1}(\mathbf{A}^T\mathbf{P}\ell + \mathbf{P}_b\ell_b)$ (10)

Variance factor: $\hat{\sigma}_0^2 = \dfrac{\mathbf{v}^T\mathbf{P}\mathbf{v} + \mathbf{v}_b^T\mathbf{P}_b\mathbf{v}_b}{r}$ (11)

The solution is iterative.

Experimental results 1: Newspaper



- **object:** a newspaper
- **scanner:** stereoSCAN^{3D} (Breuckmann)
- **average point spacing:** ~150microns.

This example:

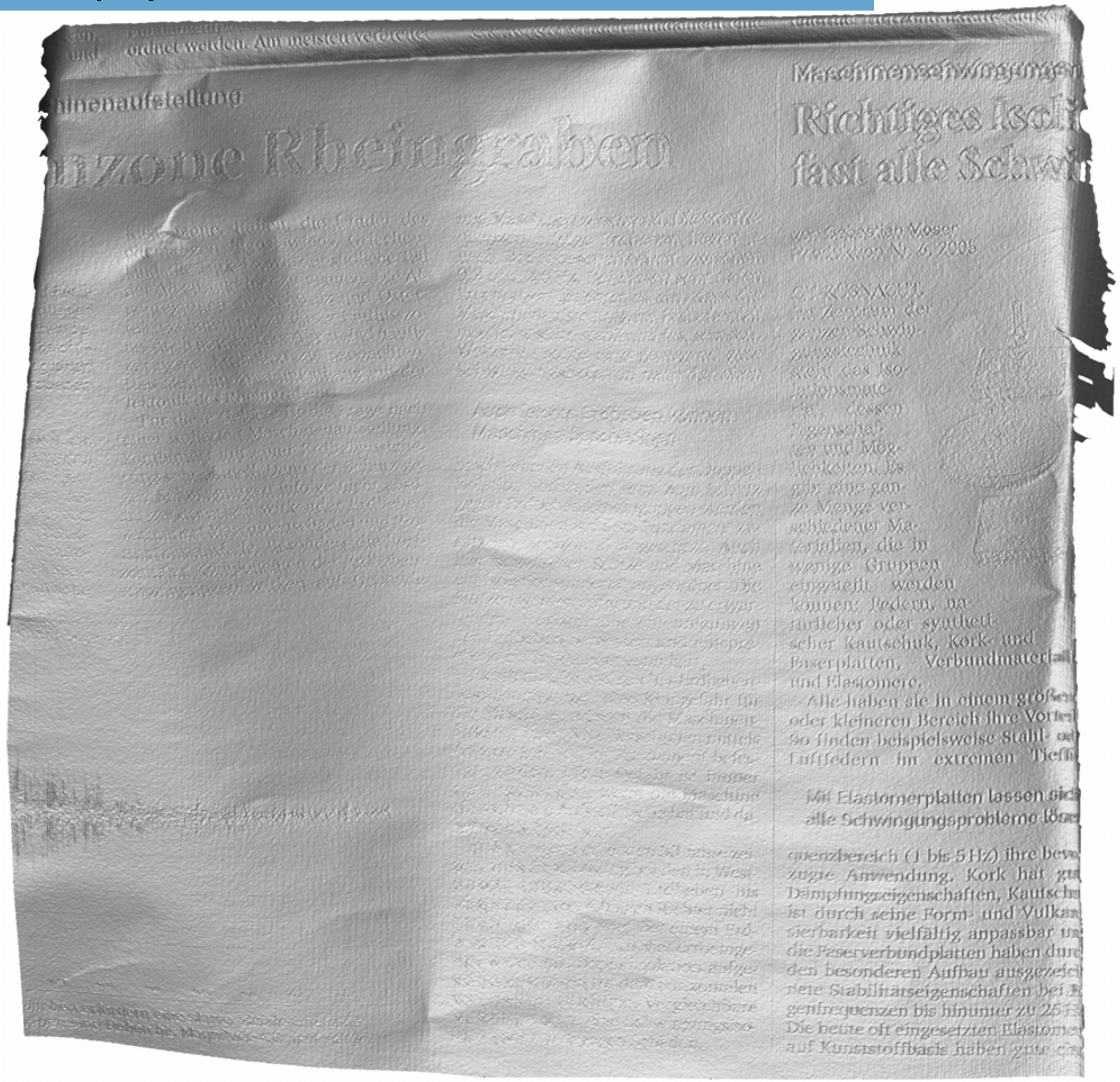
- Difficult case due to little surface information
- Little changes in surface curvature, almost a plane

Data set is courtesy of Breuckmann GmbH (Germany), http://www.breuckmann.com

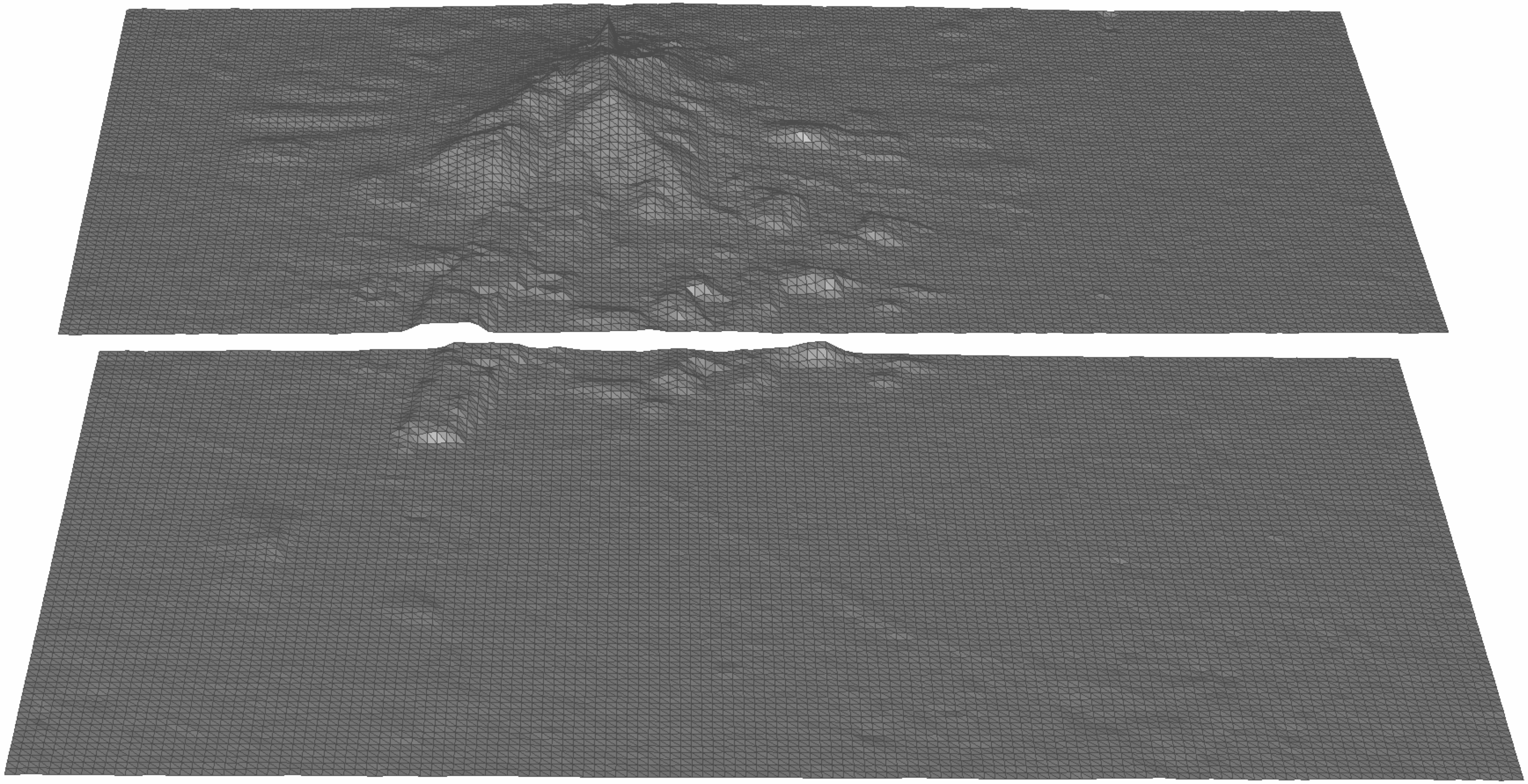
Experimental results 1: Newspaper

The result of the LS3D is successful.

No. points	377K
No. iterations	13
Sigma naught	**11.3 µm**
Time	36.7 sec.



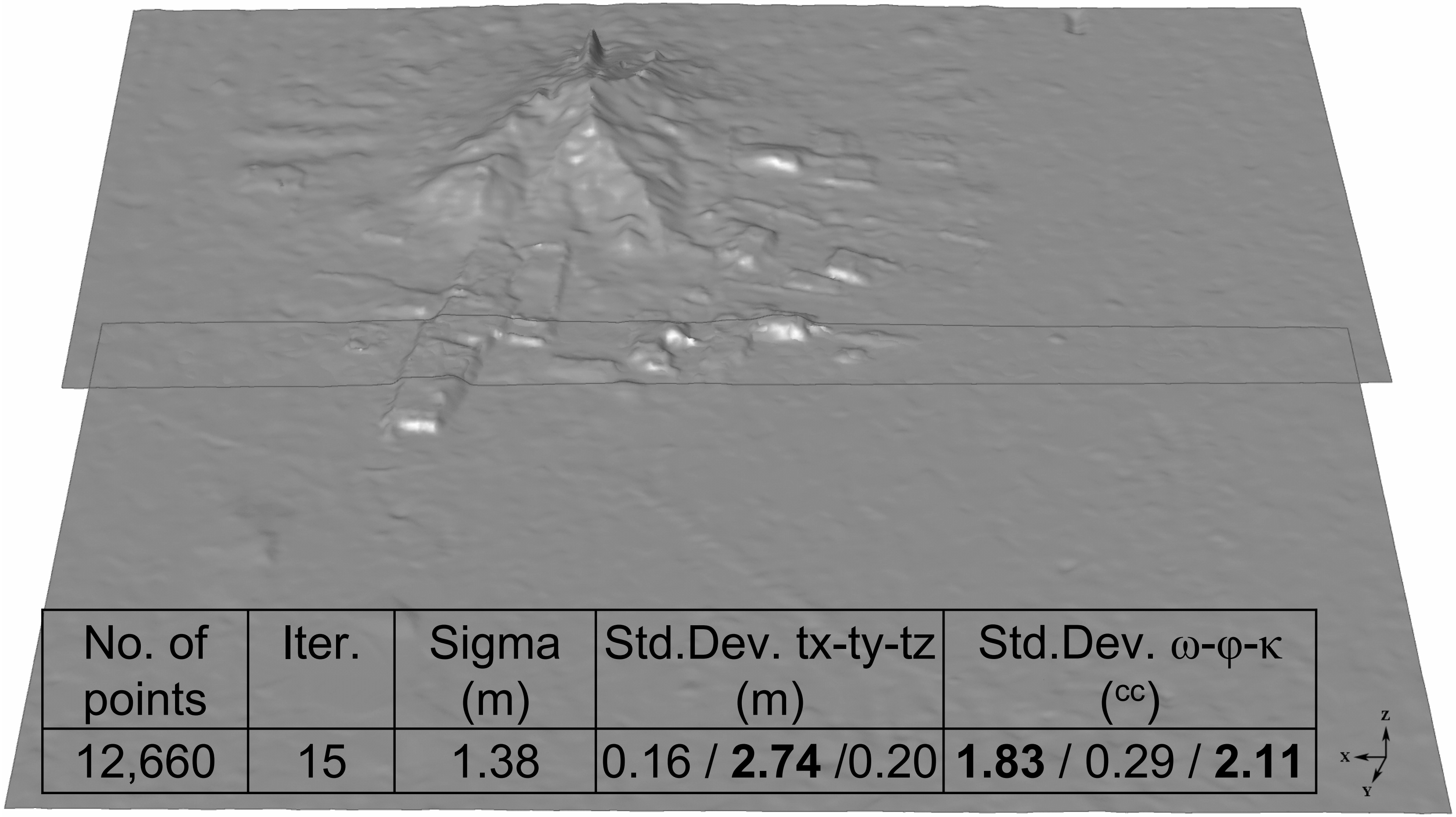
Experimental results 2: Tucume



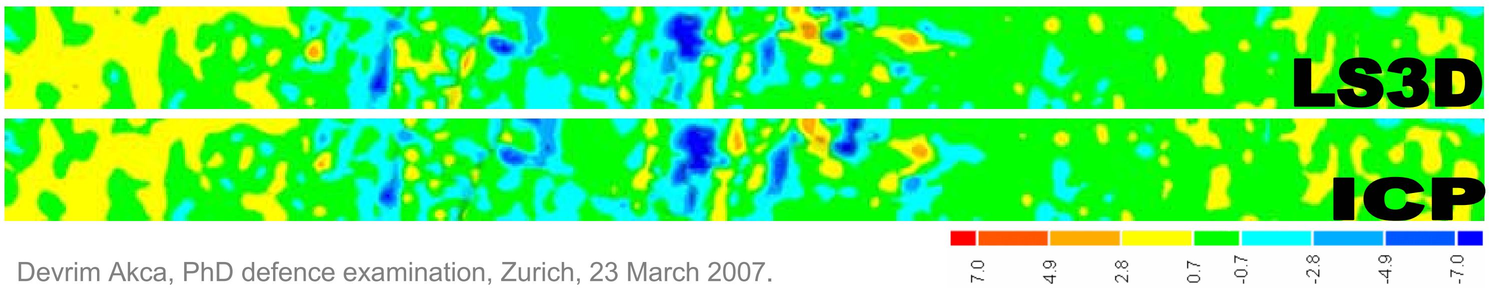
- Two **photogrammetrically** derived DTMs of an area in Tucume (Peru),
- Horizontal resolution is 5 meters,
- **<u>This example:</u>** Difficult case due to very narrow overlap along Y-direction

Data set is courtesy of Martin Sauerbier (ETH Zurich)

Experimental results 2: Tucume



No. of points	Iter.	Sigma (m)	Std.Dev. tx-ty-tz (m)	Std.Dev. ω-φ-κ (cc)
12,660	15	1.38	0.16 / **2.74** /0.20	**1.83** / 0.29 / **2.11**



(1.34 m)

Geomagic Studio
(1.42 m)

Experimental results 3: Weary Herakles

- Cultural heritage application
- 3D modeling of the lower part of a marble Herakles statue
- In the Antalya Museum

This example shows:

- Co-registration of multiple surfaces

Experimental results 3: Weary Herakles

- Digitization in the Antalya Museum
- Breuckmann optoTOP-HE coded structured light system
- 1 ½ days on site work with 67 scans
- 83.75M points in total

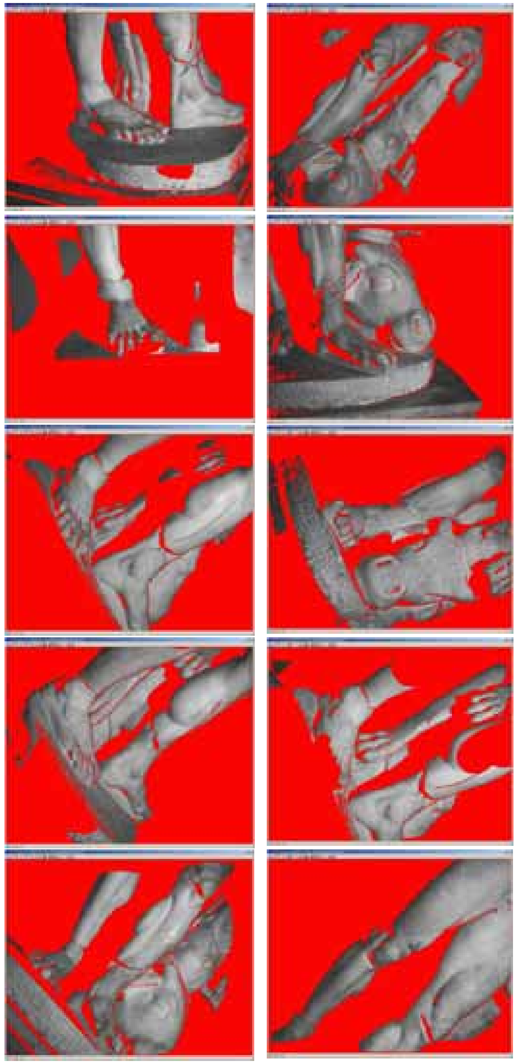
Experimental results 3: Weary Herakles

234 consecutive pairwise LS3D matching. The average sigma naught is **81 microns**.

Example: Registration of 1st and 2nd scans
Note: 3x3 down-sampling for better visualization

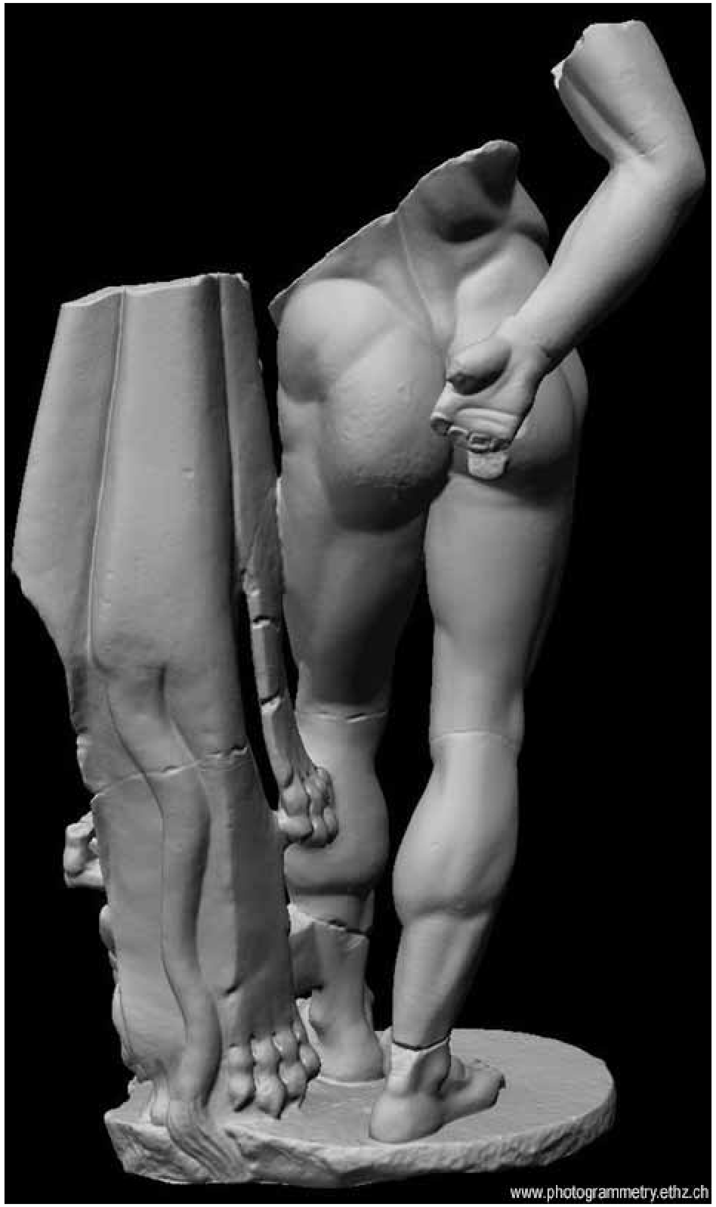
Experimental results 3: Weary Herakles

Global registration with the block adjustment by independent models solution
Sigma naught **47 microns**, in agreement with the system specifications

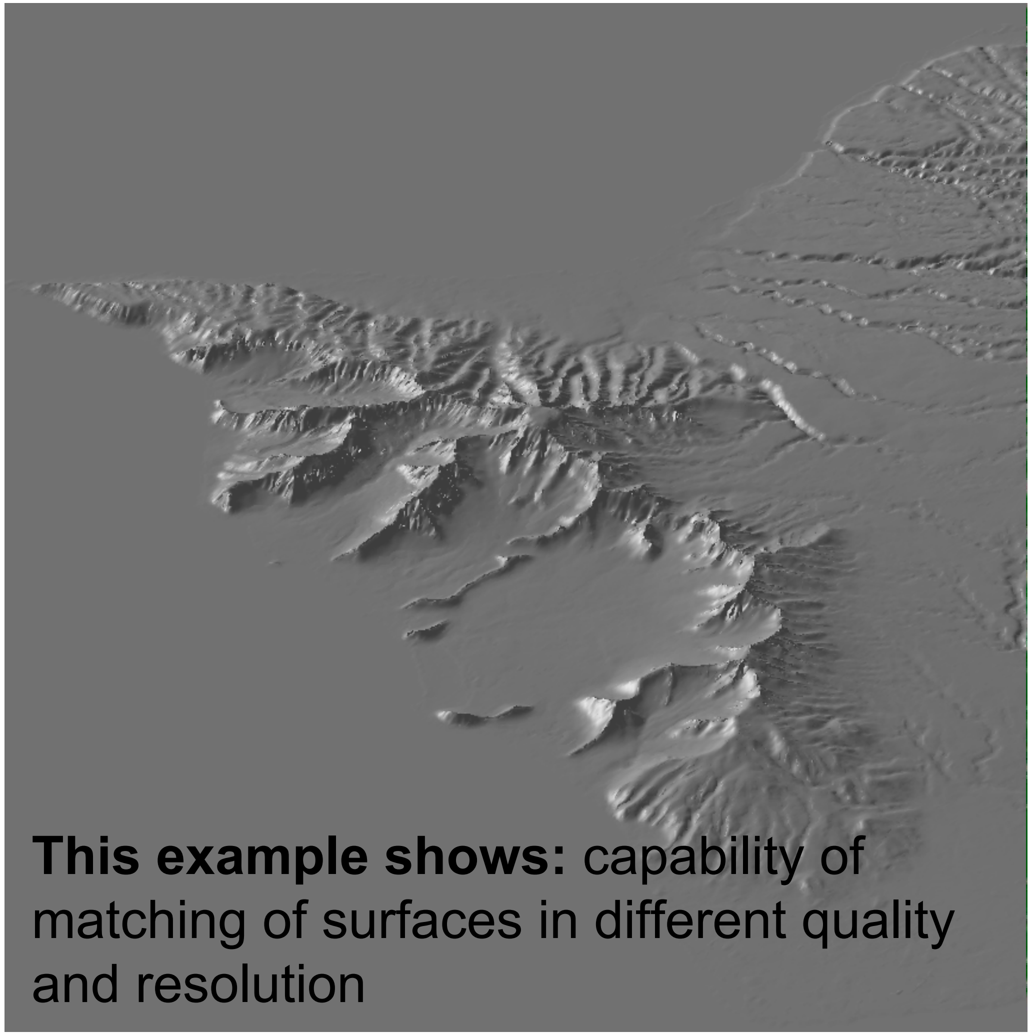


Example: Registration of first 10 scans
Note: 3x3 down-sampling for better visualization

Experimental results 3: Weary Herakles



Experimental results 4: Filling the data holes of SRTM C- DEMs



A cooperative project:

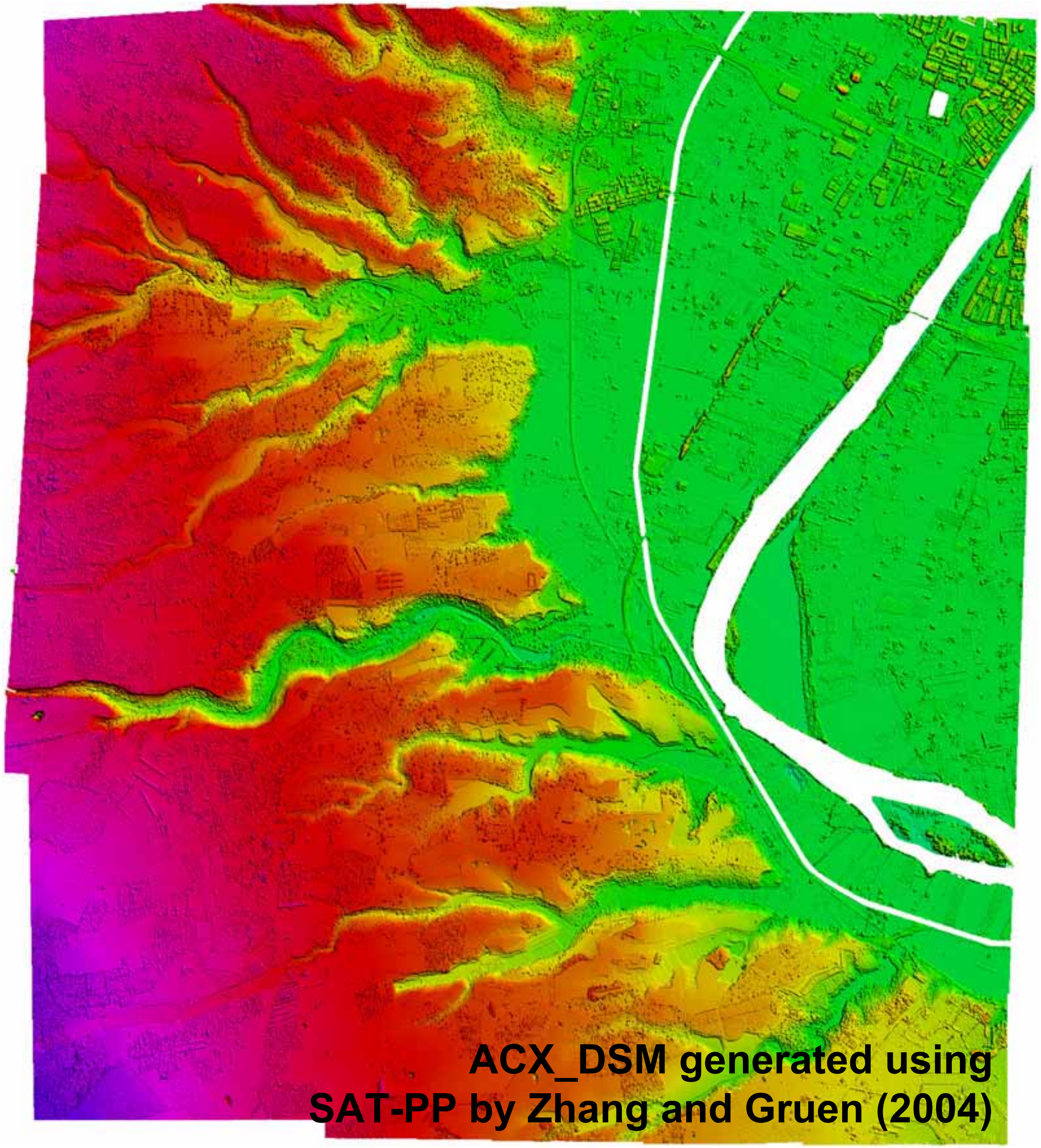
JEPPESEN swissphoto ETH

- Jeppesen: a worldwide terrain database for aviation
- Swissphoto: DB generation

- SRTM C-Band DEMs basedata,
- Data holes due to typical problems of InSAR,
- Filling the dataholes by local DEMs in any available quality/resolution,
- Correction of the reference frame differences (translation and rotation) by the LS3D

- SRTM TerrainScape™

Experimental results 5: Accuracy evaluation of DMC's DSMs



ACX_DSM generated using SAT-PP by Zhang and Gruen (2004)

Task: Quality evaluation of the DSMs derived by DMC digital airborne camera imagery

DMC DSMs

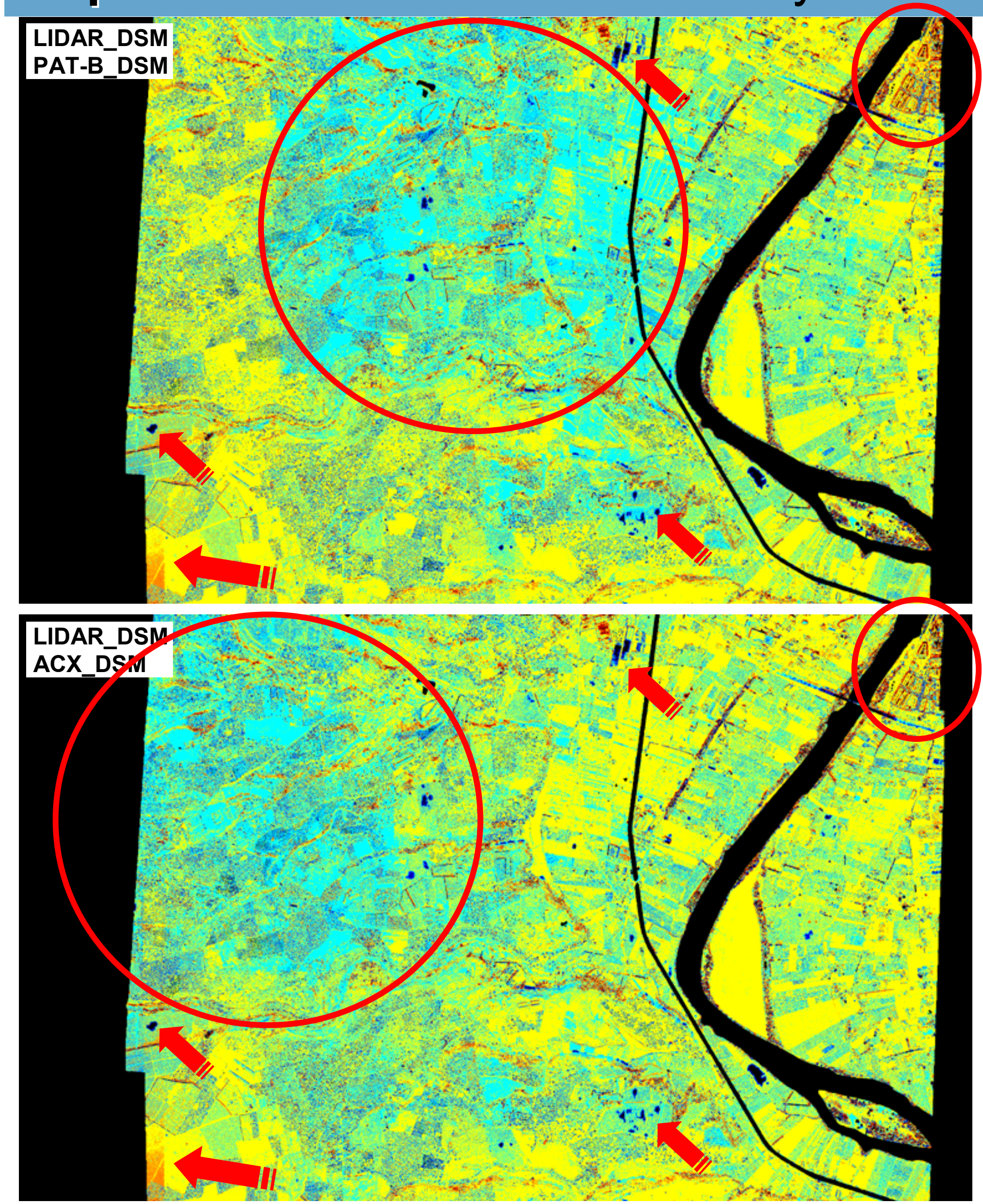
- 28 images in 4 parallel strips

PAT-B -> SAT-PP -> DSM (1m)
ACX -> SAT-PP -> DSM (1m)

LIDAR DSM (reference)

- Simultaneous acquisition with DMC
- Optech 3030
- 1.2 pnt/m2, interpolated to 2m grid spacing

Experimental results 5: Accuracy evaluation of DMC's DSMs

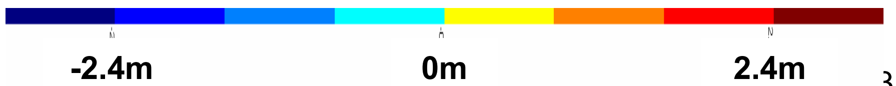


Residuals of the Euclidean distances after the LS3D matchings,

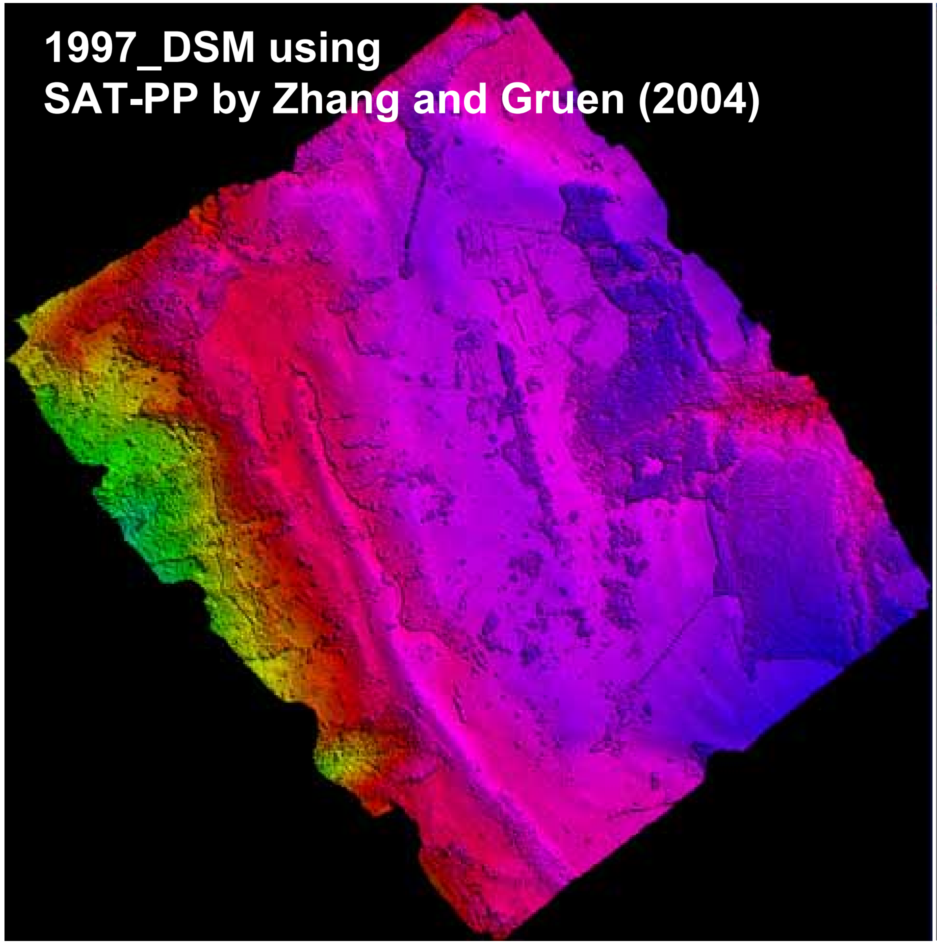
LIDAR DSM: template
DMC DSMs: search

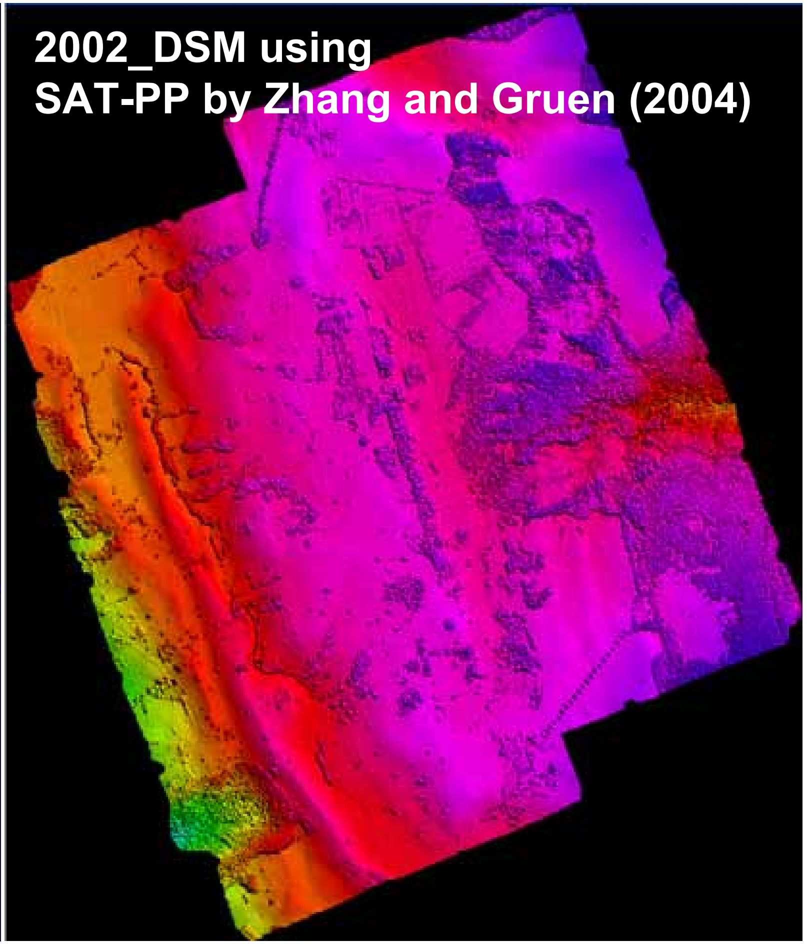
Residuals reveal many problems:

- PAT-P and ACX orientations are apparently different
- Large differences at discontinuities, at top-right, urban area
- A systematic effect at lower left, orange jump. Most possibly due to image matcher.
- Occasional errors of the image matcher



Experimental results 6: Change detection, deforestation analysis



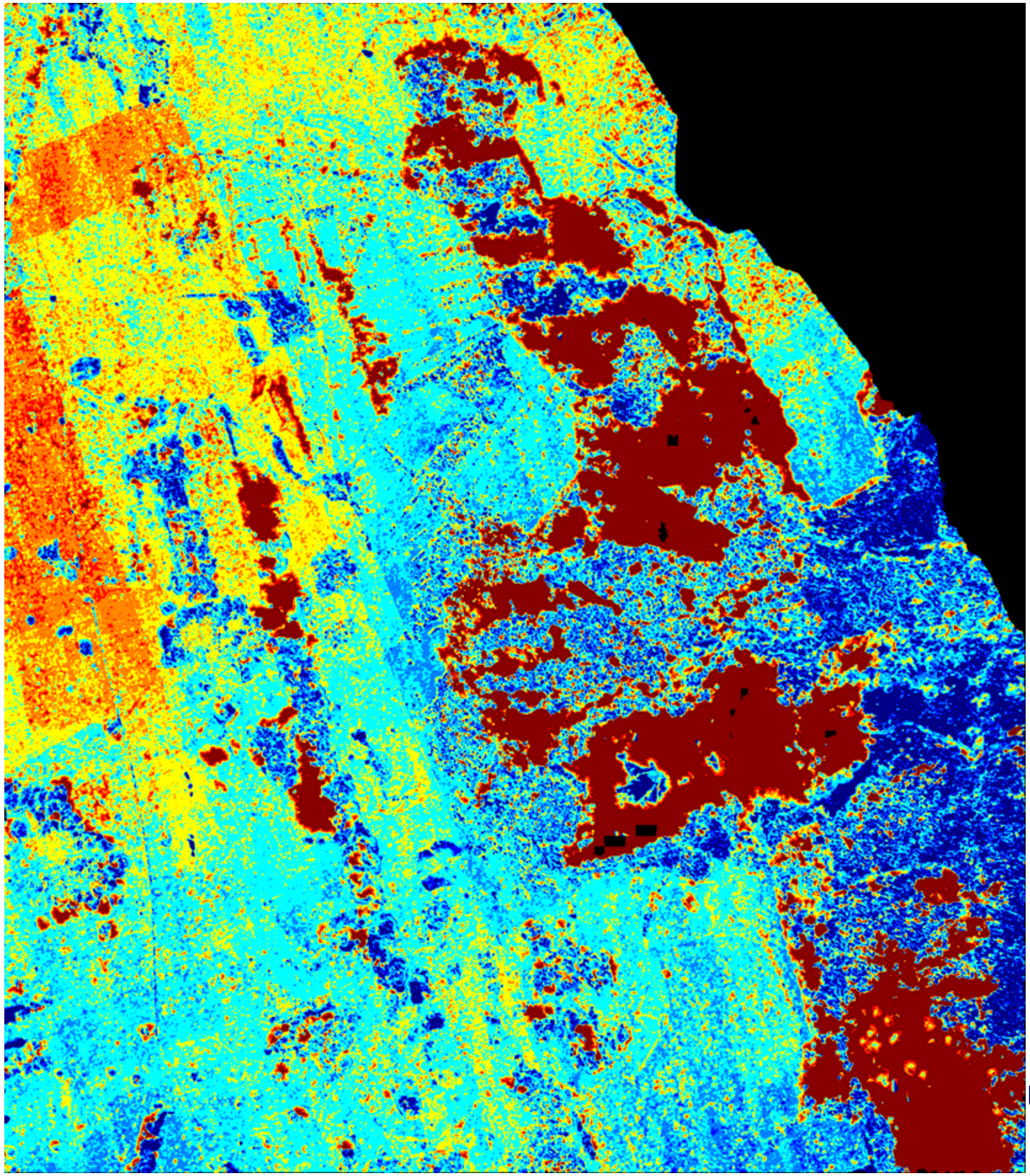


Task: Analysis of change detection and deforestation

1997_DSM and **2002_DSM**
- RC30 analog camera
- 0.5m grid spacing

2001_LIDAR_DSM
- Swisstopo
- 1-2pnt/m2 -> 2.5m grid spacing

Experimental results 6: Change detection, deforestation analysis

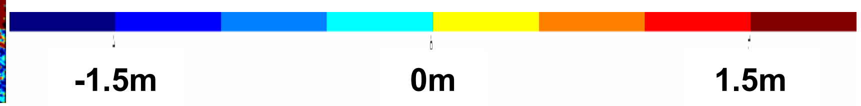


Z-components of the residuals of the Euclidean distances after the LS3D matching of

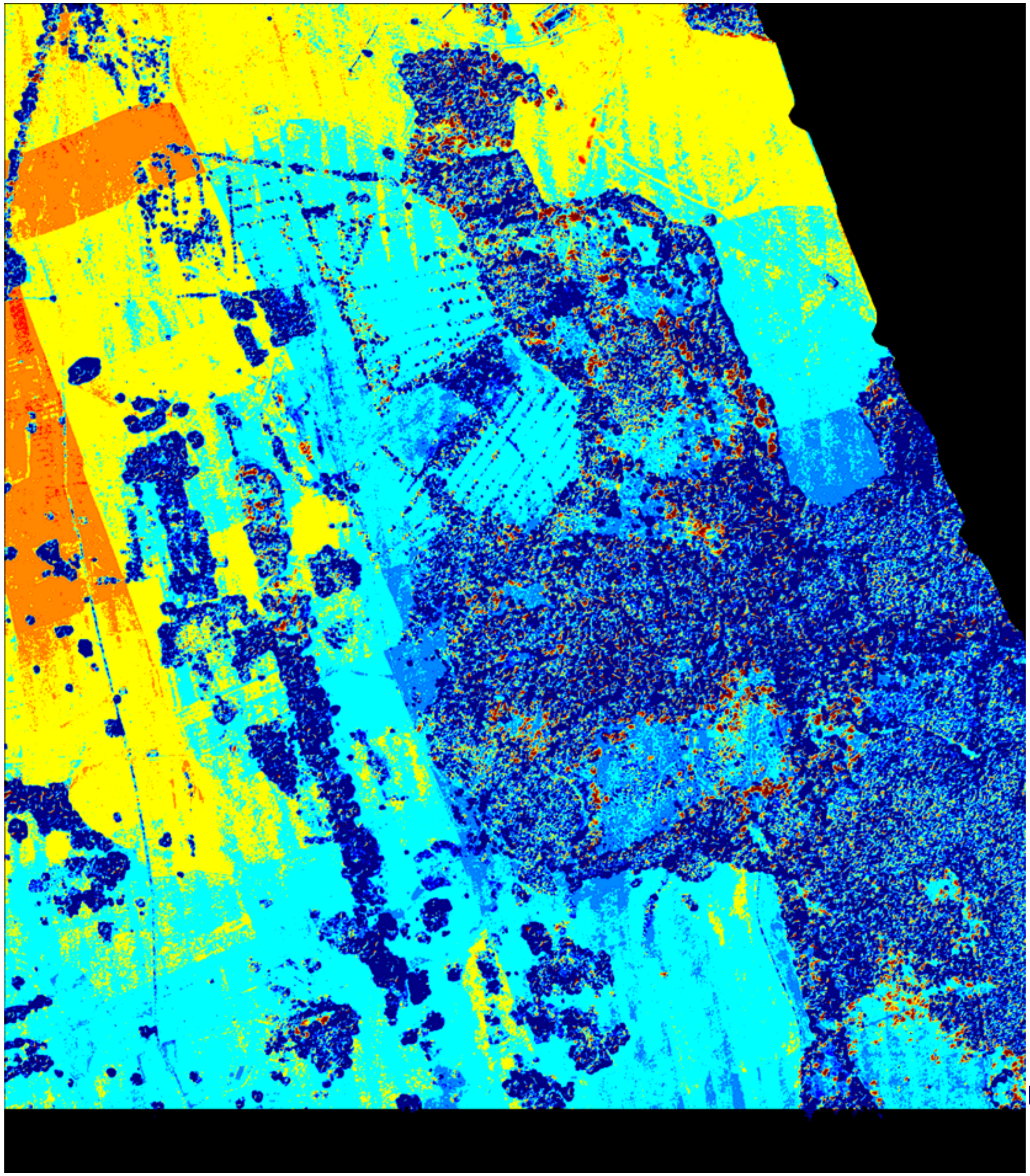
2002_DSM: template
1997_DSM: search

• **Red** areas show the deforestation!

• **Blue** areas show the growth!



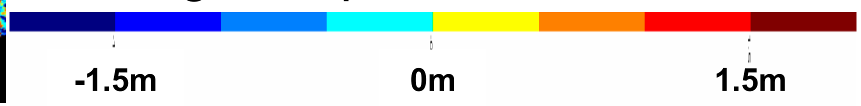
Experimental results 6: Change detection, deforestation analysis



Residuals of the Euclidean distances after the LS3D matching of

2002_DSM: template
2001_LIDAR_DSM: search

- **Small Red spots** show the loss of individual trees during 1 year
- **Blue areas** show the growth, but also including the partial penetration of LIDAR
- **Orange areas** are due to image orientation differences between two flight strips



Conclusions:

- Generalization of 2D LSM => 3D surface matching,
- Estimates 3D transformation parameters, Generalized Gauss-Markoff model,
  min = SUM(SQR(Euclidean distances))
- Non-linear model, need for initial approximations,

Advantages:

- Matching of arbitrarily oriented 3D surfaces, without using explicit tie points,
- Fully considers the 3D geometry
- Few iterations, 5-6 typically, (ICP, 20-30-50-more),
- Provides internal quality indicators,
- Capability to match surfaces in different quality and resolution,
- Flexible mathematical model for further algorithmic extensions,
- Many application areas:

  3D modeling, quality inspection, cultural heritage, accuracy analysis, change detection, etc..

Outlook:

- An automatic pre-alignment method for the initial approximations
- Higher order surface representation
- Error-in-Variables model