

## Registration of laser scanner point clouds with 3D LSM

**Devrim Akca**

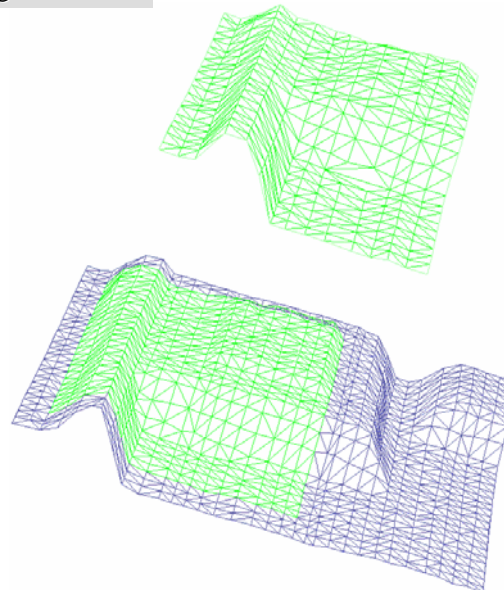
Chair of Photogrammetry and Remote Sensing, ETH Zurich

<http://www.photogrammetry.ethz.ch>

### The Subject: Registration of overlapping 3D surfaces

#### An object surface may be

- digitized:
  - + point by point,
  - + or in a different sampling pattern
- using:
  - + a laser scanner device,
  - + the photogrammetric method,
  - + or other techniques..
- acquired:
  - + from different stand-points (spatially)
  - + at different times (temporally)
- problem statement:
  - + matching the conjugate parts and
  - estimating the 3D transformation



**Contents:**

**INTRODUCTION**

**LEAST SQUARES 3D SURFACE MATCHING (LS3D)**

- **The basic estimation model**
- **Execution aspects**
  - + **Precision and reliability**
  - + **Computational aspects**
  - + **Convergence of solution vector**

**THE EXPERIMENTAL RESULTS**

**CONCLUSIONS**

**Introduction: Previous work**

**Least Squares Matching (LSM)** (*Grün, 1985*)

- Surface matching first was addressed as a straight extension of LSM

**DEM Matching** (*Ebner and Müller, 1986; Ebner and Strunz, 1988; Rosenholm and Torlegard, 1988*)

- It minimizes height differences along Z-axis by LSs (corresponds mathematically to LSM)
- It has been used for:
  - + absolute orientation of stereo models
  - + block triangulation
  - + registration of airborne laser scanner strips

**Iterative Closest Point (ICP)** (*Besl and McKay, 1992; Chen and Medioni, 1992; Zhang, 1994*)

- Iterative solution based on closed-form LS rigid transformation
- It converges slowly
- It has lack of internal quality indicators

**ETH**  
 Eidgenössische Technische Hochschule Zürich  
 Swiss Federal Institute of Technology Zürich

Photogrammetry  
**pf**  
 Remote Sensing

**Introduction: Our proposed method LS 3D Surface Matching (LS3D)**

**LS Image Matching** (Grün, 1984, 1985)

Straightforward extension

**LS Cuboid (Voxel) Matching**  
 (Maas, 1994; Maas and Grün, 1995)

another extension

**LS 3D Surface Matching**

From H.G. Maas, 1994, P+F, IGP, ETH-Zürich  
<http://www.photogrammetry.ethz.ch/research/flotomo/flotomo.html>

D. Akca, Zurich, 15 August 2005, 4<sup>th</sup> Image Sensing Seminar (ARIDA & IGP-ETHZ)

5

**ETH**  
 Eidgenössische Technische Hochschule Zürich  
 Swiss Federal Institute of Technology Zürich

Photogrammetry  
**pf**  
 Remote Sensing

**The Basic Estimation Model: Observation equations**

Two different partial surfaces of the same object

$f(x,y,z)$  : template  
 $g(x,y,z)$  : search (to be transformed)

(discrete 3D representations of the real surface)

The problem statement is estimating the final 3D transformation of search surface  $g(x,y,z)$

$$f(x, y, z) = g(x, y, z) \quad (1)$$

Considering the stochastic discrepancies, a true error vector  $e(x,y,z)$  has to be added

$$f(x, y, z) - e(x, y, z) = g(x, y, z) \quad (2)$$

Equation (2) is considered as observation equations

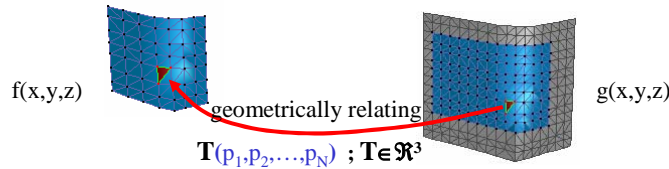
The goal function:  $[d_E d_E] = \min \quad (3) \quad d_E$ : Euclidean distance

The final location of  $g(x,y,z)$  is estimated with respect to an initial position  $g^0(x,y,z)$

D. Akca, Zurich, 15 August 2005, 4<sup>th</sup> Image Sensing Seminar (ARIDA & IGP-ETHZ)

6

**The Basic Estimation Model: Geometric relationship**



- Geometric relationship: 7-parameter 3D similarity transformation

$$\begin{aligned} x &= t_x + m(r_{11}x_0 + r_{12}y_0 + r_{13}z_0) \\ y &= t_y + m(r_{21}x_0 + r_{22}y_0 + r_{23}z_0) \\ z &= t_z + m(r_{31}x_0 + r_{32}y_0 + r_{33}z_0) \end{aligned} \quad (4)$$

- Any other kind of 3D transformation can be also used: (3D affine, tri-linear, etc..)

**The Basic Estimation Model: Functional model**

$$f(x, y, z) - e(x, y, z) = g(x, y, z) \quad (2)$$

The functional model is non-linear. It is linearized by Taylor Expansion.

$$f(x, y, z) - e(x, y, z) = g^0(x, y, z) + \frac{\partial g^0(x, y, z)}{\partial x} dx + \frac{\partial g^0(x, y, z)}{\partial y} dy + \frac{\partial g^0(x, y, z)}{\partial z} dz \quad (5)$$

*Initial state (by approximations)*
*Differentiation terms*  
 $g_x$ 
 $g_y$ 
 $g_z$   
*Numerical derivatives*

Final functional model in linearized form:

$$\begin{aligned} -e(x, y, z) &= g_x dt_x + g_y dt_y + g_z dt_z + (g_x a_{10} + g_y a_{20} + g_z a_{30}) dm + (g_x a_{11} + g_y a_{21} + g_z a_{31}) d\omega + \\ & (g_x a_{12} + g_y a_{22} + g_z a_{32}) d\phi + (g_x a_{13} + g_y a_{23} + g_z a_{33}) d\kappa - (f(x, y, z) - g^0(x, y, z)) \end{aligned} \quad (6)$$

The functional model in matrix notation:

$$-e = \mathbf{A} \mathbf{x} - \ell, \quad \mathbf{P} \quad \mathbf{A} \quad \text{design matrix} \quad (7)$$

$$\mathbf{x}^T = [dt_x \ dt_y \ dt_z \ dm \ d\omega \ d\phi \ d\kappa] \quad \text{parameter vector}$$

$$\ell = f(x, y, z) - g^0(x, y, z) \quad \text{constant vector (Euclidean distances)}$$

$$-e_b = \mathbf{I} \mathbf{x} - \ell_b, \quad \mathbf{P}_b \quad \text{The unknown parameters are added as stochastic quantities} \quad (8)$$



**The Basic Estimation Model: Mathematical model**

The total system is a Generalized Gauss-Markoff model:

$$-\mathbf{e} = \mathbf{A} \mathbf{x} - \ell \quad , \quad \mathbf{P} \tag{7}$$

$$-\mathbf{e}_b = \mathbf{I} \mathbf{x} - \ell_b \quad , \quad \mathbf{P}_b \tag{8}$$

The Least Squares solution of the joint system gives as:

Solution vector:  $\hat{\mathbf{x}} = (\mathbf{A}^T \mathbf{P} \mathbf{A} + \mathbf{P}_b)^{-1} (\mathbf{A}^T \mathbf{P} \ell + \mathbf{P}_b \ell_b)$  (9)

Variance factor:  $\hat{\sigma}_0^2 = \frac{\mathbf{v}^T \mathbf{P} \mathbf{v} + \mathbf{v}_b^T \mathbf{P}_b \mathbf{v}_b}{r}$  (10)

The solution is iterative. At each iteration:



- search surface is transformed to a new state:  $g^t(x,y,z) \Rightarrow g^{t+1}(x,y,z)$
- $\mathbf{A}$  and  $\ell$  are re-evaluated.

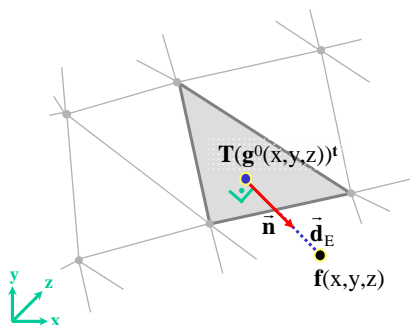
The iteration stops if each element of the alteration vector  $\hat{\mathbf{x}}$  falls below a certain limit:

$$|dp_i| < c_i \quad , \quad i = \{1,2,\dots,7\} \tag{11}$$

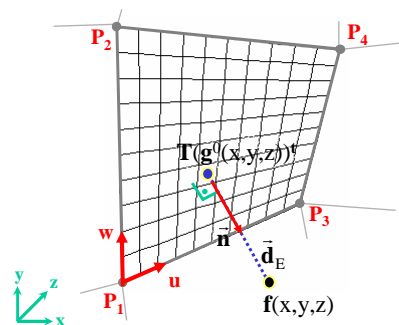
**The Basic Estimation Model: Numerical derivative terms**

Numeric derivative terms  $\{g_x, g_y, g_z\}$  are defined as local surface normals.  
 Their calculation depends on the analytical representation of search surface elements.  
 We represented them in two kind of piecewise forms optionally:

**Triangle plane representation**



**Parametric bi-linear representation**



Both of them are 1<sup>st</sup> degree  $C^0$  continuous piecewise surface representations.

### Precision and Reliability

- $\mathbf{K}_{xx}$  gives useful information on the stability of the system and quality of the data content

$$\mathbf{K}_{xx} = \hat{\sigma}_0^2 \mathbf{Q}_{xx} = \hat{\sigma}_0^2 \mathbf{N}^{-1} = \hat{\sigma}_0^2 (\mathbf{A}^T \mathbf{P} \mathbf{A} + \mathbf{P}_b)^{-1}$$

- A simple weighting scheme adapted from Robust Estimation Methods is used in order to eliminate occluded parts and large outliers

$$(\mathbf{P})_{ii} = \begin{cases} 1 & \text{if } |(\mathbf{v})_i| < K\sigma_0 \\ 0 & \text{else} \end{cases}$$

### Convergence of Solution Vector

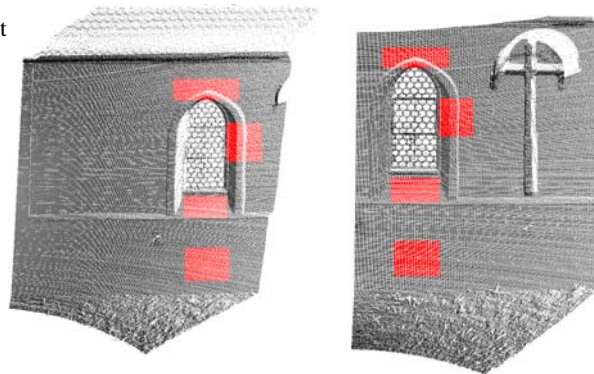
- Typical convergence rate is 5-6 iterations in a good data configuration case.
- It depends on the quality of initial approximations and data content.

### Computational Aspects

- Adjustment part is very small (Cholesky decomposition + back-substitution)  
 > 95% of the computational effort is for searching the correspondence.

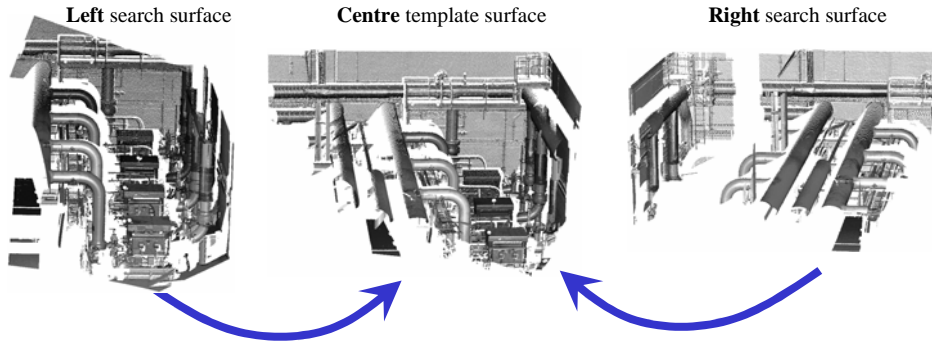
### Further Extension: Simultaneous multi-subpatch matching

- Suppose that two scans are subject to matching
- Straightforward way is matching the whole overlapping area
- Another way is matching of only some cooperative subpatches



- + Subpatches are manually selected
- + Joined to the system by the same 3D transformation parameters
- + Individual subpatches may not include sufficient information, but together they provide a **computationally effective solution**.

### Experimental Results (#1): Plant

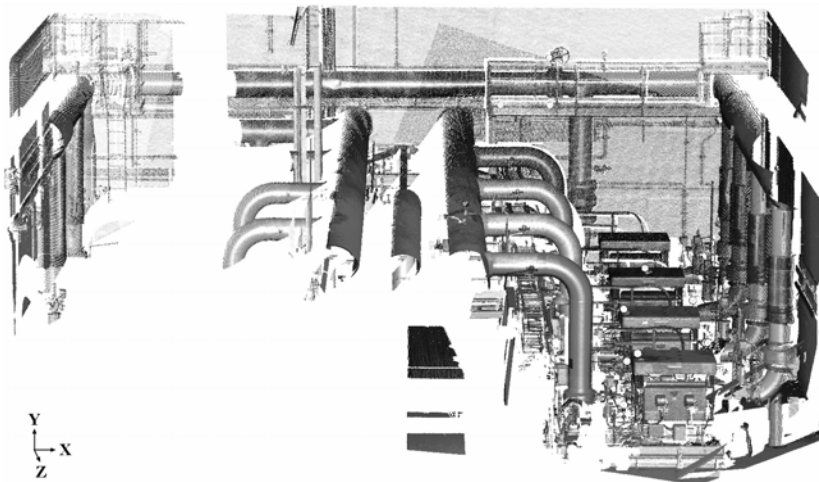


- object: plant area
- scanner: Leica HDS 2500
- average point spacing ~ 12 mm.

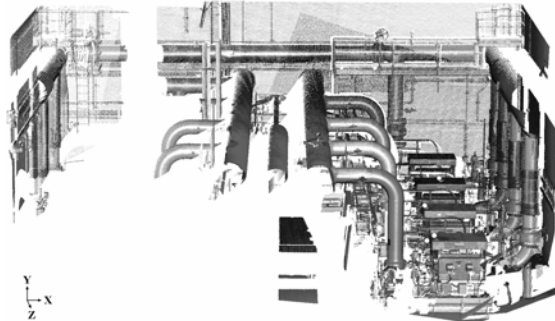
Data set is courtesy of Leica Geosystems HDS Inc.

### Experimental Results (#1): Plant

Result of LS3D surface matching method:



### Experimental Results (#1): Plant



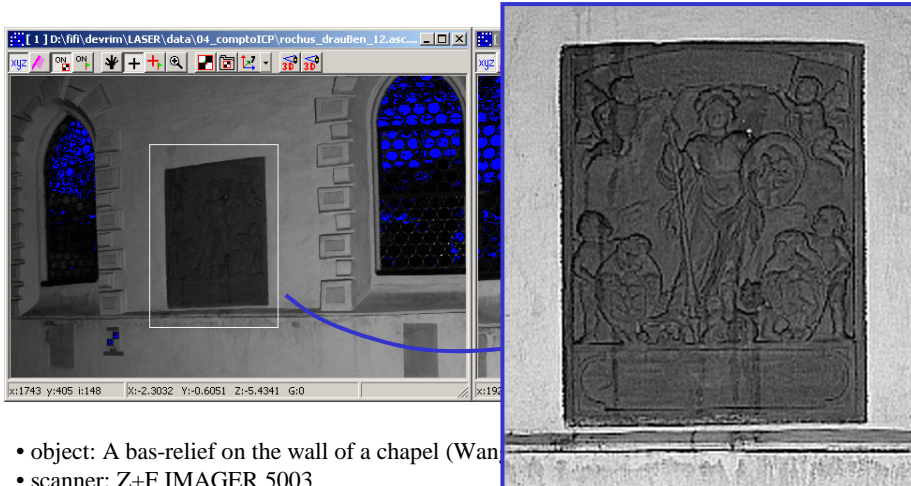
Numerical results of "plant"

#		$\Sigma$ points	Iter.	$\hat{\sigma}_0$ (mm)
I	LEFT	245041	6	<b>2.78</b>
II	RIGHT	323936	7	<b>2.54</b>

Numerical results of "plant" (multi-subpatch approach)

#		$\Sigma$ sub-patches	$\Sigma$ points	Iter.	$\hat{\sigma}_0$ (mm)
III	LEFT	5	20407	6	<b>2.11</b>
IV	RIGHT	7	37983	8	<b>2.01</b>

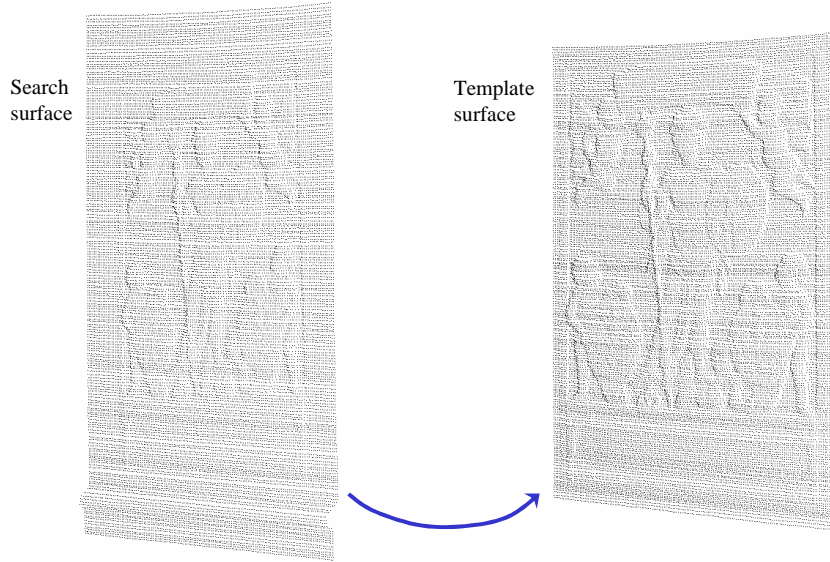
### Experimental Results (#2): Comparison to ICP



- object: A bas-relief on the wall of a chapel (Wan...)
- scanner: Z+F IMAGER 5003
- average point spacing ~ 11 mm.

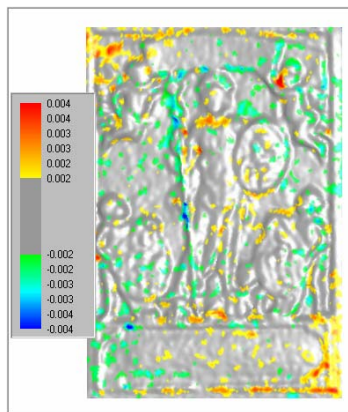
Data set is courtesy of Zoller+Fröhlich GmbH Elektrotechnik.

**Experimental Results (#2): Comparison to ICP**



**Experimental Results (#2): Comparison to ICP**

Result of LS3D surface matching method:

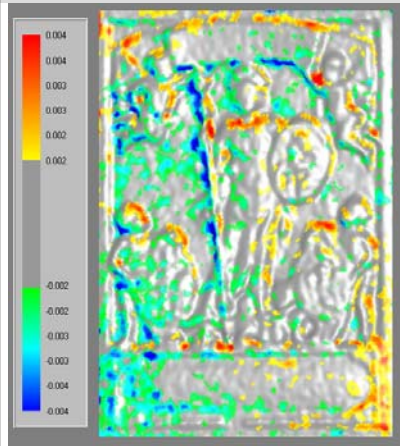


Residuals after the LS3D matching method

no. points	31520
iterations	10
spacing (mm)	~11
sigma0 (mm)	2.45

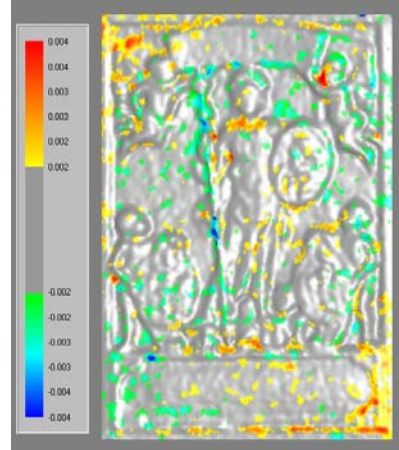


### Experimental Results (#2): Comparison to ICP



**ICP** Geomagic Studio v.4.1  
Statistical results: N/A

**RMSE: 2.55 mm**



**LS3D**

**RMSE: 2.40 mm**

Comparing distribution and magnitude of the residuals, LS3D gives slightly better results than the ICP.

### Experimental Results (#3): "Room"

Template surface



Search surface

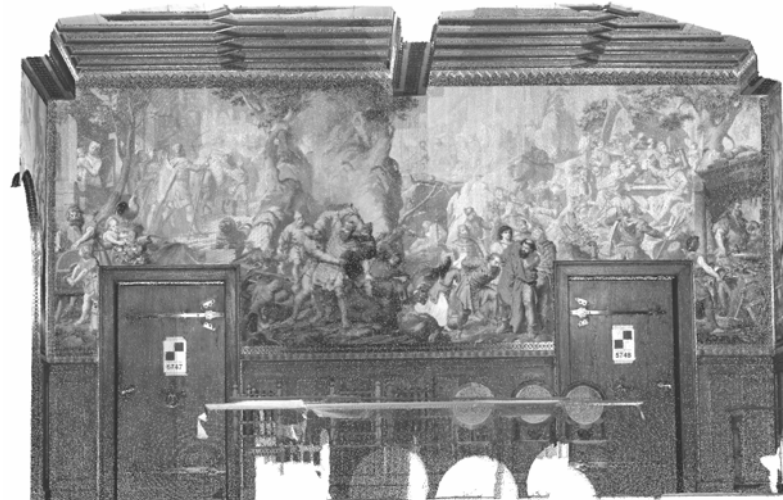


- object: a room in Neuschwanstein Castle in Bavaria (Germany)
- scanner: Zoller+Fröhlich IMAGER 5003
- average point spacing ~ 5 mm.

Data set is courtesy of Zoller+Fröhlich GmbH Elektrotechnik.

**Experimental Results (#3): "Room"**

Result of LS3D surface matching method:



D. Akca, Zurich, 15 August 2005, 4<sup>th</sup> Image Sensing Seminar (ARIDA & IGP-ETHZ)

21

**Experimental Results (#3): "Room"**



#	$\Sigma$ sub-patches	$\Sigma$ points	Iterations	Time (sec.)	$\hat{\sigma}_0$ (mm)
I	1	1,155,502	10	142.5	<b>3.69</b>
II	7	279,088	8	25.8	<b>3.60</b>

Basic model

Multi-subpatch approach

D. Akca, Zurich, 15 August 2005, 4<sup>th</sup> Image Sensing Seminar (ARIDA & IGP-ETHZ)

22

### Experimental Results (#4): “wooden Buddha”



- **object:** An Indonesian Buddha
- **material:** polished wooden, shiny!
- **scanner:** Breuckmann triTOS-HE based on fringe projection technique
- **average point spacing** ~ 0.3-0.5 mm.

Remondino et al., 2005. 3D Modeling of close-range objects: photogrammetry or laser scanning? Videometrics VIII, SPIE vol. 5665, pp. 216-225.

Data set is courtesy of Fabio Remondino (ETH Zurich) and Breuckmann GmbH.



Each scan ~1.4M, totally 21.5M points



**ETH**  
 Eidgenössische Technische Hochschule Zürich  
 Swiss Federal Institute of Technology Zürich

Photogrammetry  
**pf**  
 Remote Sensing

**Result of LS3D matching**

Tot. no. points	5.6M
Aver. no. points	4~11
Average point spacing	66~105 micron
Point cloud resolution	42 micron

-9.5M triangles

-3M triangles

D. Akca, Zurich, 15 August 2005, 4<sup>th</sup> Image Sensing Seminar (ARIDA & IGP-ETHZ) 25

**ETH**  
 Eidgenössische Technische Hochschule Zürich  
 Swiss Federal Institute of Technology Zürich

Photogrammetry  
**pf**  
 Remote Sensing

**Experimental Results (#5): "Pinchango Alto"**

Aerial Image

- area: Pinchango Alto, a pre-Inkaic site in Peru
- scanner: Riegl LMS Z-420i
- average point spacing 1~30 cm, changing with range

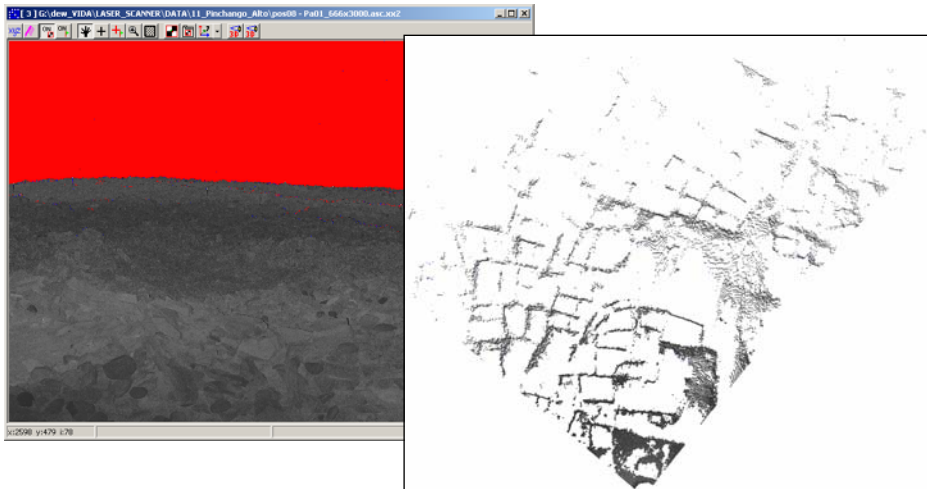
Data set is courtesy of Riegl GmbH.

D. Akca, Zurich, 15 August 2005, 4<sup>th</sup> Image Sensing Seminar (ARIDA & IGP-ETHZ) 26

### Experimental Results (#5): "Pinchango Alto"

The data set:

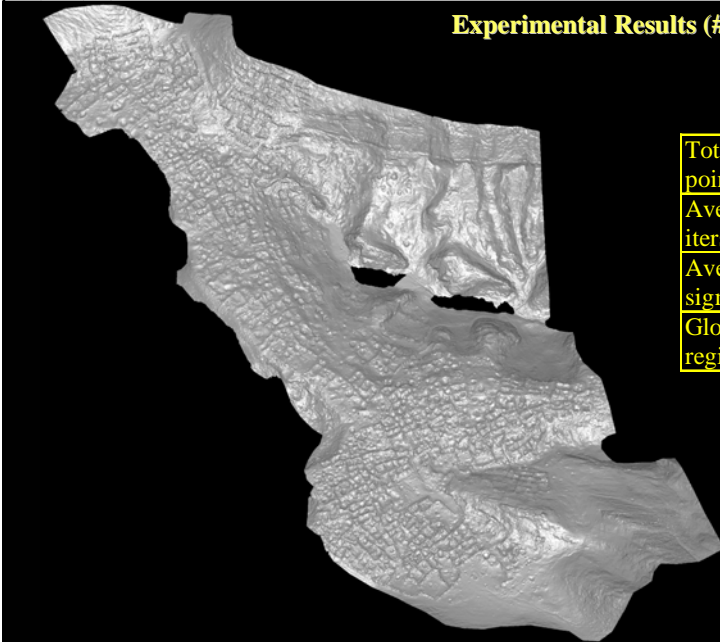
- large data set (57 scans with 144 million points)
- large occlusions due to topography



D. Akca, Zurich, 15 August 2005, 4<sup>th</sup> Image Sensing Seminar (ARIDA & IGP-ETHZ)

27

### Experimental Results (#5): "Pinchango Alto"



Tot. no. points	144M
Aver. no. iters.	~7-8
Average sigma	~1 cm
Global registration	0.5 cm

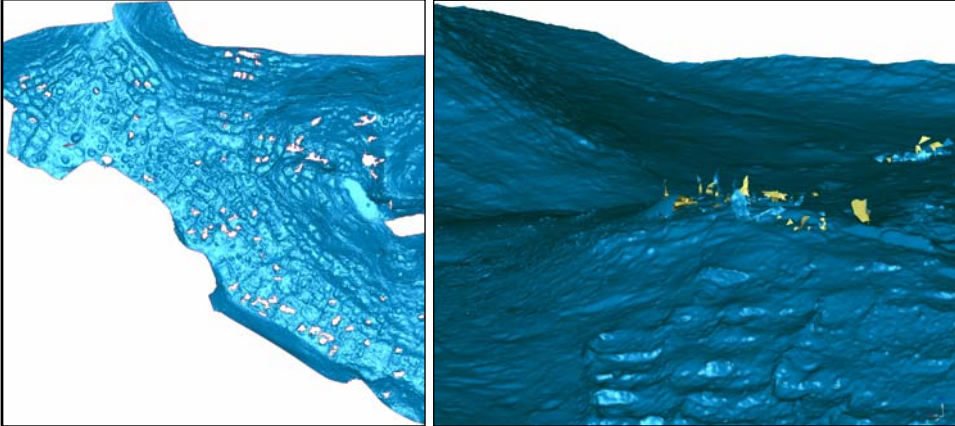
D. Akca, Zurich, 15 August 2005, 4<sup>th</sup> Image Sensing Seminar (ARIDA & IGP-ETHZ)

28

## Experimental Results (#5): “Pinchango Alto”

### Difficulties in modelling:

- Geomagic Studio 6, memory limitations in surface wrapping
- data holes due to occlusions of walls and hollows
- non-static objects on the site during 5 days field work



## Conclusions

- LS3D is a generalisation of the least squares 2D image matching concept
- Non-linear functional model, need for initial approximations
- Generalized Gauss-Markoff model for the estimation
- Monitoring capabilities for internal quality control
- It can solve the point cloud registration task without using special targets.

**Thank you for your attention!**